The impact of stochastic volatility on IM and MVA in a post LIBOR world

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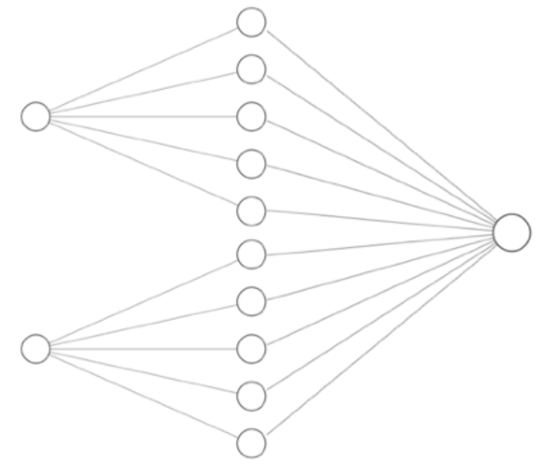
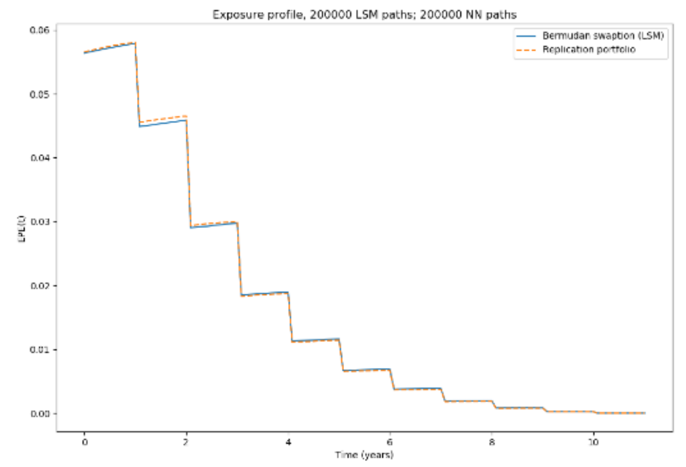
ING

PhD project

Numerical methods

1. **Semi-static replication for IR derivatives**
   - Decomposing callable IR derivatives into a portfolio of vanilla IR options.
   - *Article under review*





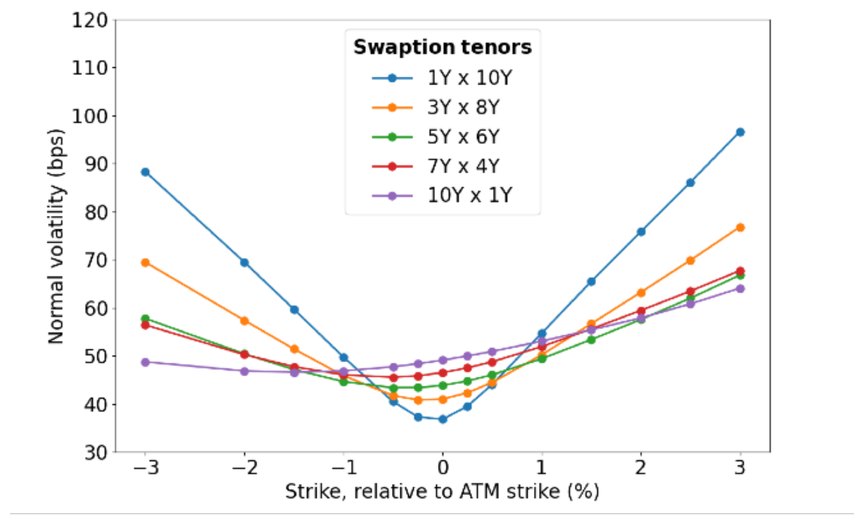
2. **Fully static replication for forward sensitivity computation**
   - Efficient prices and sensitivities along the MC path.
   - Application to CVA, IM and MVA
   - *Article in progress*

