Replicating portfolio construction for Interest Rate Risk measurement in the Banking Book

Deep dive in stochastic replication of Pipeline Risk

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Business case

Why do we need replicating portfolios?

The aim is to replicate risks in the banking book by a portfolio of instruments with well-defined risk characteristics.

mortgages	savings
loans	debt
Liquid assets	equity

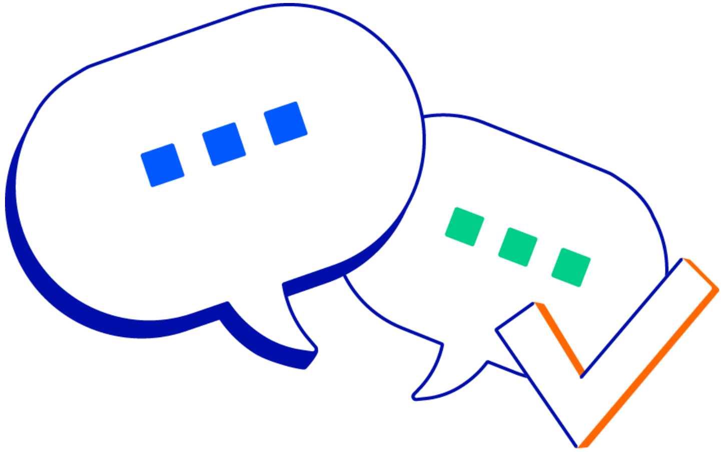
pipeline

mortgages	savings
loans	debt
Liquid assets	equity

Replicating Portfolio

Requirements of the replicating Portfolio

The Replicating Portfolio should have the following properties:



1. It has the **same risk characteristics**. For IRRBB this means BPV, SOT, EVE at Risk, all relevant value measures.
2. It is a **static portfolio** (no dynamic replication).
3. It consists of **well-defined instruments** that are standard to value (no behavioural risk anymore).
4. The replicating portfolio can be used in **production system** (QRM) and for Capital calculations.

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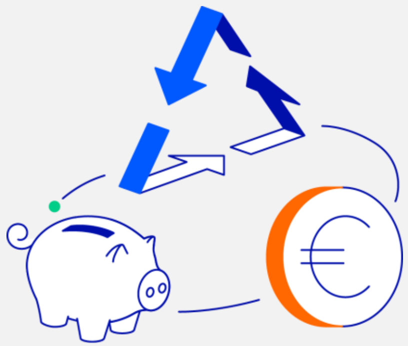
Motivation

Risk Measurement

- Main **modelling** can be outside production system
- Main **computations** can be outside production system



Hedging and Steering

- RP gives easy **insight** to the risks in the portfolio
- RP can inform the **hedging** portfolio

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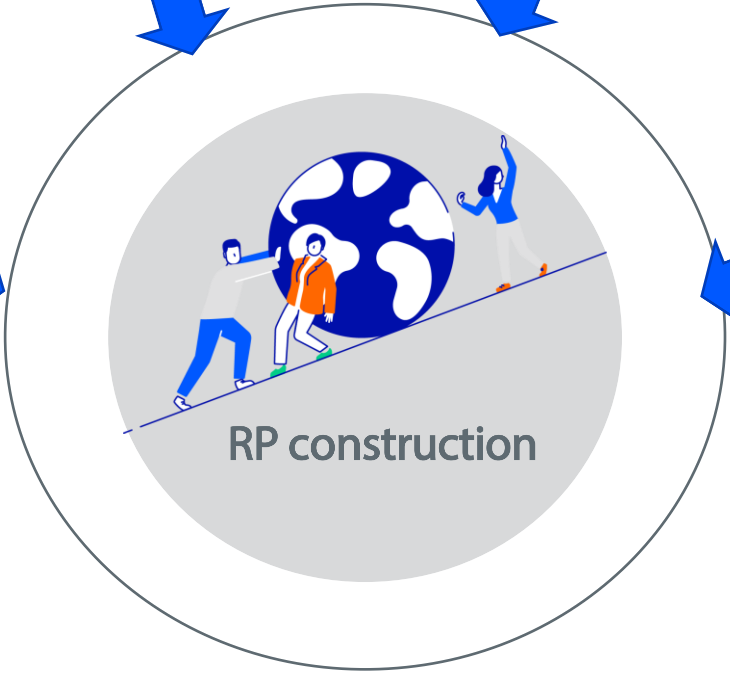
Challenges for Modelling and Users

1. How to determine the acceptance criteria? What is an acceptable level of accuracy loss?
2. What is the set of instruments to be used in the RP?
3. How close is an RP to the optimal RP? What is the cost/benefit of further research in the modelling project?
4. Different stakeholders have different requirements. E.g. Accuracy versus number of instruments.



Pipeline use case

We started with the pipeline option, since:

It is simpler option than e.g. the mortgage prepayment option

Option is short-dated

Risks are well understood and relatively easy to monitor.

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Failed attempts for pipeline

Jamshidian's trick

- Jamshidian has a nice result that allows replication of amortizing swaptions by zero-coupon bond put options
- Is exact in a Hull-White world

✖ Fails for non-parallel shocks (e.g. SOT shocks)

Matching Greeks

- Another approach is to select a number of Greeks to match, e.g. BPV's and/or Vegas.
- This becomes a linear problem and is easy to solve

✖ We did not get it working for large shocks

Pipeline use-case

Introduction in Stochastic replication

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Customer journey and pipeline risk

Definition

Pipeline risk is defined as the fluctuation in the bank's (future) cash flows due to uncertainty about the terms of settlement of offers issued to customers.

