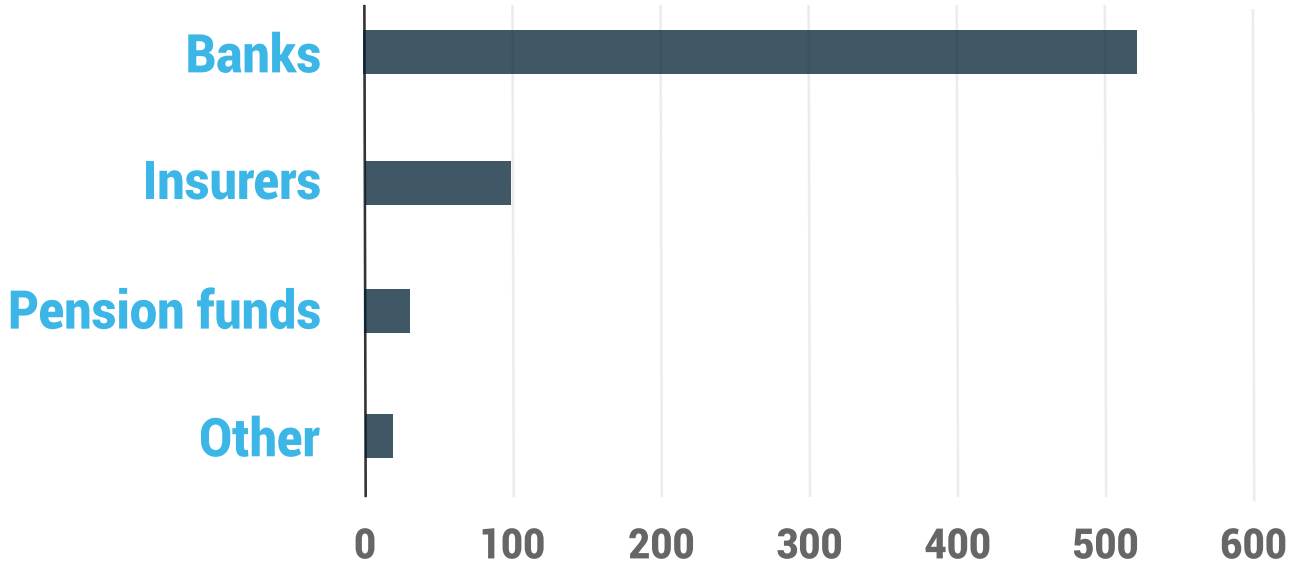
Replicating portfolios for prepayments

RISK AT WORK

Mortgage prepayment is the largest source of optionality for Dutch financial institutions

… yet most of them cannot calculate their sensitivities frequently

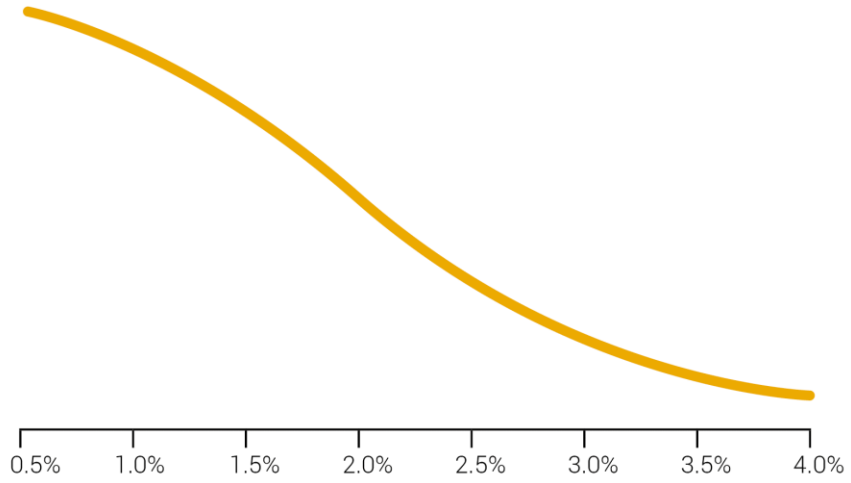
Mortgages:
Largest asset in town?