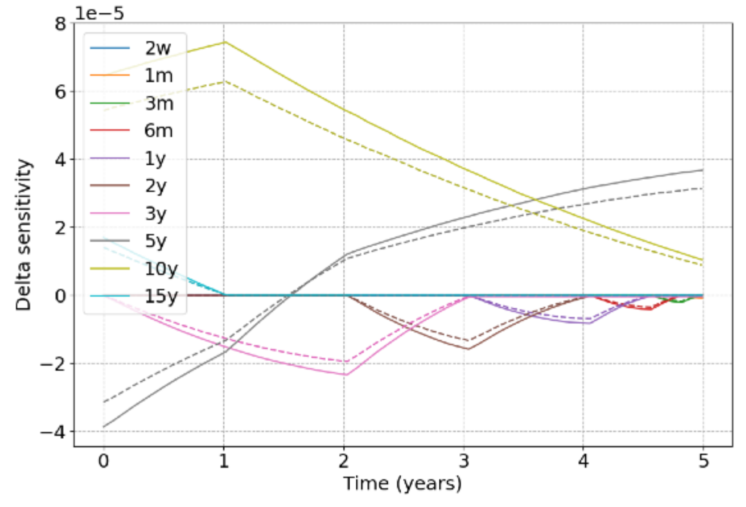
A modern approach to interest rate risk management

Model risk

Impact of volatility skew modelling on MVA

- Closed-form estimates of prices and sensitivities along the MC path for vanilla IR products.
- Impact analysis on real market data.

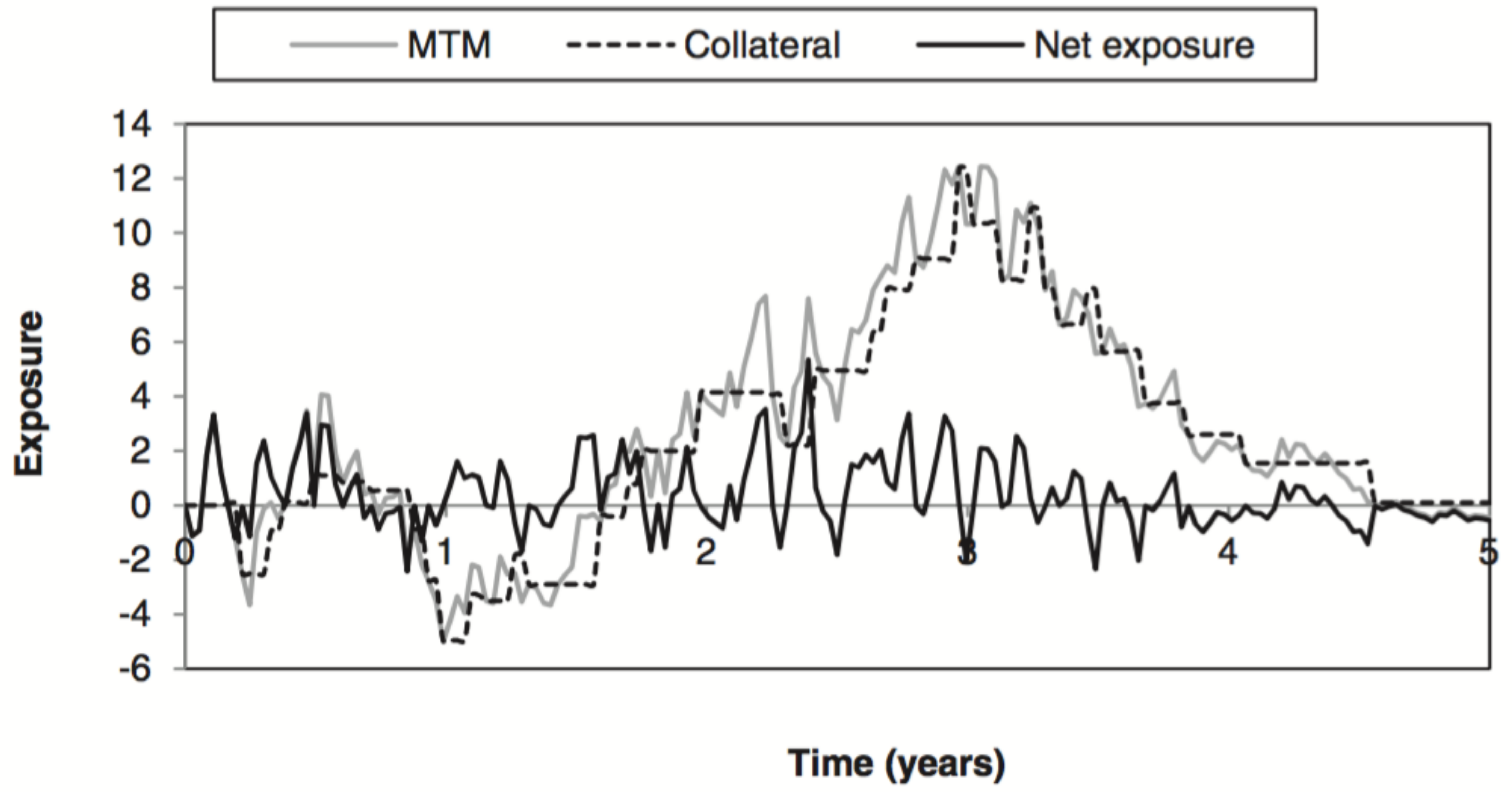




J. H. Hoencamp, J. P. de Kort & B. D. Kandhai (2022)
The Impact of Stochastic Volatility on Initial Margin and MVA for Interest Rate Derivatives,
Applied Mathematical Finance, 29:2, 141-179,
DOI: 10.1080/1350486X.2022.2156900