Client processes

Pipeline risk may arise from:

(a) new loan applications,

(b) regular repricing of existing loans, and

(c) early repricing of existing loans.

Risk types

The following risk types are identified:

(a) Linear risk: The bank is committed to the offered rate (rate lock) once a client received an official rate offer.

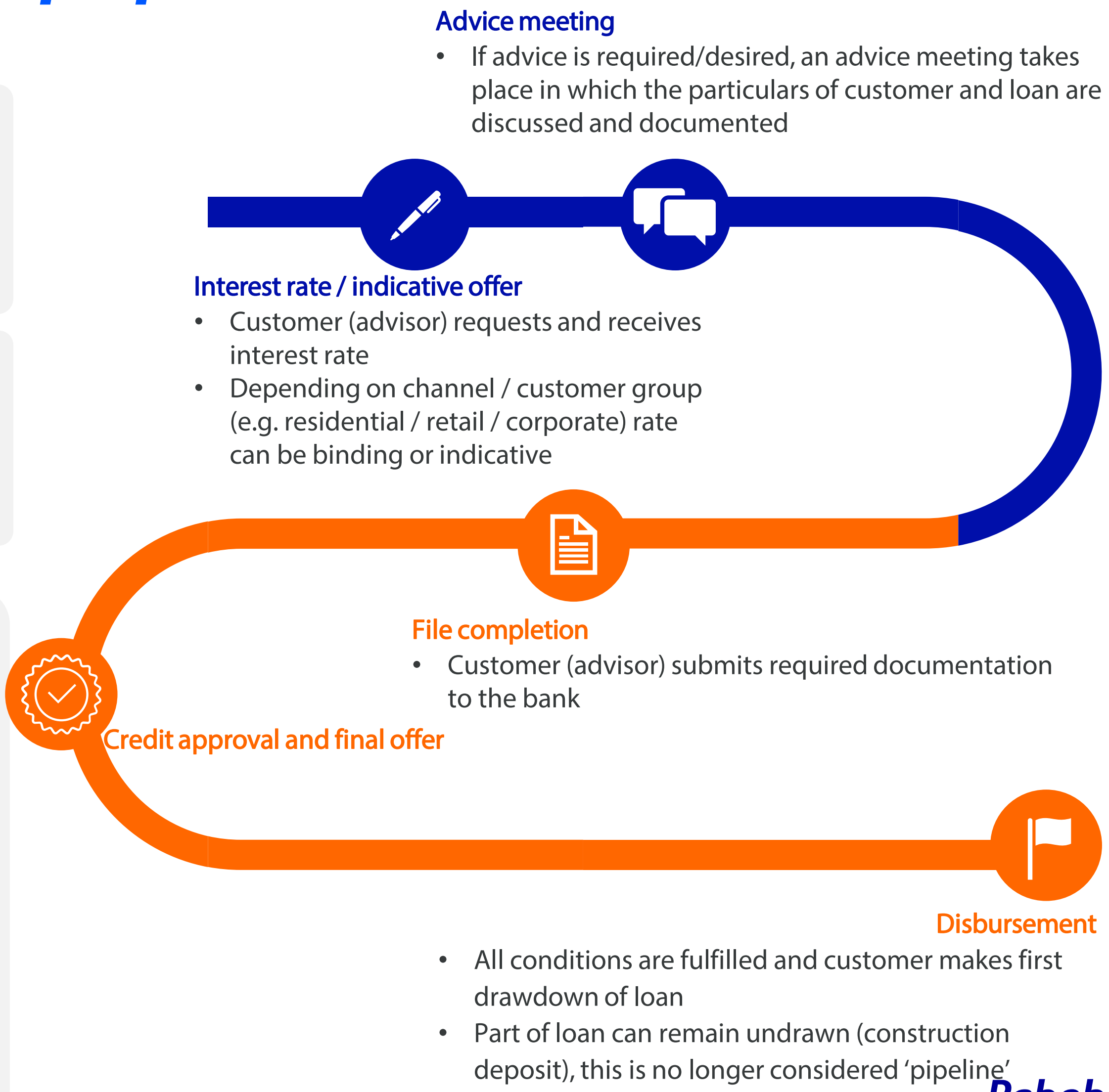
Rate lock

(b) Option risk: in case of options embedded in the offer issued to the customer (e.g. the 'floatdown' option where customer receives lowest of offer and disbursement rate), and

Float down

(c) Behavioural risk: the customer may elect not to take out the offer or have certain choices (e.g. repricing tenors) to make.

