TopQuants Newsletter

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Editorial

Dear reader,

The TopQuants team presents the first issue of our 2016 newsletter series. As always we cordially invite all readers to contact us with your ideas and submissions. Anything that is relevant to our quant audience, is more than welcome! Since 2016 marks the fifth anniversary of TopQuants, keep a look out in your mailbox, on Twitter (@topquants), and on our webpage, for new events. We are working very hard to make our events bigger, better, and quantier!

The current issue will kick off with the summary of the Autumn Event that was held at the DNB in November 2015. We witnessed another record-breaking event, attracting over 160 quant professionals, who arrived to Amsterdam not only from within the Netherlands, but also from other destinations in Europe. The agenda was similar to all previous Autumn Events organized by TopQuants: two rounds with six parallel sessions. Every year the range of covered topics is becoming broader, this time the discussed problems ranged from credit and liquidity risk analysis, valuation adjustments, the impact of negative rates, modelling of the commodity prices, asset-backed trading in energy markets to risk management through open source and web technologies.

This is followed by a summary of the Quant Careers 2015 event, at which three former students Nathan Meibergen (TU Delft), Marcin Rybacki (Tilburg University) and Sina Zolnoor (Free University of Amsterdam)) battled it out against each other in order to decide who is the winner of the Best Quant Finance Thesis Award 2015. See page 4 to find out who claimed the title of the best young quant in the Netherlands in 2015.

The next article is a summary of yet another TopQuants event - a *Boom Bust Boom* screening, hosted by EY. The documentary produced by a member of Monty Python Terry Jones and economics professor and entrepreneur Theo Kocken explains why bubbles exist and financial crashes happen, and why human behaviour needs to be considered when creating economic policies.

Hereafter we take a look at an uncharted territory of negative rates and their impact on pricing models, which is an extended summary of one of the talks at the last Autumn Event by Veronica Malafaia (ING Bank).

Another interesting article was delivered to us by Laurens Kolkman from KPMG, the host of the upcoming Spring Event, who discusses how FinTech Start-ups might take over the Financial System.

This issue also includes a white paper written by Frank Pardoel, Hans Heintz & Pim Poppe (RiskQuest), which deals with penalties related to early prepayment in an ALM framework.

The final article is a summary of another presenter at the Autumn Event - Bert-Jan Nauta, who discusses how liquidity risk can be included in the valuation of assets through a simple model.

Our next newsletter will contain coverage of the upcoming Spring Event, kindly hosted by KPMG. We hope you will enjoy reading this newsletter and we look forward to seeing you at the upcoming TopQuants event(s).

On behalf of the TopQuants team,

Marcin Rybacki

TopQuants Autumn Event—2015

Current topics in modelling

The 2015 TopQuants Autumn Event was hosted by De Nederlandsche Bank (DNB), the Dutch central bank, at their headquarters in Amsterdam. We witnessed another record-breaking event, attracting over 160 quant professionals, who arrived to Amsterdam not only from within the Netherlands, but also from other destinations in Europe. This proves an increasing significance of events organised by TopQuants, but also shows that our organisation has earned recognition outside the Netherlands.

The agenda was similar to all previous Autumn Events organised by TopQuants: two rounds with six parallel sessions, each with a seating capacity of approximately 30 attendants per session. Every year the range of covered topics is becoming broader, this time the discussed problems ranged from credit and liquidity risk analysis, valuation adjustments, the impact of negative rates, modelling of the commodity prices, asset-backed trading in energy markets to risk management through open source and web technologies.

A welcome speech, on behalf of TopQuants, was given by Marieke van der Klip, who briefly outlined the goal and the history of the organisation and also expressed gratitude to DNB for being so kind to host the event. This was followed by a warm speech by Jan Sijbrand, Executive Director and Chairman for Prudential Supervision at the DNB. He was happy to see so many professionals present at the event, and emphasised his attachment to the quant society – he also holds a PhD in mathematics. He pointed out the importance of technical and mathematical skills in the financial industry these days.
Kees de Graaf (UvA) and Sarunas Simaitis (Right Random Decisions) gave a presentation on Smile and default: the role of stochastic volatility and interest rates in counterparty credit risk. They outlined the challenges of calculating Credit Value Adjustments (CVA) and Counterparty Credit Risk (CCR). The authors used Finite Difference Monte Carlo as a fast and accurate method to compute risk measures such as Expected Exposure (EE), Expected Positive Exposure (EPE) or Credit Value Adjustment (CVA). The response of the audience after the presentation was very enthusiastic and was followed by a lively discussion.

Cyriel de Jong (KYOS) focussed on general characteristics of energy price movements and the valuation of energy assets in his presentation titled Asset-backed trading strategies in energy markets. He highlighted the main principles of dynamic hedging of energy assets and explained numerical techniques for the derivation of delta hedges, analysing dynamic hedging strategies and backtesting methods.

The presentation from EY by Diederik Fokkema, Floris van der Loo, Guusje Delsing, Nathan Meibergen and Jan Willem Timmer was a discussion on the results of a survey amongst eleven banks on the application of xVA for x in {C,D,F,L,K,M,X, etc.} and the results obtained by Guusje Delsing, Nathan Meibergen (winner of the 2015 edition of the Best Quant Finance Thesis Award) and Jan Willem Timmer in their Master's theses. All three (former) students dealt to some extent with CVA and DVA, both of which are valuation adjustments to take into account the default of either of the two involved parties within a bilateral derivative contact.

Veronica Malafaia (ING) gave an overview on the impact of negative interest rates on derivatives pricing models. She explained that many interest rate models are based on lognormal dynamics, which prove to be less useful in the negative rates environment. Veronica pointed out that banks' models need to be ready for negative rates. The so-called displaced diffusion approaches are an important step in this direction. She made an important remark that "simple truths" do not hold any longer in negative rate scenarios and hence the users of models must critically question their intuition.

Rob Sperna Weiland (UvA) talked about Liquidity Risk in the Sovereign Credit Default Swap. Rob was the winner of 2014 edition of Best Quant Finance Thesis Award and this time again he made an excellent impression on the TopQuants' audience. He noted that the Euro crisis has put the default risk of sovereigns into a new perspective. Nevertheless, historical sovereign defaults are still too scarce to allow for any meaningful statistical backtest or quantitative model development. The crisis not only resulted in increased default risk, also liquidity in sovereign debt became much more relevant. In his research Rob used CDS spread data to provideestimates of default probabilities, correcting for the presence of liquidity risk.

There were two presentations from DNB. Ryan van Lamoen gave a speech on Testing for bubbles in asset prices: evidence from QE and other applications. He focused on testing for bubbles in asset prices and the implementation of monitoring tools, which are of great importance for policy purposes. He proposed an alternative test to identify explosive price behaviour and the corresponding episodes in which they occur. He put special emphasis on the impact of Quantitative Easing in the Eurosystem on equity prices and government bonds, since there is a possibility that this unconventional monetary policy may drive equity prices and government bond yields further away from their fundamental drivers.

The talk by Pieter van Zwol concerned the main challenges of the current approach for measuring counterparty credit risk exposures (SA-CCR), which will be effective by January 2017. During the presentation he analysed the differences with the new SA-CCR using example derivative portfolios.

Gerben de Zwart (APG) and Johan Duyvesteyn (Robeco) conducted an empirical analysis of the term structure in the volatility risk premium in the fixed income market by constructing long-short combinations of two at-the-money straddles for the four major swaption markets (USD, JPY, EUR and GBP). Their findings indicate that both delta-vega and delta-gamma neutral straddle combinations earn positive returns that seem uncorrelated, suggesting that the term structure is affected by both jump risk and volatility risk. They showed evidence that these results are robust for macroeconomic announcements and the specific model choice to estimate the risk exposures for hedging.

Lech Grzelak (Rabobank) gave a more technical, yet very interesting presentation on arbitrage–free volatility parameterizations with stochastic collocation. He argued that with a large number of market volatility quotes, it is natural to express them in terms of a parametric form so that the whole range of strikes can be explained by only a few parameters. For several years a market standard for volatility parametrisation has been the well-known Hagan formula – the so-called SABR model, which is very easy to implement through expansion formulae. The approximations themselves unfortunately can lead to densities that are not arbitrage-free - especially for very low strikes they can become negative. The speaker introduced a technique to determine an arbitrage-free density implied by Hagan's formula using the stochastic collocation method. The principle is to determine a few collocation points on the implied survival distribution function and project them on a polynomial of an arbitrage-free, Gaussian, variable. The proposed method proves to be very fast and straightforward to implement as it only involves 1D Lagrange interpolation and inversion of a linear system of equations. The presentation was very interactive, given its technical character the attendants posed multiple questions, which were later very thoroughly addressed by the presenter.