Collateral: Variation Margin

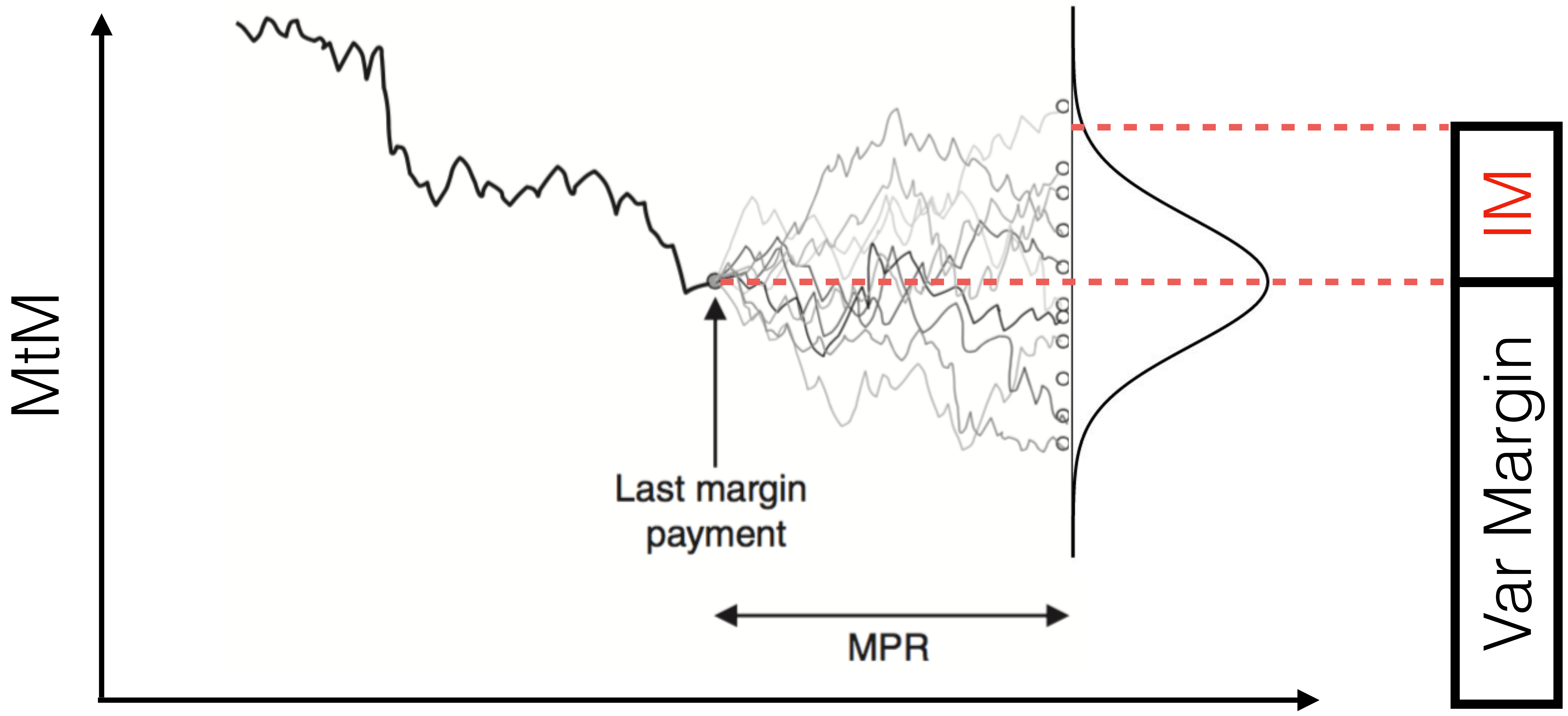
- Reduces exposure (EPE) en thus risk charges (CVA).
- Reflects the MtM of the trade.