Chooser Settlement



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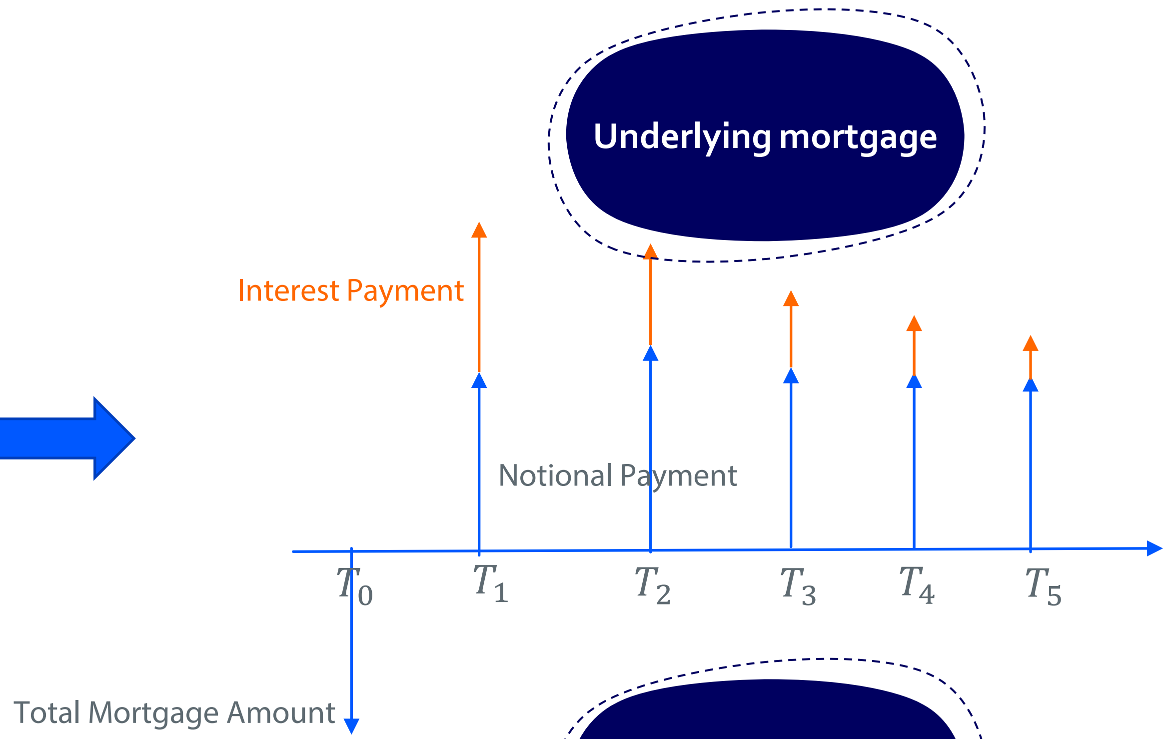
Linear Risk

Replication of rate lock offers

Forward start: T_0 > tomorrow

Cash flow: interest payment + contractual repayment (notional)

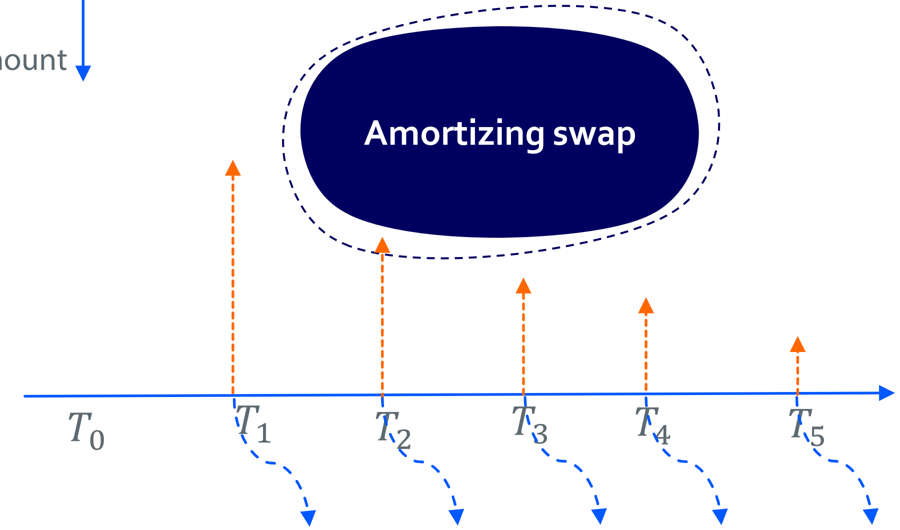


Forward start: T_0 > tomorrow

Cash flow: fixed leg payment + floating leg payment

Different cash flows from the mortgage

Different CF's but at T_0:
PV Mortgage =PV Amortizing swap



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Non-Linear Risk

Replication of float down offers

- Clients' option to pay: Minimum(Rate_offer_date, Rate_T_0)
- Payoff at T_0, for the bank, in addition to a receiver $\text{fwd\_}ASwap$:

$$\boldsymbol{Min\left(0, PV_{T_0}^{M} - PV_{Offer}^{M}\right) = -Max\left(0, PV_{Offer}^{Aswap} - PV_{T_0}^{Aswap}\right)}$$

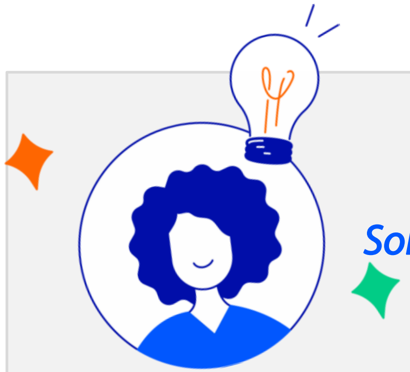
- $PV_{T_0}^{M}, PV_{Offer}^{M}$ the present values of the mortgages at T_0 and on the offer date with their corresponding interest rates
- $PV_{T_0}^{Aswap}, PV_{Offer}^{Aswap}$ the present values of the amortizing swaps at T_0 and on the offer date with their corresponding interest rates