In his talk Bert-Jan Nauta (RBS) went beyond the by now common approach to Funding Valuation Adjustments (FVAs). In his view funding needs to depend on the liquidity of the position that needs to be funded. He presented a model which allows to relate the funding required for a certain position to its liquidity. In this context, liquidity risk is to be understood as the risk for an event to occur which forces a bank to liquidate some of its assets. If such a liquidity stress event occurs, the bank cannot simply roll over its funding any more. Under certain assumptions about the distribution of stress events, the presenter's model gives an optimal funding strategy for assets of various liquidity levels. It can thus be applied to motivate a reasonable (read cost efficient) compromise in between the extreme cases of overnight funding and full term funding of positions.

Steffen Pang (Zanders) and Mitchell Ponder (Zanders, VU) explained that corporates focus on the stability of monthly, quarterly or (semi-)annual cash flows. A key risk is formed by price volatility of commodities. This talk presented an econometric model to capture this risk, which neatly integrated several quantitative techniques ranging from Principal Component Analysis, a Kalman filter to GARCH. Volatility in commodity prices form a key risk in Financial Risk Management of Corporate Risk Management. Therefore, the presented model provided fundamental insight in the commodity price risk a corporate is facing.

In his talk on Using open data and open source for next generation of risk models Philippos Papadopoulos (OpenRisk) focused on two different aspects, i.e. data types (and data representation) and application of open source code to risk management. Philippos gave an introduction about some well-known data formats such as CSV, XML and JSON, illustrating their advantages and disadvantages. Then he gave some insight on data standards, highlighting how lack of them is probably the biggest data related issue in finance: in fact, lack of standardisation is what creates the majority of the data issues in finance. Data formats that allow to deal with big amounts of data already exist (i.e., XML – very powerful, even if it is difficult to work with it). In fact, different data standards are used depending on the purpose (i.e., XBRL for financial reporting and SDMX for historical time series). Then the presenter explained the idea behind REST API, which allows each user to retrieve data necessary from specific URLs in different formats according to users' preferences. He provided a description of what it is meant by model; this involves three parts: (i) abstract model, (ii) source code and (iii) model instance. The idea is to have all these three components linked to URLs accessible to users. The only difference between (i), (ii) and (iii) is that while (i) and (ii) are static metadata, it is possible to send queries to (iii) in order to retrieve documentation, code, etc. An example of this was given by the OpenRisk Dashboard. It allows to retrieve statistical data from the ECB warehouse, to visualise it and to use it within risk models.

After all parallel sessions were finished the participants of the event were invited to attend a math quiz organised by the Dutch Mathematical Olympiad. Quintijn Puite (Wiskunde Olympiade) explained the rules and hosted the quiz. Everyone was given two sheets with letters A and B printed on them, which had to be used to give the correct answer. For every question the available time was 20 seconds. When the time was over, everyone still in the game had to raise their voting sign in the air. If the answer was incorrect the voting sign had to be returned. The riddles touched upon topics such as calendars, expansions, pets, odometers and ski lifts. Everyone had a lot of fun and the four remaining contenders were awarded with gifts sponsored by the Dutch Mathematical Olympiad.

The lively event was concluded by drinks and snacks sponsored by the event host. TopQuants are thankful to the DNB for sponsoring and hosting the event. We appreciate all efforts by the speakers and the quant audience for making this another successful TopQuants event.

Quants and Emotions - Quant Careers 2015 Minisymposium

On the 6th of November 2015 TopQuants, in cooperation with EY and Quants@VU, organised the second edition of Best Quant Thesis Award. The formula was the same as in the previous year. At first candidates had to submit their theses. 7 made it to the semi-final. Finally the top 3 competitors were selected to pitch, at the symposium, for the victory and the title of the best young quant in 2015, in front of the jury and the invited guests.
The event was opened by Diederik Fokkema, the president of TopQuants, and Svetlana Borovkova (Associate Professor at Free University of Amsterdam). Diederik joked that although two of the final three theses were supervised by members of the TopQuants board, there were no further similarities with FIFA.
The growing popularity of the contest (33 submissions this year) was emphasised, as well as very high quality of the theses, which made the choice extremely difficult for the jury – a revote was needed to reach a final conclusion on the candidates that wrote the top three theses. The contestants represented all top Dutch universities with quantitative programs. Also the range of research topics was very wide – from the valuation of exotic options, XVA, credit risk to risk management in pensions as well as financial econometrics.
The opening lecture was given by Nathan Attrell, the head of European Business, Hedge Funds and Proprietary Trading of Thomson Reuters, who spoke about the role of social media, psychology and sentiments in investment decisions. He started with a brief history of Thomson Reuters – a company founded in the fifties of the nineteenth century which offered services in transmitting stock market quotations via a telegraph and which grew to a major multinational mass media and information firm in the world. Further, Nathan explained how analysing social media can lead to successful investment strategies. He gave examples, based on events happening in China or Greece, how one can draw conclusions, from the magnitude and tonality of sentiments posted in social media, that could lead to investment decisions. At the same time Nathan pointed out that this is not always as straightforward as it may seem. According to research people have a tendency to overreact to negative news and underreact to positive news, hence an appropriate filtering mechanism should be put in place.
After Nathan's presentation it was time for the three pitches of the top three contenders. The first one was given by Sina Zolnoor (Royal Bank of Scotland and Free University of Amsterdam), whose thesis covered the topic of managing a liquidity portfolio in an optimal way in terms of risks and returns, while meeting regulatory requirements. One of the conclusions was that periphery government bonds play a prominent role within the asset allocation, whereas many banks do not seem to include them in their liquidity portfolios. He was followed by Nathan Meibergen (EY, TU Delft), who investigated the impact of modelling recovery rates on CVA. His model for recovery rates mutually calibrates to both senior and subordinated debt and applies a correlation between default and re-

covery. Nathan showed that with this model CVA decreases, hence being less conservative than when the usual assumption of a constant recovery is used. As Svetlana mentioned, that's already two out of three candidates stating that banks are potentially being too conservative.
Marcin Rybacki (Cardano, Tilburg University) gave the final pitch. His research addressed the issue of the principles for benchmark curves construction in the Liability Driven Investment framework, trying to address some of the problems that arise with regulatory discount curves such as the ultimate forward curve.
While the jury, consisting of Hristina Lokvenec-Guleska (NN), Marcus Hemminga (RiskCo), Mary Pieterse-Bloem (APG), Bastiaan Verhoef (Van Lanschot Bankiers) and Roger Lord (Cardano), retreated in order to reach a decision, the final talk was given by Nelly Litvak - a mathematics professor at University of Twente and a bestselling author. She presented her new book "IQ to love" (see here or here) , where she gives advice to smart and quantitatively-focused men (aka "nerds") on how to find and keep a relationship. Before kicking off her talk, she asked the audience a few questions. One was whether anyone in the audience wanted to date Katy Perry. The response was very enthusiastic. We have not yet heard if any quant has, after Nelly's talk, attempted to date Katy Perry, but please let us know if you have. And Katy… you're more than welcome at our next event!
Nelly explained that the book had a rather mathematical structure, each statement was followed by a theorem and finally a proof. The presentation consisted mainly of the examples and tips from the book and also contained interactions with the audience – volunteers were asked to step forward. The first situation showed that women love to complain and want to receive compassion, and not necessarily look for a road map whereas guys with technical backgrounds tend to be focused on providing solutions. In the second scene quants were asked to describe their job to a girl on a hypothetical date which proved to be more difficult than perhaps anticipated, because, as Nelly pointed out, technical tasks are difficult to explain and might sound boring, so the only solution is to do the homework and come prepared.
The general message of Nelly's presentation was very positive – intelligent people are sexy, they only need confidence, which they can find after reading the book "IQ to Love". Confidence is key!
Soon after Nelly's presentation the results were announced. Rob Sperna Weiland, the winner of the 2014 award, declared Nathan the winner of the second edition, followed by Sina (second) and Marcin (third).
TopQuants takes this opportunity to convey our hearty congratulations to the winners and would like to thank all the participants for making it an interesting event. Further, we would like to extend our gratitude to the two speakers, Nathan Attrell and Nelly Litvak, for taking the time to present to our audience. We look forward to seeing you all at a future TopQuants event!