Source: J. Gregory, *The XVA Challenge: Counterparty Risk, Funding, Collateral, Capital and Initial Margin*, John Wiley & Sons, 2020

Collateral: Initial Margin

- Cover against market changes after default, when outstanding positions are not yet settled, but variation margin (collateral) is no longer updated
- Reflects the 99% VaR on a 10-day time window



Source: J. Gregory, *The XVA Challenge: Counterparty Risk, Funding, Collateral, Capital and Initial Margin*, John Wiley & Sons, 2020

SIMM: standard initial margin model

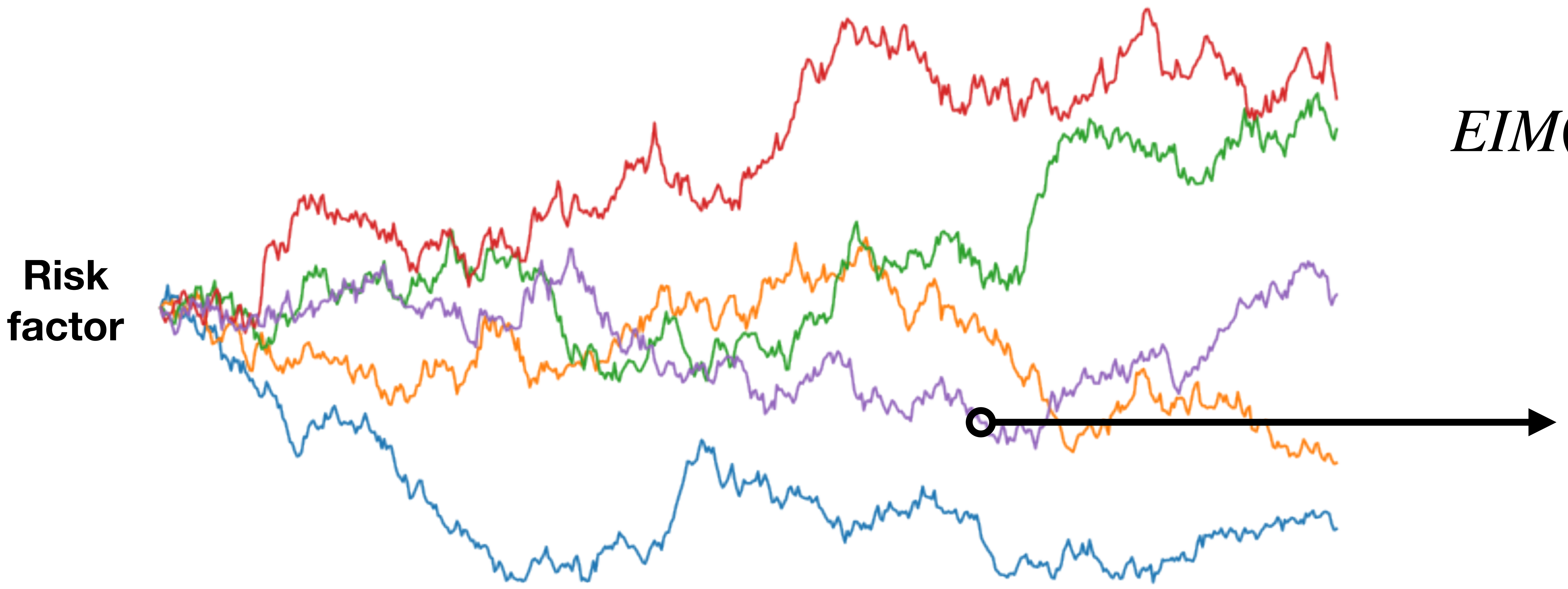
- $\text{IM}(t) = \sum_{g \in \mathcal{G}} \text{Margin}_g(t), \qquad \mathcal{G} = \{\text{Delta, Vega, Curvature}\}$
- $\text{Margin}_g(t) = \text{VaR}^{10d}_{99\%}(t) \approx \Phi^{-1}\left(99\ \%\right) \times \text{std}_{10d}\left(V_t\right)$
- $\text{std}_{10d}\left(V_t\right) \approx \sqrt{(\partial \mathbf{V})^\top \boldsymbol{\Sigma}\ \partial \mathbf{V}}$
    - $\partial \mathbf{V}_t = \left[\frac{\partial V(t)}{\partial \theta_1(t)} \quad \cdots \quad \frac{\partial V(t)}{\partial \theta_k(t)}\right]$
    - $\boldsymbol{\Sigma}$: covariance matrix of underlying risk-factors $\left\{\theta_1, \ldots, \theta_k\right\}$