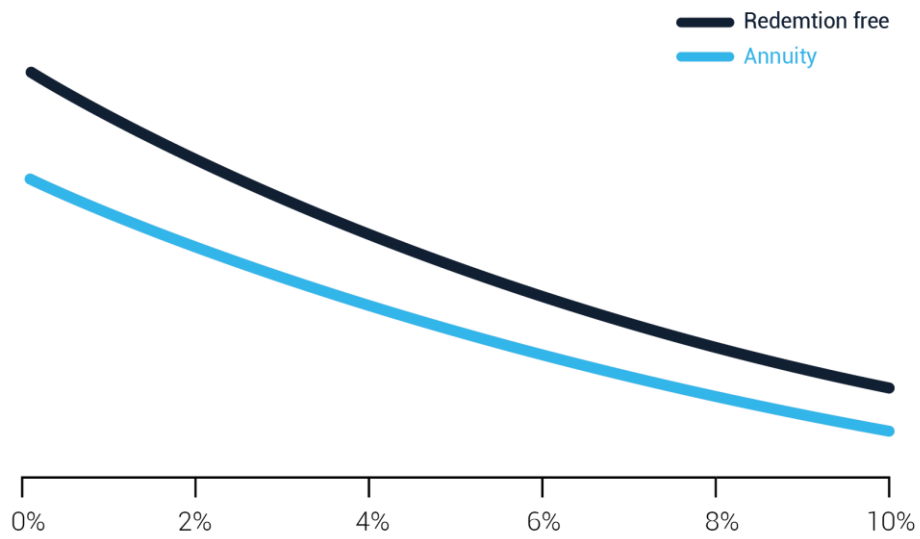
Outstanding retail mortgage loans (2018) in EUR bln. Source: DNB

Client behaviour
causes short convexity

Observed prepayment rates as function of refinance rates

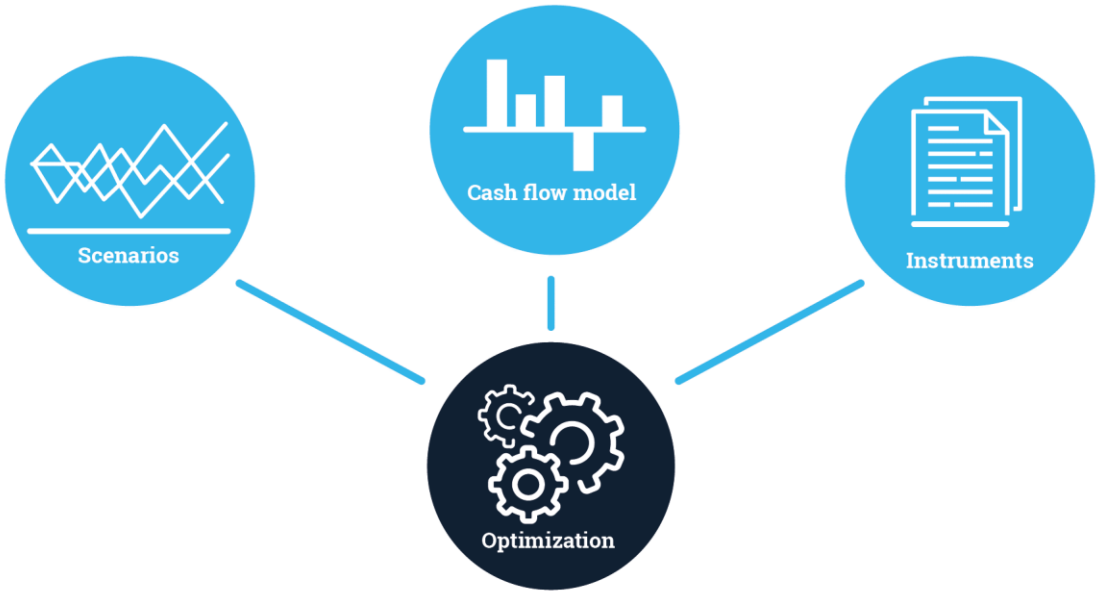


Duration as function of prepayment rates



Daily simulation not feasible in most systems

- Long-dated option
- Large portfolios
- Complex option
- Numerically expensive evaluations



Summary

- Find a portfolio of vanilla/easy-to-evaluate products that have the same characteristics as some underlying portfolio
- Establish replicating portfolio infrequently; revalue the replicating portfolio as often as you want

Process

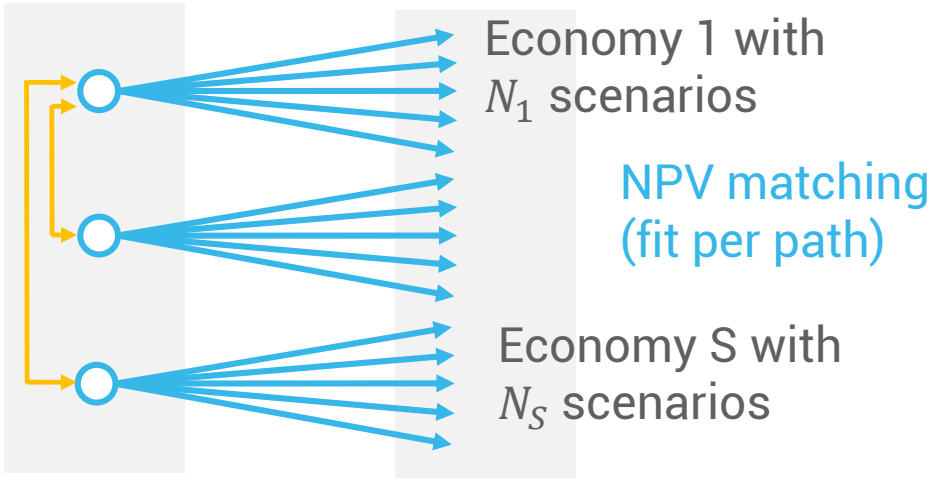
Simulate a cash flow model under various scenarios. Then solve an optimizaton problem to find the weights of a set of replicating instruments