Replication: selling an amortizing receiver swaption with the strike being equal to the interest rate on the offer date

Replicating Portfolio Compression

Problem

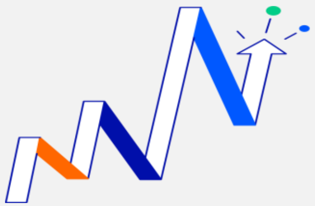
- Large number of instruments
- Computational constraints for valuation under multiple scenarios
- Non-standard market instruments for steering



Solution

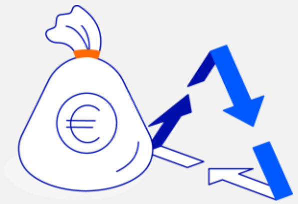
- Via stochastic replication we aim to find a compressed RP consisting of market standard instruments that has the same present value and/or sensitivities.

Stochastic replication in a nutshell



Scenarios

The scenarios that serve as inputs for the optimization problem are stochastic and together generate multiple paths of the evolution of state variables of interest. For the Pipeline use this is the swap rate. The goal here is cover a broad range of the potential payoffs.

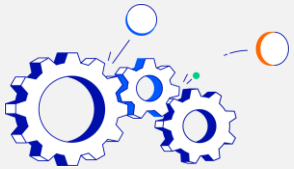


Cash flow calculation

The cash flow model models the underlying product being replicated. For Pipeline, the cash flows are generated on option expiry date assuming physical settlement and using the swap rates calculated under various scenarios to estimate the payoff

Candidate instruments

The replicating instruments define the instruments that are allowed to be part of the replicating portfolio. It should also be possible to generate cash flows for the instruments under the scenario.



Optimization

The optimization problem defines how a good a replicating portfolio is. There are many possibilities on to formulate the optimization problem. Some examples of choices to make are whether to optimize on the NPV of the cash flows or whether to include of sensitivities in the cost function to ensure matching of greeks.

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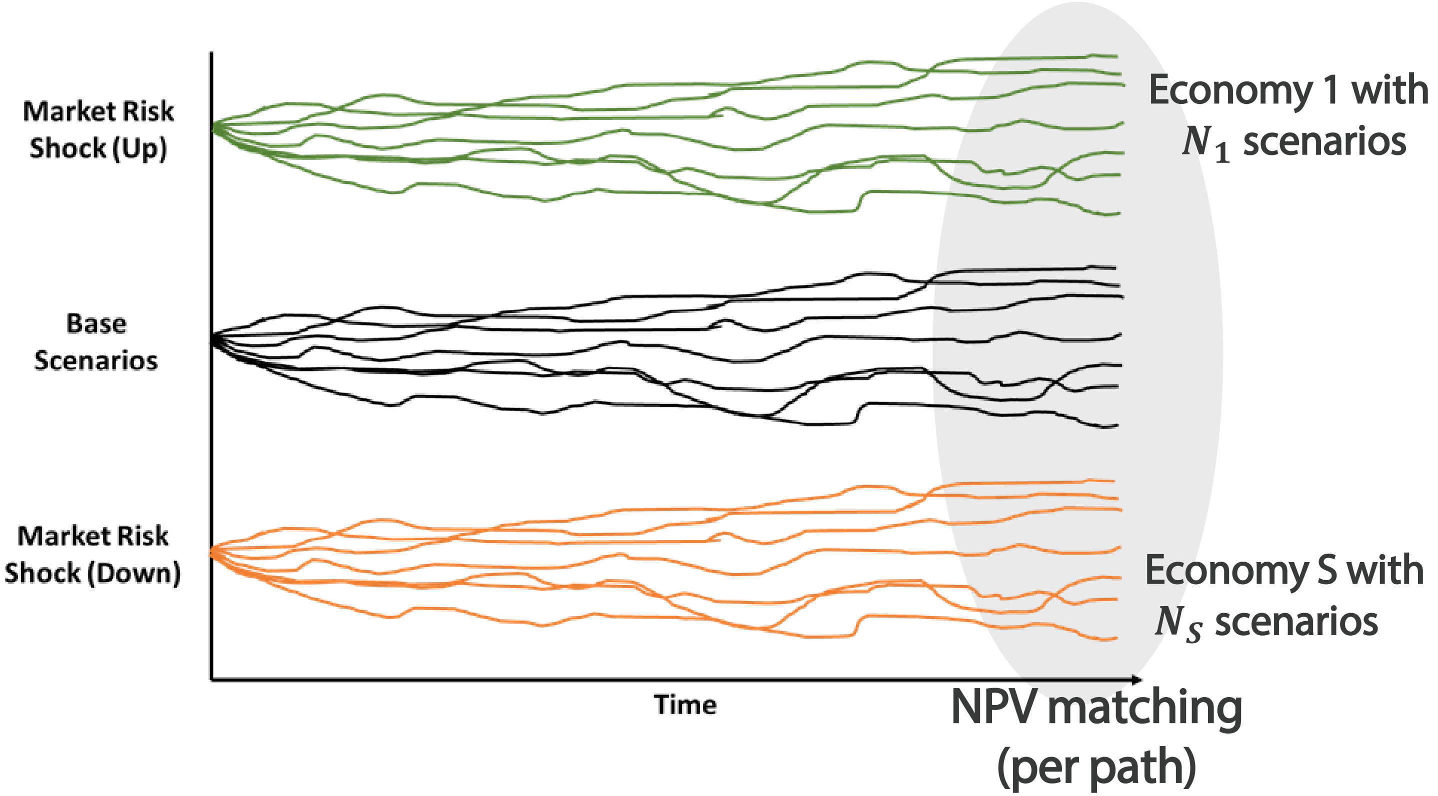
Scenarios