Boom Bust Boom - a TopQuants screening

On Wednesday, the 3rd of February 2016, EY, in their headquarters in Amsterdam, hosted the screening of a documentary that explains why bubbles exist and financial crashes happen, and why human behaviour needs to be considered when creating economic policies.
A welcome speech was given by Diederik Fokkema, the President of TopQuants, who was very glad to see that so many members of the quant society arrived to watch the screening.
"Boom Bust Boom" is the result of a meeting between writer, director, historian and Python Terry Jones and economics professor and entrepreneur Theo Kocken. The film is co-written by Jones and Kocken. It is co-directed by Terry Jones with son Bill Jones and Ben Timlett, AKA Bill & Ben Productions. It combines live action, animation and puppetry featuring high profile advocates such as John Cusack, leading experts including Nobel Prize winners Daniel Kahneman, Robert J. Shiller and Paul Krugman. It represents a global movement to change economics through education and encourage debate. "Boom Bust Boom" proposes a simple idea – let's adapt economics to human nature and have a unique look at why economic crashes will keep happening. Terry Jones says of the film: "If you don't know what happened to the economy in 2008 you will do by the end of this film. I am hoping it will encourage debate on how we can adapt the economic system to human nature, instead of idly hoping humans will change."
As Theo Kocken points out, "The people in the street are the ones who directly or indirectly pay for and suffer from financial follies. It is crucial the public as a whole understands what went wrong and what they can do to try and stop it happening again. We want to support economic students around the world who struggle with the fact that what is taught at universities doesn't link to reality."
After the screening Roger Lord, the Secretary of TopQuants, invited the special guest of the event Theo Kocken to the stand, who shared his personal experiences during the making of the movie and also an-

swered many questions from the audience. Below we cite only a few of them.

How did the collaboration with Terry Jones start?

Theo and Terry met via Rob Buckman, an old friend of his who had worked together with members of Monty Python on several occasions. Together they made a short movie in honour of Cardano's 10-year anniversary, titled "Risky Business and the Business of Risk" (see part 1 here and part 2 here). When he became a professor at the Free University of Amsterdam it was Theo's goal to introduce diversity and pluralism into economics. Since Theo and Terry (Rob Buckman passed away in 2011) shared many similar ideas, they decided to make a movie together that would educate and entertain at the same time.

What were the insights from the audience?

Theo recalled a story of a professor at the VU who recommended to stop showing the film because the students might feel uncomfortable. This request met with an immediate response of the rector of the university who said that being uncomfortable is the very essence that drives science.

After the crisis has there been any change in the mainstream education in economics?

Theo explained that after the crisis many people began to understand that they need to learn from behavioural finance and other cognitive sciences, as well as from complexity theory like agent-based modelling and network theory. Yet, he admitted, there is no integrated theory combining behavioural finance and classical macroeconomics. Still the theory gives too little attention to the role of debt and speculation.

If you could name the most misleading economic model still used in practice, what would that be?

According to Theo volatility is a very good example of a measure that is perceived as a risk metric, whereas the conclusions drawn based on it are often incorrect. Volatility often diminishes when real underlying risks grow. Similar observations can be made about correlations, which in times of crisis can jump from 0 to 1.

What about China? We can currently observe a big Ponzi scheme over there, in particular on the stock and housing market. Would it be valuable to show the movie there as well?

Theo recognises the policies introduced by Chinese authorities as fairly prudent, however we should keep in mind that China faces the burden of a large debt, which may be too heavy for its society to carry. He argued that currently, also in Europe, people believe that debt is a factor determining growth, which is in fact a delusion, especially in the light of a shrinking society. It is a fact that everyone wants to grow with the boom and benefit from it. Hence, the role of the regulator is crucial to not let speculation and debt drive the boom into a bubble. Theo argued that they should be the ones who step on the break.

The purpose of quantitative easing and lowering rates is to boost the economy, but it looks like speculation, does it not?

In Theo's view current policies of the American and European central banks seem like a bet. Financial assets are completely detached from the real economy and this creates inequality. Non-innovative companies can fund themselves practically for free, while small, innovative ones do not have access to that funding. This does not seem like a very robust environment for the economy to recover.

After all questions were answered, Diederik thanked Theo for his time and the fact that TopQuants were given this opportunity to introduce "Boom Bust Boom" to the quant society. After the session everyone headed to the drinks, snacks and further discussions about the future of economy.

TopQuants would like to express their gratitude to EY, which has been so kind to host the screening.

Find out more about the film at boombustclick.com, the accompanying educational initiative to drive awareness of the global financial 'boom and bust cycle'.

Since the screening the movie has become available on iTunes and Netflix, though not yet in their Dutch stores. Therefore, within the Netherlands the documentary can still be viewed on NPO Doc, via this link.

Impact of negative rates on pricing models

by Veronica Malafaia
(ING Bank - FI/FM Quantitative Analytics, Credit and Trading Risk)

Negative interest rates have made the headlines in the recent years, as they are being imposed by many rich-world central banks to revitalise the economy and are taking the markets to mostly uncharted territory. We discuss here what are the main implications on pricing models and some of the many challenges for risk management.

Negative rates?

Low interest rates are expected to contribute significantly to the economic recovery, by encouraging spending and investment, as they reduce the incentive to save and make borrowing cheaper. The current low inflation environment is however pushing central banks to further decrease their key short-term rates to negative levels [1].

The implications are manifold. On a higher level, it can be argued that negative rates can lead to lower profits and to capital erosion due to the asymmetry in the rates offered on the deposits (which banks are reluctant to set as negative) and the rates received on mortgages (which are typically linked to a benchmark floating rate) [2]. Although there is the general belief that there is a lower limit beyond which rates will not decrease further, it is far from clear what the exact value actually is [3].

Negative rates have also triggered implicit floors on mortgages and asset-backed bonds [4], due to clauses preventing negative coupons. CSA agreements were as well object of an intense discussion, namely whether the collateral poster should have to pay interest in case of a negative interest rate, which ultimately lead ISDA to define the negative interest rate protocol [5].

Pricing models

Besides the implications above on the pay-off, derivatives pricing is affected by negative rates also via the pricing- and market data-models. An obvious remark is that all models assuming a lognormal dynamics for the underlying interest rate are not suitable for a negative rate environment.

Until recently, the Black model was the market paradigm for interest rate options, with the move to shifted lognormal/normal models being triggered by the low rate values. There is however no consensus yet on which of the models to choose [6]. Although the shifted lognormal process seems a more natural candidate given the conviction that there is a lower limit for interest rates, the normal model can be argued to be a more pragmatic choice given the uncertainty on the actual value of this limit.

The interpolation of swaption volatilities is another patent example of the impact of negative rates. The market convention so far has been the formula by Hagan et. al. [7]. It corresponds to an expansion of a stochastic volatility model (SABR) that however fails to work well for high volatility, long maturities and deeply out of the money options. The potential implied negative density probability at low strike high expiry options is particularly relevant in the present low rate environment.
Market consensus is far from being reached, with several approaches proposed in the literature, for instance deriving better analytical approximations from the SABR model or improving directly Hagan's implied density, among many others (see [8] and references therein).

The impact of negative rates on the most popular interest rate models that are used to price more complex derivatives, such as the Hull-White or the Libor market models is more limited. The Hull-White model allows naturally for negative rates, with the old question of how realistic is the implied lower level for the rates becoming more relevant now. For the Libor market model, both lognormal/shifted lognormal formulations are possible and similar questions arise. One commonly used approach for calibration of these models is based on analytical approximations for volatilities/correlations, which should naturally be changed if a lognormal dynamics is no longer assumed.

Risk management

The implications for risk management are also wide, with the potential need to change many of the pricing systems and market data transformation algorithms which were tailored to lognormal models. The calculation of sensitivities, ultimately used for P&L explanation and limit setting, might need to be revisited as well, especially if calculated by the bump-and-revalue method of the underlying market data, instead of referring directly to market instruments (see Figure 1). The shift methodology used to generate historical scenarios for the value-at-risk calculation might require also a review in light of the low rate environment, just to cite some of the most relevant examples.