Replicating portfolios provide a solution

Replicating Portfolios: Scenarios

In general

Train and validate RP on wide range of scenarios



Choice of scenarios depends on purpose (capital calculations, hedging, …)

For prepayment

- Can model interest rates, mortgage spreads, housing market activity, … essenstially anything need for a prepayment model

Replicating Portfolios:
Cash flow model

In general

- Any model that can take as input a simulation path generated from a scenario set and that can turn it into a set of cash flows works

For prepayment

- Possible to path-wise subtract cash flows from cash flows with some floored/minimum prepayment rate

Replicating Portfolios:
Replicating instruments

In general

- Candidate assets based on underlying product characteristics (maturity, moneyness, …)
- Allow for position constraints to improve portfolio intuition
- Trade-off between quality-of-fit and simplicity of replicating portfolio

For prepayment

- Instruments chosen, depend on goal:
  - Simplify mortgage portfolio -> mortgage as replicating instrument
  - Capital calculations -> no real limitations
  - Hedging -> tradeable instruments
- For best fit, should reflect that prepayment leads to path-dependent option

Replicating Portfolios: Optimization problem

In general

$$\min_{\boldsymbol{\omega}} \sum_{s} \left\{ \frac{\lambda_P^s}{N_s} \sum_{n=1}^{N_S} \left\| NPV_n^s - \sum_k \omega_k \, I_{n,k}^s \right\|_\alpha + \lambda_G \left\| \sum_n \left(\Delta NPV_n^S - \sum_k \omega_k \Delta I_{n,k}^S \right) \right\|_\alpha \right\}$$

Per path error term **Greek error term**

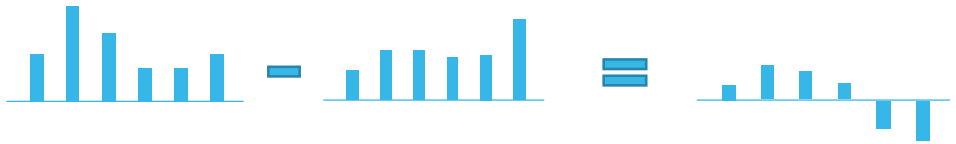
Potentially add:

- constraints on individual instrument weights ω_i
- Trading costs to promote sparsity
- Balance sheet constraints

Replicating Portfolios:
Optimization problem

For prepayment

- **Isolate prepayment option** by subtracting (per path) the cashflows under minimal prepayment from the path's cash flows



Removes matching the discounting risk on the fixed cash flows from the replication and focuses the replication on the prepayment option only!

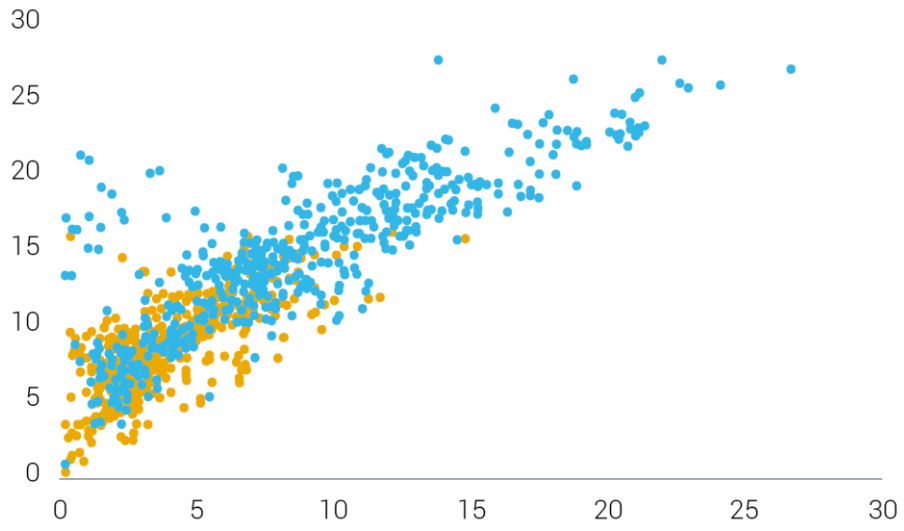
- Instead of one NPV per path, compute **Bucketed NPVs** per path