For robust replications, train and validate RP under a wide range of scenarios:

- Base scenarios
- Sensitivities: Parallel Movements, BPV shocks, Principal Components
- Stressed Events: 1-in-x scenarios, higher vols



Training Sets:
Use wide range of scenarios to create robust replication portfolio under various market

Validation Sets:
Check RP quality using out-of-sample sensitivity sets (e.g. test against overfitting).

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Candidate Instruments

Typical **replicating instrument set** consists of vanilla swaps and swaptions.

Select **suitable candidate assets** based on underlying **product's characteristics**:

- Moneyness of underlying embedded options
- Key maturities of the underlying products

Portfolio intuition can be increased by restricting the instrument combinations:

- Adding position constraints per instrument (category), e.g. only long receiver swaptions

Though the optimization routine (by construction) will find the best fit given a set of replicating instruments, **using suitable candidate assets and constraints** will largely increase:

- Portfolio intuition.
- Computational efficiency.

Optimization: Quadratic Program

Given stochastic cash flows, the optimal Replicating Portfolio obtained via QP as:

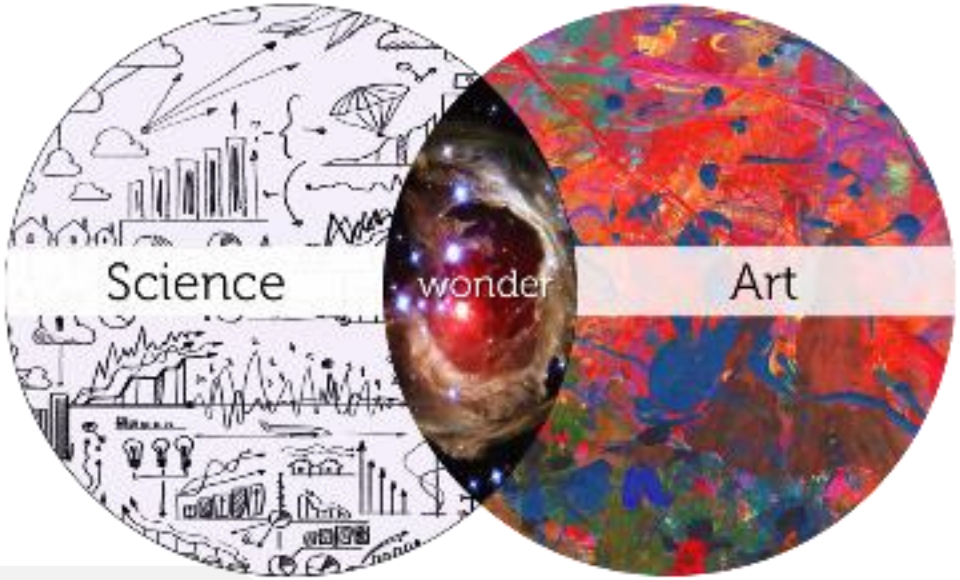
$$\min_{\mathbf{w}} \sum_{s=1}^{S} \left\{ \lambda_P^s \cdot \underbrace{\frac{1}{N_s} \sum_{n=1}^{N_s} \left(NPV_n^s - \sum_{k=1}^{K} w_k \cdot a_{n,k}^s \right)^2}_{\text{Average Squared (path) diff}} + \lambda_G^s \cdot \underbrace{\sum_{n=1}^{N_m} \left(\Delta NPV_n^s - \sum_{k=1}^{K} w_k \cdot \Delta a_{n,k}^s \right)^2}_{\text{Average Squared sensitivity diff}} \right\}$$

Path penalty (λ_P^s), Greek penalty (λ_G^s)

Can be solved directly using Quadratic programming techniques:

- Quadratic programs generally harder to solve than linear programs.
- The squared deviation is commonly used in statistics as part of least squares estimations of econometric models.
- Advantage is that no additional constraints are required per scenarios (compared to the LP formulation) and the problem size hence only depends on the number of instruments.