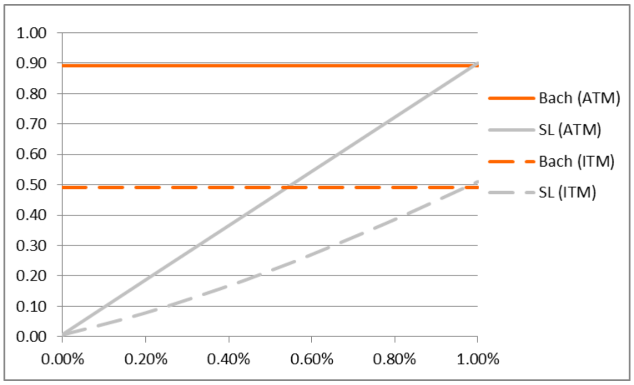


Figure 1: *Lognormal and shifted lognormal vega values for a 5Y put option against normal vega, for shift sizes up to 1%. The ATM level is 0.01%. The shifted lognormal and normal volatilities were calibrated so that the option prices corresponded to the Black price with 50% volatility. Vega was calculated by bumping and revaluing, with a shift size of 1bp in the normal case and 1% for all the others.*

To conclude, negative rates have profound implications on the economy, pricing models and risk models, many of them still unknown. The only consensus so far is that negative rates are here to stay.

References:

[1] Everyone likes negative interest rates now, World Finance (May 2015).

[2] Why negative interest rates have arrived – and why they won't save the global economy, The Economist (February 2015);
Negative rates to shake up financial system, say experts, Financial Times (February 2015).

[3] Negative creep, The Economist (February 2016);
Europe's negative rates considered to have further to fall, Financial Times (November 2015);
The death of cash, Bloomberg (April 2015).

[4] Negative rates halt payments in European asset-backed bonds (April 2015).

[5] Dealers threaten to charge for 0% CSA floors, Risk net (October 2014);
Negative Eonia rate causes CSA headaches, Risk net (September 2014).

[6] Negative rates: dealers struggle to price 0% floors, Risk net (November 2012).

[7] P.S. Hagan et. al., Managing smile risk, Wilmott magazine (2002).

[8] L. A. Grzelak and C. W. Oosterlee, From arbitrage to arbitrage-free implied volatilities, Journal of Computational Finance (March 2016);

A. Antonov at. al., Mixing SABR model for negative rates (January 2015);

P. S. Hagan and D. Kumar, Arbitrage-free SABR, (January 2014);

J. Andreasen and B. Huge, Expanded forward volatility, Risk (January 2013).

Bank-less future: how FinTech Start-ups might take over the Financial System

by Laurens Kolkman
(Transaction Services, KPMG)

Since the crisis in 2008, the Financial Technology (FinTech) is again experiencing an enormous rise and it is not the first time that technology is an important disruptor in the financial world as we know it.

Earlier our Fintech team did an exercise of the mind when they wrote a blog on the bank without employees. Although that would already be a hard to imagine development, when we look at the start-up FinTech investment market, there might be no banks in the future at all.

Technology and financial crisis

Technology caused both positive and negative developments, but technology has been making waves in the evolution of the financial system. In 1967, it was the first ATM and financial calculator that led to the big shift from analogue to digital financing which would change the way we handle money forever. Twenty years later, in 1987, it were the first financial trading computers, that according to many, were the cause of the next disruption in the 'money business': the "Black Monday" market crash. Almost 30 years later, when the financial crisis of 2008 started, again many pointed their finger at the underdeveloped technological infrastructure that could not keep up with the increasing complexity of the financial system. More recently in 2010, algorithmic trading (high-frequency trading), which is a form of electronic trading, caused the Flash Crash were the American stock market experienced the highest intra-day drop in history.

Where innovative technology is thus often the cause of economic disruptions, it is also the bridge to restore confidence in that same financial system. Prior to the most recent crisis, this bridge was built by the established financial players such as banks and financial regulators. But in 2008, another group of people stood up to help and reinvent the financial system: Start-ups.

The rise of FinTech start-ups

The most recent financial crisis created a coherence of conditions that shifted the attention from financial institutions to financial start-ups. Consumers lost their trust in banks and the urge to have more control over one's own finances arose. At the same time, we become busier by the day and besides knowledge, we do not have the required time to manage our own financials, regardless of how important we find this. And exactly that conflict is where the rising FinTech start-ups come into play. But why are these small start-ups able to challenge those established financial institutions?

Power is where the money is at

The above is mainly from a consumer perspective, but also businesses and capital providers are looking for new alternatives. The 2008 financial crisis also resulted in stricter regulations for capital markets that did not benefit the efficiency for capital providers. Again, FinTech start-ups jumped into these negative side-effects and came up with innovative services and products so that businesses could meet both their own, and their customer's needs. This combination of interest from both consumers and businesses, ultimately led to a year after year exponential increase of investments in this sector and provided the much needed capital and attention for the FinTech start-ups in order to be a strong competitor for the established financial system.

FinTech deal explosion

In the past year in Europe alone, the number of FinTech start-up exits (an acquisition/merger or IPO) quadrupled from 11 to 44 deals. Globally, almost $14 billion was invested in over 821 FinTech companies. Proof that not only consumers are looking for alternative financial service providers, is evident from the fact that the number of unique investors that invested in FinTech start-ups rose from 223 to 894 in the last five years. Furthermore, these deals are not only pure money making transactions but also prove to be strategic: non-financial companies like Google, Intel, Salesforce and global players like SalesBank, Naspers and Ping An insurance are significantly active in the FinTech mergers & acquisitions (M&A) market. Google Ventures alone, made 37 FinTech deals in the past five years.

A war of many

Knowing that FinTech start-ups are becoming more important in the financial sector and knowing why this is happening, one question is still remaining: how? Exactly the above smashing numbers explain how FinTech start-ups are becoming so immensely popular. Since the 2008 financial crisis early stage (seed capital) investments in FinTech start-ups increased from $328 million to $1.3 billion in 2015. The increased availability of funding, in combination with the decreasing cost to set-up a new business (in 2000 this would cost you around $5 million, in 2015 not even $5k), led to an explosion of new FinTech start-ups all focusing on specific financial services. While before the crisis, the financial system was controlled by a few big banks, nowadays thousands of companies within this market are rivaling, specializing and being better at the many products and services that the big players all used to provide by themselves. And it is not just happening at one place, it is happening all around the world: CBinsights has even unbundled all services and products from both a US and a European bank and tied a FinTech start-up to all of them.

More is coming

While investments and M&A deals within the payment sector of FinTech exponentially grew and skyrocketed in 2015, the adoption of it has only just begun. It is estimated that by 2017, 50% of all payments will be done by mobile phone. The same goes for money lending. Crowdfunding platforms and other peer-to-peer lending platforms increased in popularity amongst start-ups as an alternative for traditional bank lending. Only recently, consumer lending businesses like LendingClub and Prosper, who provide algorithm based interest rates and higher returns for individual investors, received attention from large investors. Furthermore, as the world becomes more connected and the Internet of Things will increase the amount and different kinds of data that can be used and analyzed online, the FinTech sector could achieve huge advantages of these developments when adapting their financial products and services to the daily interests and choices of individuals. As the number of FinTech M&A deals rose by 66% in 2015 and the army of innovative FinTech start-ups is only increasing, more deals within this sector is expected for the coming year. In fact: January 2016 already counted 112 FinTech deals, covering a staggering $7 billion.

FinTech start-ups are owned by individuals who deliver what customers want, instead of what the system needs or requires. It is them who enable individuals to manage their own financials more closely in a way they choose and they enable businesses to handle capital more efficiently and directly with their customers. They allow businesses, wealthy people and even common individuals to invest in each other, without mediators. All of the above explains why so much money is going into this market and why even non-financial businesses like Google are only starting to shift their capital to FinTech. To remain in this competition, banks will also need to adapt to, and more importantly, change the financial system that becomes more dependent on data and technology by the day. So knowing how FinTech startups become increasingly important within the financial system, it is only time that tells us when banks as we know them will be superfluous and technology takes their place.