$$BucketedNPV_n(t_1, t_2) = PV_n(\{CF(s) | t_1 \leq s < t_2\})$$

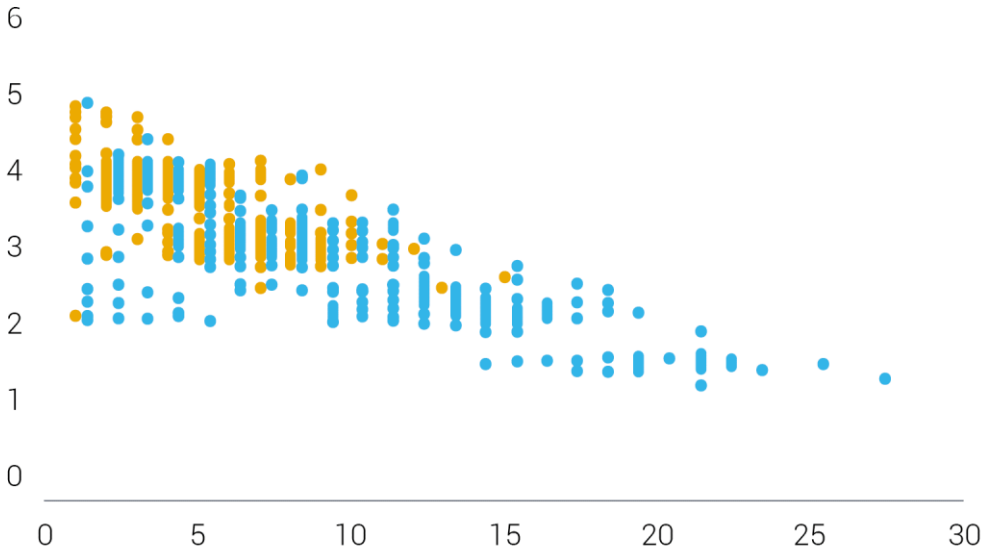
Allows for NPV matching, CF matching or anything in between. Useful for matching accruals!

Example: simulated mortgage portfolio

Generated mortgage portfolio of 1,000 mortgages with a notional of 1,000,000 each



Time to refix vs time to maturity



Mortgage coupon versus time to maturity

Example: prepayment model and scenarios

Prepayment model

- Prepayment rate =
  max{ 0%, min(6%, refinance incentive) }
- Isolate prepayment option, i.e. for each path subtract cash flows with 0% CPR from the generated cash flows

Scenarios

- Three scenarios:
  - Base = starting with curve at end of generative process
  - Down -10 bps
  - Down -50 bps

Example: replicating instruments and optimization problem

Replicating instruments

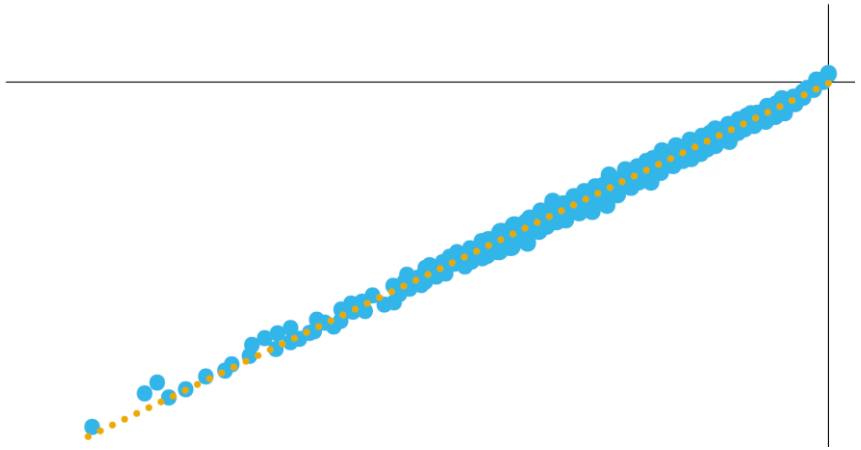
- Zero coupon bonds
- Cash settled swaptions

Optimization problem

- Single NPV bucket (no accrual matching)
- Balance between NPV fitting and sensitivity matching
- Quadratic error function
- Fixed cost per instrument added to promote sparsity/robustness

Example: results

Pathwise fit



Quality of fit metrics

	NPV		Abs Sensitivity				
Scenario	Product	RP	Product	RP	Sensitivity error	R^2	RMSE
Base	-11,650,836	-11,663,643				99.00%	782,120
-10bps	-12,728,659	-12,734,295	-1,077,822	-1,070,652	-0.70%	99.10%	806,186
-50bps	-17,571,965	-17,573,442	-5,921,129	-5,909,799	-0.20%	99.20%	923,102

Note that from statistics one can also make estimates for gamma and 1-in-200 optionality losses

Q&A

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