Stochastic Replication: As much Science as Art



The Science (Toolkit)

- *Create a robust, generic platform for stochastic replication.*
- *Allow for a rich optimization toolkit*
- Perform out-of-sample testing of replicating portfolio

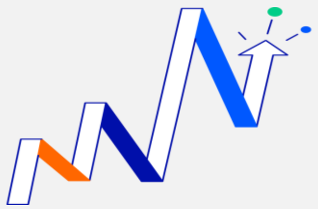
The Art (Business Knowledge)

- Train portfolio using wide range of scenarios
- Proper selection of instruments
- Use appropriate optimization settings, e.g.:
  - Position constraints
  - Increase intuition and prevent overfitting

Balance quality of fit versus portfolio intuition

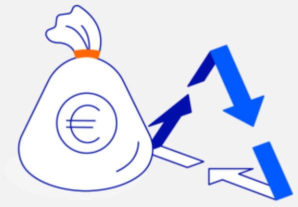
Case study

Replication of 10 amortizing swaptions into vanilla swaptions



Scenarios

- 1-factor Hull-White with $\sigma = 0.02, \alpha = 0.1$
- Economies: base curve + SOT curves
- 500 paths per economy



Cash flow calculation

- Physical settled swaptions
- Cash flow appears on the option expiry date and calculated as

$$CF = PV\ NotionalPayments * Payoff$$

where,

$$Payoff = Strike - SwapRate$$

Candidate instruments

- Vanilla swaptions
- Option expiry for each week of the first 3 months
- Tenor: 7Y, 8Y, 10Y, 12Y, 15Y, 20Y, 25Y
- Strikes falling in the range of strikes of AS: [0.028, 0.031]
- Allow only long positions in optimization

Optimization

- Optimize only on NPVs and without assigning weight per economy

$$\min_{\mathbf{w}} \sum_{s=1}^{S} \left\{ \frac{1}{N_s} \sum_{n=1}^{N_s} \left(NPV_n^s - \sum_{k=1}^{K} w_k \cdot a_{n,k}^s \right)^2 \right\}$$

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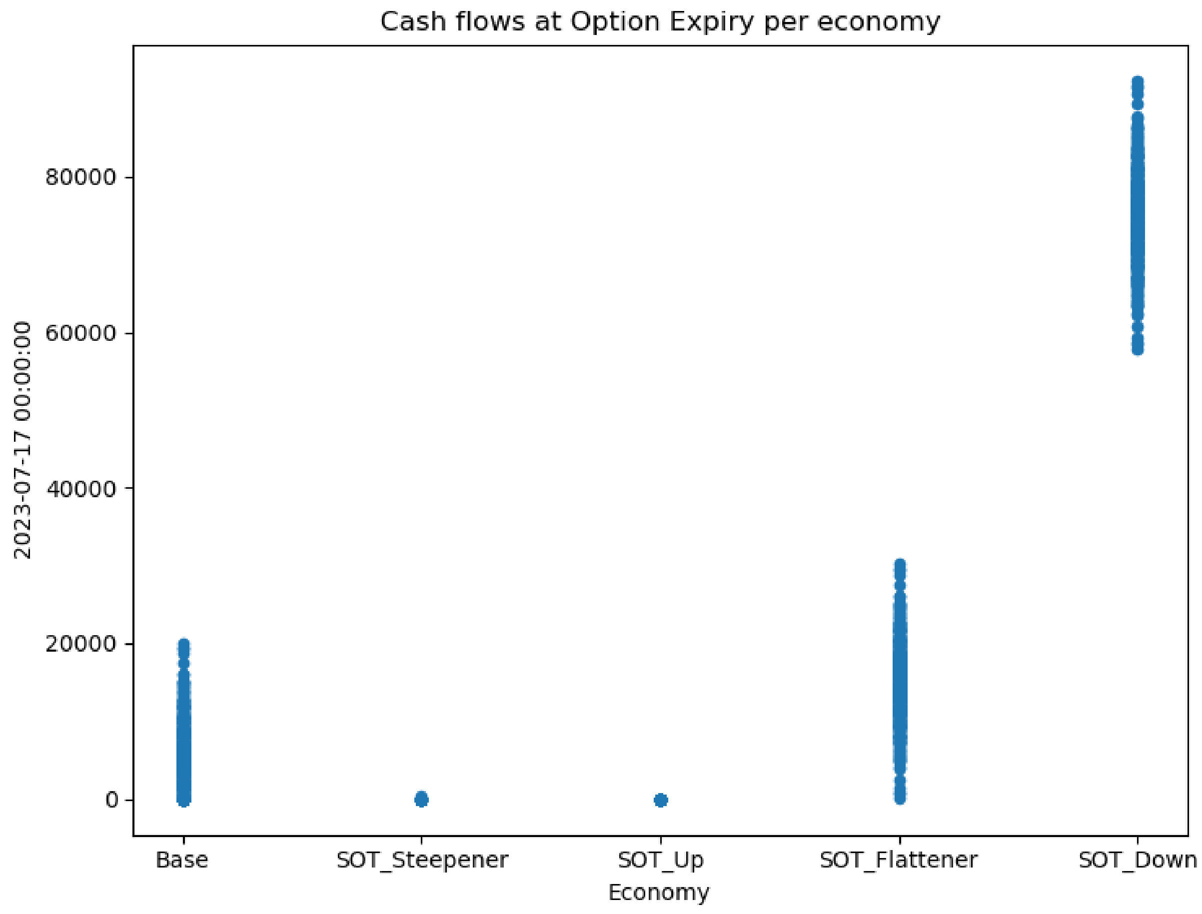
Cash flows example

- In this example, the offer start dates equal 30/6/2023 and the option expiry 17/7/2023.
- For paths where the simulated swap rate is smaller than the strike rate, payoff =0. Therefore, the option is out-of-the-money and the cash flows equal to 0.
- For SOT Up scenario, the base scenario is shocked with 200bps Up and therefore almost all paths end out-of-the-money.
- For SOT Steepener, the short end of the curve is shocked upwards, therefore again most paths end out-of-the-money.