ALM Risk Penalty Methodology

by Frank Pardoel, Hans Heintz & Pim Poppe (RiskQuest)

In short, Asset Liability Management (ALM) within a retail bank comes down to managing interest rate risks stemming from saving accounts and mortgage loans. Saving accounts (or other types of bank accounts) allow clients to deposit or to withdraw their money. These accounts are marked as liabilities for the bank. Mortgage loans on the other hand, are loans sold by the bank to clients for financing (residential) property. The (residential) property itself is used as collateral. Mortgage loans are marked as assets for the bank. In an ideal world a bank uses the saving accounts to finance the mortgage loans. In absence of duration mismatches, withdrawal of savings, defaults on mortgage loans, prepayments and interest rate fluctuations, a risk-free strategy could be defined. The strategy uses the savings accounts for financing the mortgages. The bank pays the account holder a certain saving rate. At the same time, the bank requests a mortgage rate equal to the saving rate plus a spread. In this strategy, the bank earns the spread.

Due to different market and non-market (or behavioral) uncertainties, setting-up such a risk-free strategy does not hold. This implies a funding gap, i.e. other sources of funding are required to fund the mortgage loans implying interest rate risk. As a consequence ALM departments are occupied with interest rate risk management, maturity matching, treasury and (external) funding. In this white paper, one of these uncertainties on the radar of ALM is selected. The white paper will consider the risk related to missing interest rate income as a consequence of prepayment events. A prepayment event is a repayment of the notional of the loan other than mentioned in the contract. In some of the prepayment events, banks compensate themselves for the missed interest rate income by applying penalties to their prepaying clients. Since penalty methodologies are far from transparent and calculating incorrect penalty amounts or the inability to explain the calculations may lead to reputational damage, the theory and intuition is considered hereafter. Understanding the steps underlying the penalty calculations, enables management and front office to explain the matter in a transparent way to clients, higher management and regulator.

Main takeaways:

- The inability to explain the rational behind penalty amounts may lead to reputational damage;
- There does not exist a uniform correct penalty methodology;
- Penalty amounts charged to clients will heavily depend on the underlying assumptions and understanding them is crucial;
- Linear, bullet and level paying loans with similar loan characteristics will lead to different penalty amounts;
- Although the risk seems negligible, the implementation of an erroneous penalty methodology may result in missed cash inflows;
- RiskQuest has extensive knowledge both in modelling, data-cleansing and documentation with a special attention to the field of ALM.

Introduction

At origination of a mortgage loan, the mortgage holder agrees with the bank on the terms of the contract. The contract terms are binding and of importance in case of relocation of the mortgage holder, a (partial) prepayment or an interest rate reset. The most important characteristics of a mortgage loan captured in the contract are: notional of the loan, amortization type of the loan, reconsider period, (fixed) term of the loan, interest rate fixed period, mortgage coupon and the annual amount that one can prepay each year.

Three characteristics that are of particular interest for the topic in this white paper are: the prepayment type, the amortization type and the franchise amount. The prepayment type tells the bank the incentive of the client to repay on the mortgage loan. In general, one identifies the following prepayment events in the Dutch mortgage market: relocations, (external) refinancing, partial prepayments (referred to as curtailments) and reconsider events. The last event refers to reconsidering the offered mortgage rate during a predefined reconsider period.

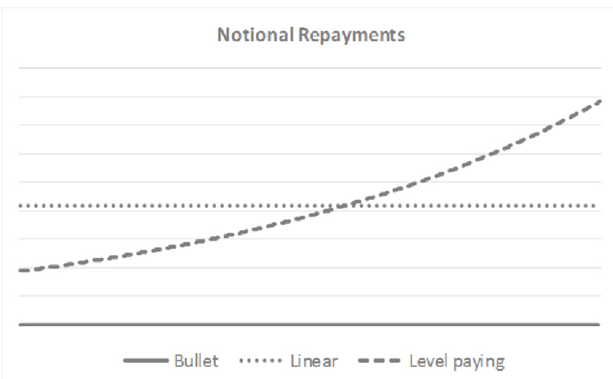
The amortization type indicates according to which payment scheme the loan will be repaid. The most well-known amortization types are bullet, linear and level paying[1]. Figure 1 shows the notional repayment and interest payment schemes for each of the amortization types. Note that for level paying loans the sum of the notional repayment and interest payment is constant for

[1] The Dutch terms are aflossingsvrij (bullet), lineair (linear) and annuïtair (level paying).

each period in time. The total periodically payment has the form of an annuity.

The franchise is the annual amount that one can repay on top of the contractual repayments without being penalized. The franchise amount is a fixed percentage of the original outstanding notional. In other words, the bank gives the client the option to annually prepay a fixed percentage of the original outstanding notional, free of charge, e.g. 10%. This embedded option – written to the client by the bank – has an economic value. In order to determine the value of the embedded options within a mortgage contract, a stochastic model for (future) interest rates is required which is not in scope of the current white paper.

A prepayment exceeding the franchise amount will, under certain conditions, be penalized. The next section examines the methodologies to determine the height of the penalty. It also provides illustrative examples.



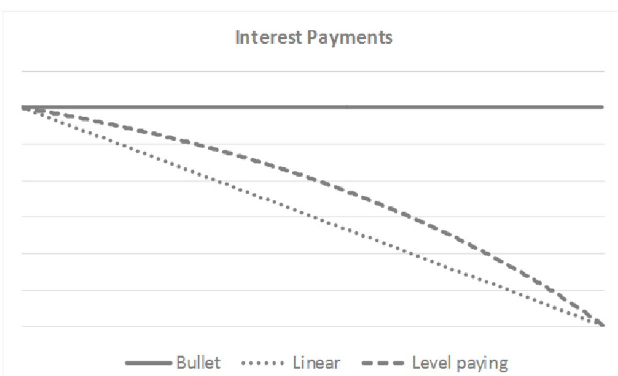


Figure 1: *Upper the notional repayment scheme of a loan under the three different amortization types. Lower the interest payment scheme of a loan under the three different amortization types.*

Missed (interest) cash flows

In the most straightforward scenario, the client repays the contractual notional and interest payments in accordance with the mortgage contract. In that case, the bank has full transparency with respect to the future cash inflow. Thereby, the bank can perfectly monitor the interest rate risk metrics and set-up an appropriate hedge upfront. In reality, prepayment events introduce uncertainty with respect to the future cash inflows and hence risk for the bank. If a client prepays on his loan, the cash flow profile with the future interest income of the bank changes. Apart from the direct impact on the interest income of the bank, the hedge strategy is also affected.

In the Netherlands, a mortgage holder is allowed to foreclose the mortgage loan in case of relocation without being penalized. If one sells the (residential) property, the mortgage loan can be foreclosed without financial consequences[2]. The same holds for reconsider events. In case of (external) refinancing and partial prepayments however, a compensation is requested by the bank. Note hereby that the partial prepayment amount should exceed the franchise amount. Whenever multiple partial prepayment occurs within a year, the franchise amount is adjusted after each event. The events and the consequences from a penalty perspective are depicted in Figure 2.

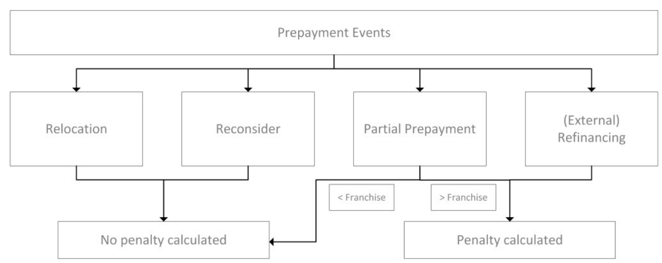


Figure 2: *Overview of the different prepayment events and the financial consequences from a penalty perspective.*

An incentive for external refinancing presents itself whenever the mortgage rates, quoted in the market, drop below the contract coupon rate. Conditional upon a lower mortgage rate regime, refinancing a mortgage loan will be beneficial for the client in two ways. First, based on the original fixed term period, the contract coupon after (external) refinancing is lower compared to the original contract coupon. Second, since the original fixed term period exceeds the remaining fixed term at moment of prepayment, the new contract coupon will be based on a shorter fixed term. For example, suppose a client

[2] An exception is when the (residential) property decreased in value. In that case, the remaining loan amount after selling the property has to be compensated

obtained a mortgage bullet loan with a 10-year fixed rate period and a contract coupon of 5.00%. After five years, the client decides to refinance the loan internally. The current quotes for the 5-year and 10-year mortgage loans are respectively 4.00% and 4.50%. Although the current 10-year rate is lower than the contract coupon, the contract coupon after refinancing will be 4.00% since the remaining contractual fixed rate period is 5 years. The interest payment and notional repayment will be adjusted accordingly.