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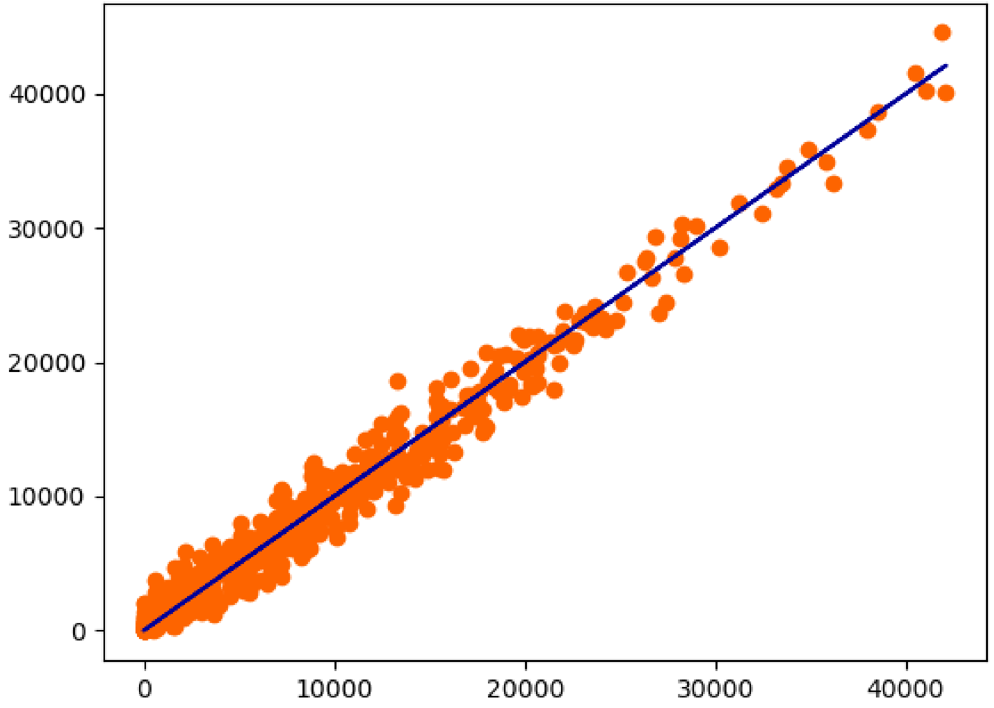
Example bad behaviour (1/2)

When we use only base scenario in the optimization

- To assess the performance of the RP:
  - We can use statistical measures like R2, adj R2, etc.
  - We can value both RPs analytically using the Black model and compare their values. This can be seen as an additional out-of-sample test.
- When only base scenario is used in the optimization, we have high R2 (97%) but low performance for extreme scenarios

	Black PV AS RP	Black PV VS RP	Rel diff
Base	-10,907	-11,009	1%
1bp Up scenario	-10,492	-10,568	1%
1bp Down scenario	-11,334	-11,463	1%
SOT Up scenario	-16	-7	-55%
SOT Down scenario	-175,602	-206,726	18%
Steepener	-587	-452	-23%
Flattener	-34,464	-39,610	15%



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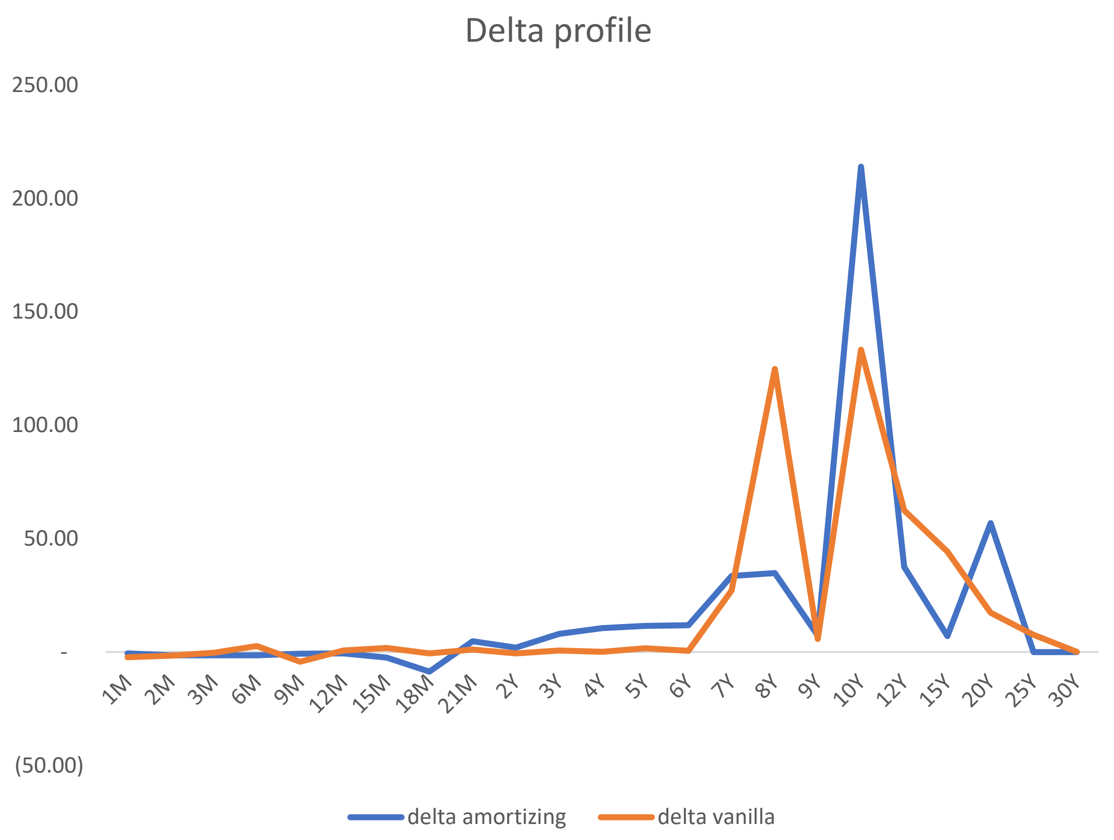
Example bad behaviour (2/2)