The previous example demonstrates the impact on the bank its interest rate profile in case of a refinancing. According to the contract, the bank receives interest cash flows of 5.00% for a period of ten years. Due to a refinancing event, the cash flows after year five diminish by 1.00% as depicted in Figure 3. Note that the rates first have to be converted in monthly rates since mortgage payments usually occur monthly[3].

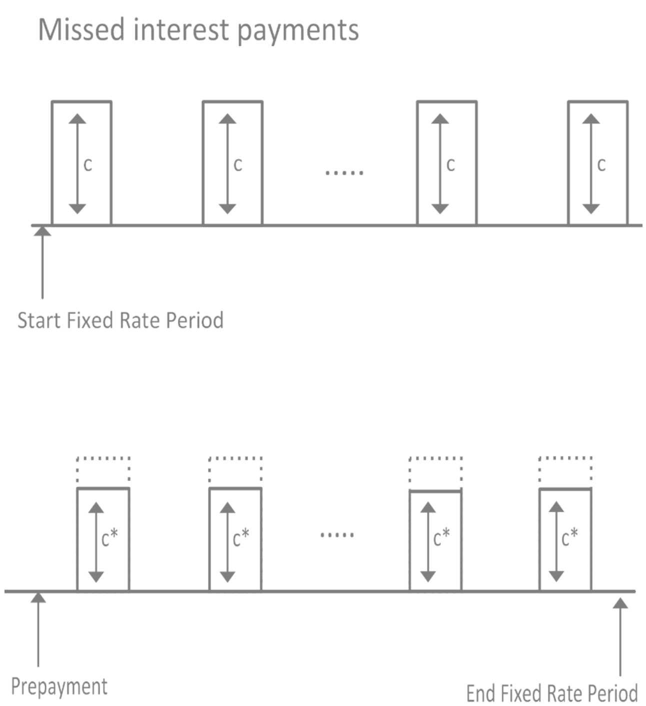


Figure 3: *Illustration of the methodology underlying the penalty calculations; where c is the original monthly contract coupon and c* is the monthly mortgage rate after the prepayment.*

The penalty expression for a bullet loan

In case of a bullet loan the penalty amount the bank will charge its client, equals the present value of the missed future interest cash flows. The reasoning is as follows: suppose the bank has to sell a new similar mortgage loan immediately after the prepayment event, then it will only be able to realize an interest income equal to the current mortgage rate times the notional of the loan. Hence, the lost interest due to the prepayment event needs to be compensated for by the prepaying client. One can express the penalty for a bullet loan at time t in a mathematical form as follows:

$$P_{bullet}(t) = \sum_{k=1}^{M} N(t)\Delta I(t) d(t)^k$$

Here P(t) is the penalty amount at time t, N(t) is the prepaid notional amount at time t, d(t) is the discount factor, M are the remaining payments within the fixed rate period and ΔI(t) is the difference in interest payments due to the prepayment event. For bullet loans it holds that ΔI(t) =c-c*(t); with the monthly contract coupon rate c and monthly mortgage rate after prepayment equal to c*(t). After rewriting and using the closed form expression for geometric series, one finds for the normalized penalty expression for bullet loans, i.e. N(t)=1:

$$P_{bullet}(t) = \left(c - c^*(t)\right)\sum_{k=1}^{M} d(t)^k = \frac{\left(c - c^*(t)\right)d(t)\left(1 - d(t)^M\right)}{1 - d(t)}$$

Refer to the earlier example, c=0.41%, c*(t)=0.33%, M=60 and d(t)=d=0.9967. The normalized penalty amount is 0.0435, i.e. for each 1 EUR penalty-bearing prepaid notional, the client has to pay 0.0435 EUR.

Note that the future cash flows are discounted against rate c*(t) and not the risk-free rate. Moreover, a flat interest curve is used instead of a forward curve. The amortization period of a mortgage loan is assumed to be 30 years. The penalty-bearing notional refers to the prepaid notional amount minus franchise amount. Each bank determines its own conditions. The conditions are captured in the terms of the mortgage contract. Therefore, differences may arise based on different assumptions.

The penalty expression for other types

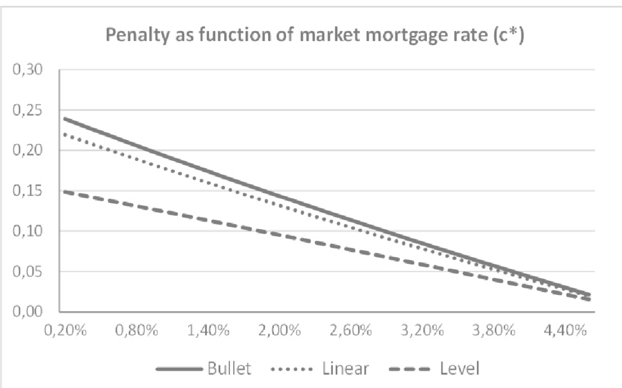
Besides bullet loans, linear and level paying loans are the most common types. Until now the notional repayments have not been considered for calculation purposes where these are zero for bullet loans. For linear mortgage loans these are not of interest for the penalty calculation, since the notional repayments are not dependent on the contract coupon and hence equation [1] is still valid. Although the value of ΔI(t) depends on k. For level paying mortgage loans, the notional repayments depend on the contract coupon via the so-called annuity factor a(c)[4]. Equation [1] is extended with the notional repayment

[3] Annual rates are converted into monthly rates by using the equation $(1+r_{annual}(t)) = (1+r_{monthly}(t))^{12}$ or in terms of the monthly rates $r_{monthly}(t)=(1+r_{annual}(t))^{1/12}-1$. Sometime people use the shortcut $r_{monthly}(t)=r_{annual}(t)/12$.

which penalty calculation accordingly. The normalized penalty equation (or penalty per EUR penalty-bearing notional) for level paying loans can be expressed as an analytical solution as follows:

$$P_{level}(t) = \sum_{k=1}^{M}\left(\frac{N(t)}{a(c)} - \frac{N(t)}{a(c^*)}\right)d(t)^k = \frac{\big(a(c^*) - a(c)\big)d(t)\big(1 - d(t)^M\big)}{a(c)a(c^*)\big(1 - d(t)\big)}$$

When one uses the parameter values of the example again, however this time for amortization types level paying and linear, the normalized penalty amounts are respectively 0.0309 and 0.0401.



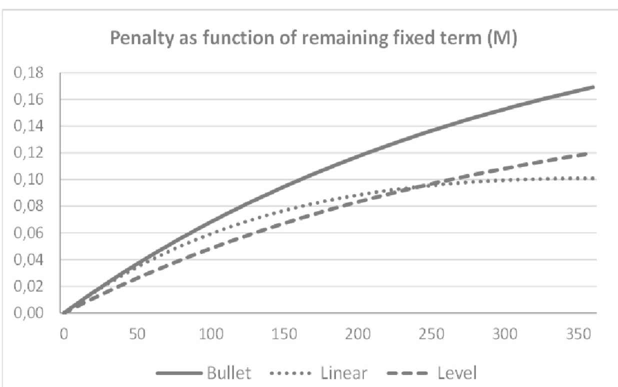


Figure 4: *Left shows the penalty per EUR penalty-bearing notional as a function of the remaining payments within the fixed rate period. Right shows the penalty per EUR penalty-bearing notional as function of the current mortgage rate in the market, i.e. mortgage rate after prepayment. The penalty on the y-axis and the parameter c* or M on the x-axis.*

Penalties for the level paying and linear loan with the same characteristics are respectively 29% and 8% lower compared to the bullet loan. To give the reader more intuition of the sensitivities, the penalty per EUR penalty-bearing notional are plot as function of the mortgage rate after prepayment c* (upper graph) and as a function of the remaining payments M (lower graph).