Accurate value matching vs bad delta profile

- The objective function includes only matching on NPVs and not matching on sensitivities.
- If we include more extreme scenarios in the optimization (SOT Up and Down, Steepener, Flattener), we observe that the Black values match accurately, but not the delta profile.

	Black PV AS RP	Black PV VS RP	Rel diff
Base	-10,907	-10,604	-3%
1bp Up scenario	-10,492	-10,188	-3%
1bp Down scenario	-11,334	-11,031	-3%
SOT Up scenario	-16	-8	-47%
SOT Down scenario	-175,602	-175,507	0%
Steepener	-587	-551	-6%
Flattener	-34,464	-34,256	1%



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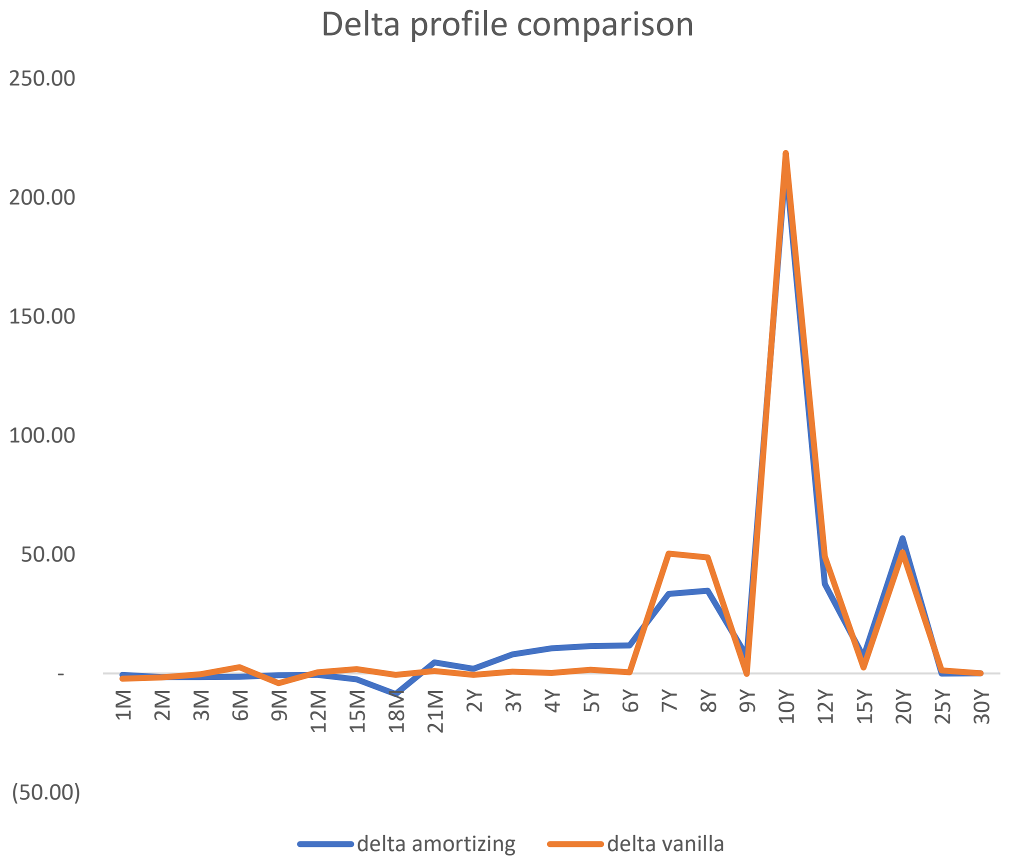
Successful replication example

Accurate value matching and accurate delta profile by shocking individual tenors in the optimization

- We enrich the input economies by adding scenarios, where individual tenor points of the base curve are shocked
- This is an alternative way to produce accurate delta profile instead of changing the objective function

	Black PV AS RP	Black PV VS RP	Rel diff
Base	-10,907	-10,595	-3%
1bp Up scenario	-10,492	-10,180	-3%
1bp Down scenario	-11,334	-11,021	-3%
SOT Up scenario	-16	- 9	-46%
SOT Down scenario	-175,602	-175,557	0%
Steepener	-587	-557	-5%
Flattener	-34,464	-34,425	0%



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Thank you for your attention

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