The dependency of the penalty amount towards the market mortgage rate is intuitively straightforward. The smaller the mortgage rates observed in the market, the larger the penalty, with a limit around c*=0.00%. For the penalty as function of the remaining number of payments, one observes that the larger the remaining payments within the fixed rate period, the larger the penalty as expected. An interesting observation however is that the linear and level paying penalty intersect. The intersection is caused by the difference in amortization scheme and hence outstanding notional. Before the intersection the outstanding notional (and hence the penalty) of the linear loan exceeds the outstanding notional of the level paying loan. After the intersection the effect is vice versa.

Conclusion

The penalty formulas for the different amortization types give an expression for the penalty amount per EUR penalty-bearing prepaid notional. The differences in the amortization schemes result in different penalty expressions accordingly. Topics related to penalty calculations, not discussed in this white paper but very interesting are for example, the construction of the mortgage rates, including or excluding the net payment at the end of the fixed rate period in the penalty calculations, hybrid amortization types and prepayments that happened within a month.

The white paper introduces the environment of prepayment events, the penalty methodology, the penalty expressions and the intuition behind the formulas. Having a clear understanding of the penalty calculations decreases reputational risk. Moreover, it contributes to a transparent framework. RiskQuest understands the mathematics behind interest rate risk calculations, e.g. penalty calculations, and at the same time is able to explain the theory in common language.

4The annuity factor determines the monthly (annuity) payment of a level paying mortgage loan. The annuity factor is calculated as: $a(c) = \frac{1-(1+c)^{-T}}{c}$ where T is the remaining amortization period. Note that this is the combined interest and notional repayment.

Valuation with Liquidity Risk

by Bert-Jan Nauta

Abstract
This contribution is based on the presentation I gave on the TopQuants autumn event in November 2015. I discuss how liquidity risk can be included in the valuation of assets through a simple model.

Introduction
Liquidity risk is one of the main risks for banks. Indeed, banks have developed many tools to manage and model liquidity risk such as collateral management tools, intraday liquidity monitoring, balance sheet stress testing, liquidity buffers, etc. To support these many quantitative models have been developed to get a better understanding of liquidity risk, such as behavioral models for savings, mortgages (especially prepayments), models for collateral (out)flows, etc.
However, there is currently no methodology for the inclusion of liquidity risk in the valuation of a bank's assets. Of course, banks price liquidity and liquidity risk through an internal pricing mechanism, called funds transfer pricing (FTP). This is typically done, by taking a risk-free curve adding a spread to cover the bank's funding spread and add a spread for liquidity buffer costs. However, this only determines an internal rate, which does not (and should not) enter the valuation of assets.
Hence, there is a risk, liquidity risk; that is not included in the valuation assets. One of the basic concepts of finance is that risks that affect the return of an asset should be valued. There exist well-known modelling techniques for market risk (Black-Scholes and extensions) and credit risk (Merton model, reduced form models). However not for liquidity risk.
This summary may be a little unfair. In recent years, banks have developed models to include their funding costs in the valuation of derivatives. These approaches acknowledge that liquidity comes at a cost and include this in the value of derivatives. Nevertheless, there are two problems with these approaches:

1. they violate some basic finance theorems, such as the Modigliani-Miller theorem, that essentially states that the value of an asset is independent of its funding,
2. these approaches are not risk-sensitive. The funding costs of a liquid government bond are the same as of an illiquid structured note.

For these reasons, I believe a valuation methodology that includes liquidity risk needs to be developed. Here I will introduce a simple model that captures some of the essential features involved.
This note is based on [1], [2] and [3].

Liquidity Risk Model
Before developing the liquidity risk model, it is useful to discuss the definition of liquidity risk. Various definitions of liquidity risk exist. However, since the focus here is on the valuation of assets, only the risk that affects the pay-off or cash flows needs to be considered. One subtle aspect is that the illiquidity of an asset does not affect the cash flows of the asset itself. Nevertheless, its illiquidity may affect the cash flows of the holder of the asset. When the holder of the asset is forced to liquidate an asset, the holder will get a lower return for an illiquid than for a liquid asset. Later on, an effective pay-off will be introduced to capture this effect.
From this discussion, it is clear that liquidity risk only affects the valuation of assets when it leads to a forced liquidation. Hence, the following definition of liquidity risk is used: Liquidity risk is the risk for an event to occur that would force a bank to liquidate some of its assets. I will call such an event a liquidity stress event (LSE).
The first step is to model LSEs. A simple model for an LSE is depicted in Fig 1. In normal periods there is affluent liquidity and any asset can be funded at any term. At a random time τ an LSE occurs with a random duration t_{event}. During the LSE, it is not possible to obtain funding for an asset. If an asset was funded at the time of the LSE at a term t_{fun} that is equal or larger than the duration of the LSE there is no problem. However if the funding term is shorter than the duration of the LSE, then the funding cannot be rolled over, and the asset needs to be liquidated. When the bank liquidates the asset, it will not receive its fair value. Instead, a stressed value will be received. I model this stressed value as a liquidation value *LV* times the fair value. The liquidation value is the fraction of the fair value that the bank receives when it liquidates an asset in an LSE. The liquidation value is specific per asset. For a perfectly liquid asset (cash) it will be $LV=1$. For illiquid assets, the liquidation value may be close to zero.
Before turning to the equations, I discuss the funding strategy. I assume an asset is funded at a single funding term. This funding is continuously rolled over (as long as there is no LSE).

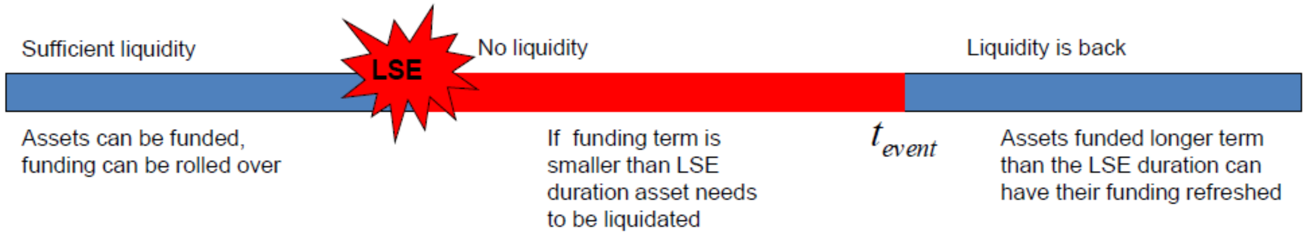


Figure 1: *An LSE.*

Since funding long term is more costly than funding short term there the bank would prefer to fund short-term. However in our liquidity risk model (and in reality) funding short-term increases liquidity risk. Later on, I will show that in this model the bank needs to balance funding costs and (expected) liquidation losses.

Finally, I will obtain the funding rates from the index LIBOR curves in a multi-curve framework. These curves are build from swap quotes and other interest rate products. Typical curves are the OIS, 1M, 3M, 6M and 12M curves, see Fig. 2 for an example from Bloomberg. I will denote the continuously compounded 3M interest rate for maturity T by $r_{3M}(T)$. I assume the bank has a way to interpolate the index so that $r_t(T)$ is available for any index t.

Figure 2: *Multiple curves from Bloomberg on 12 april 2016.*

Derivation

Consider a bullet loan that pays 1 at maturity T. From the description of an LSE in the previous section, an effective pay-off of this loan can be derived. If there is no LSE or if the funding term is sufficient, the pay-off of the loan is 1 at maturity T. If there is an LSE and the funding term is insufficient, the pay-off is the fair value of the loan times the liquidation value at the time of liquidation. This may be summarized by the effective pay-off

$$\begin{aligned} \text{Effective Pay-off} &= 1 && \text{at time } T \text{ if } \tau + t_{\text{fun}} \geq T \\ &= V_{\text{rf}}(\tau + t_{\text{fun}}) LV(t_{\text{event}}, t_{\text{fun}}) && \text{at time } \tau + t_{\text{fun}} \text{ if } \tau + t_{\text{fun}} < T \end{aligned}$$

The risk-free value $V_{rf}(t)$ is the value of the cash flow without liquidity risk,

$$V_{\text{rf}}(t) = e^{-r_{\text{OIS}}(T-t)}$$

I use a simple piece-wise linear function for the liquidation value,

$$LV(t_{\text{event}}, t_{\text{fun}}) = \max(1 - c(t_{\text{event}} - t_{\text{fun}}) I_{t_{\text{event}} > t_{\text{fun}}}, LV_{\text{min}})$$

where LV_{min} is the minimum liquidation value $0 \leq LV_{min} \leq 1$ and $c > 0$.

I model the time of the LSE τ and its duration t_{event} as random. I use an exponential distribution with intensity λ for τ and a lognormal with mean μ and volatility σ for the duration of the event t_{event}.

After some calculation (and an expansion to first-order in $(r_{t(fun)} - r_{ois})T$ and λT the funding-term-dependent value of the bullet loan is

$$V_0(t_{\text{fun}}) = e^{-r_{\text{OIS}}T} \left\{1 - (r_{t_{\text{fun}}} - r_{\text{OIS}})T - \lambda(T - t_{\text{fun}}) \left[1 - \mathbb{E}[LV(t_{\text{event}}, t_{\text{fun}})]\right]\right\}$$

Where

$$\begin{aligned} \mathbb{E}[LV(t_{\text{event}}, t_{\text{fun}})] &= N(\mu, \sigma; \log(t_{\text{fun}})) \\ &+ (1 + c t_{\text{fun}}) N(\mu, \sigma; \log(t_{\text{fun}}), \log(t_m)) \\ &- c e^{\mu + \sigma^2/2} N(\mu + \sigma^2, \sigma; \log(t_{\text{fun}}), \log(t_m)) \\ &+ LV_{\text{min}}(1 - N(\mu, \sigma; \log(t_m))) \end{aligned}$$

where $t_m = t_{fun} + (1 - LV_{min})/c$ and N is the cumulative normal. The first term in the formula for $V_0(t_{fun})$ can be interpreted as (additional) funding costs

$$FC = (r_{t_{\text{fun}}} - r_{\text{OIS}})T\,.$$

The third term can be interpreted as liquidity costs

$$LC = \lambda(T - t_{\text{fun}})[1 - \mathbb{E}[LV(t_{\text{event}}, t_{\text{fun}})]]\,.$$

The value of the bullet loan can be obtained by optimizing over the funding term

$$V_0 = \max_{t_{\text{fun}}} V_0(t_{\text{fun}})\,.$$

The optimal funding term is the funding term that maximizes the value. I will denote the optimal funding term by t_{liq}, since it can be interpreted as the liquidity horizon of an asset:

$$V_0 = V_0(t_{\text{liq}})\,.$$

Note that maximizing the value of V_0 is equivalent to maximizing the funding costs plus liquidity costs. Hence the optimal funding term is the term that maximizes *FC+LC*.

Results

In this section I show the results for three examples: a liquid asset, a less liquid asset, and an illiquid asset. The specifications are shown in Table 1.

asset	c	$LV_{\min}$
liquid	0.5	0.9
less liquid	2	0.5
illiquid	1000	0.0

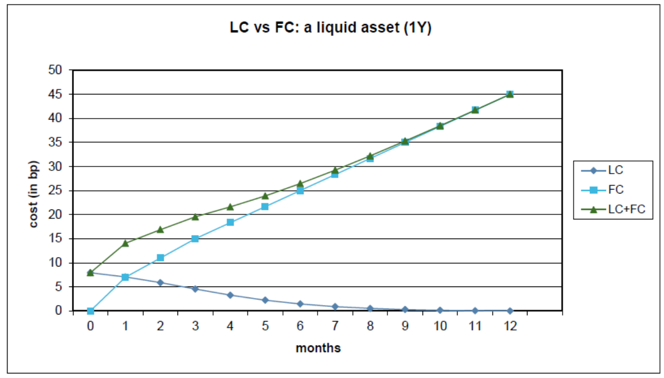
Table 1: *Parameters for a liquid, less liquid and illiquid asset.*

In Fig 3 the resulting liquidity costs as a function of the funding term are shown for these three assets. The funding costs as a function of the funding term is the same for each asset. For the liquid asset the optimal funding term is overnight (ON) as this minimizes the sum of funding costs and liquidity costs. For ON funding the funding costs are zero (as these are defined relative to ON), and the liquidity costs are 8bp. in this example. For the less liquid asset the optimal funding term is 6 months. The funding costs at this optimal funding term are 25bp and the liquidity costs 7bp. The illiquid asset has an optimal funding term of 9 months in this example. Its funding costs are 35bp and the liquidity costs 4bp. The results are summarized in Table 2.

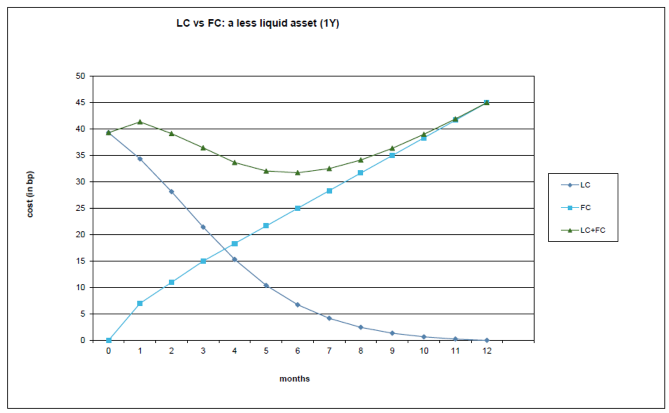
asset	t_{liq}	FC (in bp)	LC (in bp)	FC +LC (in bp)
liquid	ON	0	8	8
less liquid	6 months	25	7	32
illiquid	9 months	35	4	39

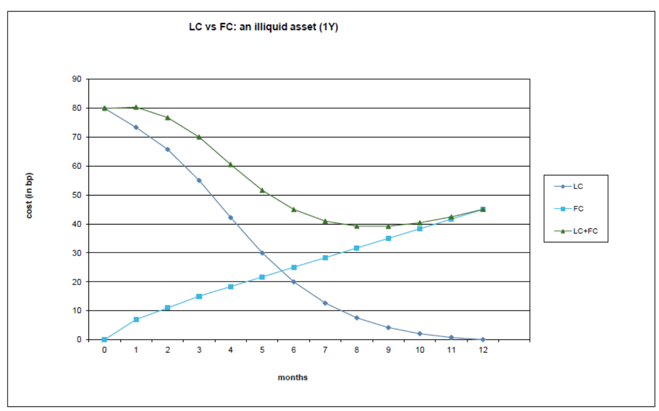
Table 2: *Optimal funding term, funding and liquidity costs for a liquid, less liquid and illiquid asset example.*

(a)



(b)





(c)

Figure 3: *Funding costs and liquidity costs as a function of the funding term for (a) a liquid asset; (b) a less liquid asset; (c) an illiquid asset.*

Summary

I have argued that liquidity risk should be accounted for when valuing assets. A simple model for liquidity risk was introduced that has an analytical solution for bullet loans.

The main features of the model and resulting valuation model are summarized below:

- The model generates two corrections to the risk-free fair value that can be interpreted as funding costs and expected liquidation losses.
- The model allows estimating the optimal funding term. The optimal funding term is the result of minimizing the funding costs and expected liquidation losses. The optimal funding term may be interpreted as the liquidity horizon of the asset.
- The discount rate references the optimal funding term.
- The value of an asset depends on its liquidity.

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References

1) Nauta B. J., *FVA and Liquidity Risk*, Available at SSRN: http://ssrn.com//abstract=2373147

2) Nauta B. J., *A Model for the Valuation of Assets with Liquidity Risk*, Available at SSRN: http://ssrn.com//abstract=2687936

3) Nauta B. J., *Multi-Curve Discounting*, Available at SSRN: http://ssrn.com/abstract=2767470

Upcoming TopQuants Events

Complexity theory and financial regulation

The event will take place on Thursday 12 May at 15:00 and will be held at the Dutch KPMG headquarters in Amstelveen. At the event, we are very happy to announce that three of the co-authors of the recent Science publication "Complexity theory and financial regulation" will be present.

Prof. dr. Cars Hommes (Economic Dynamics, University of Amsterdam), prof. dr. Hans Heesterbeek (Theoretical Epidemiology, Utrecht University) and dr. Diego Garlaschelli(Theoretical Physics, Leiden University) will explain, each from a different angle, how complexity theory may help anticipate and manage future crises.

Traditional economic theory has not been able to explain, or even predict, the near collapse of the financial system and its long-lasting effects on the global economy. Their paper discusses recent insights and techniques that offer potential for better monitoring and management of highly connected and interdependent financial systems.
Please register using this link.