MVA: margin valuation adjustment

- MVA is the expected lifetime cost of posting IM.

$$MVA \approx \sum_{i=1}^{N} FS(t_i) \times EIM(t_i) \times \Delta t_i$$

$$EIM(t) = \mathbb{E}^{\mathbb{Q}}\left[D(0,t) IM(t)\right]$$

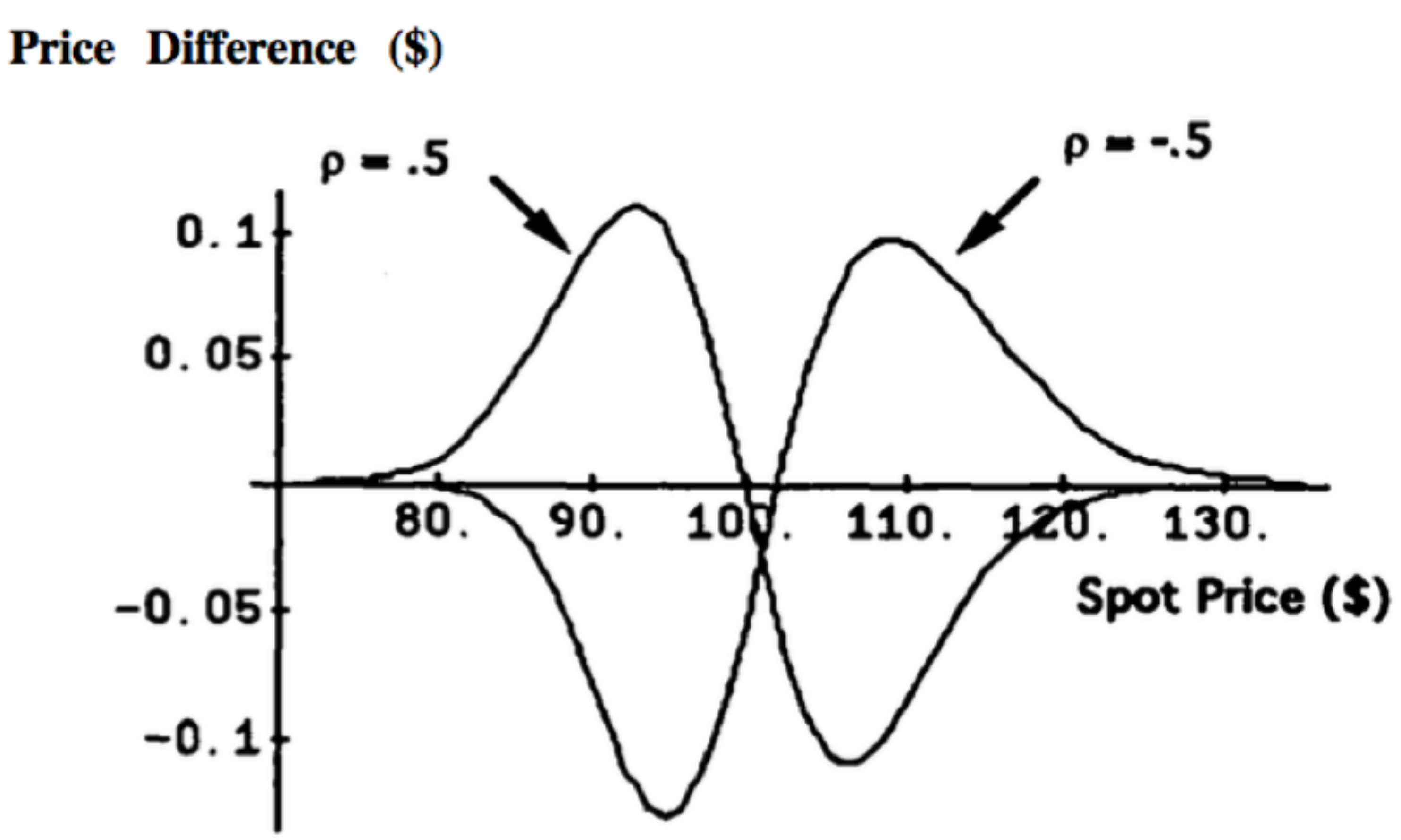


$$\left\{ \begin{array}{l} \dfrac{\partial V(t)}{\partial R_t} \\ \dfrac{\partial V(t)}{\partial \sigma_t} \end{array} \right.$$

Research question 1

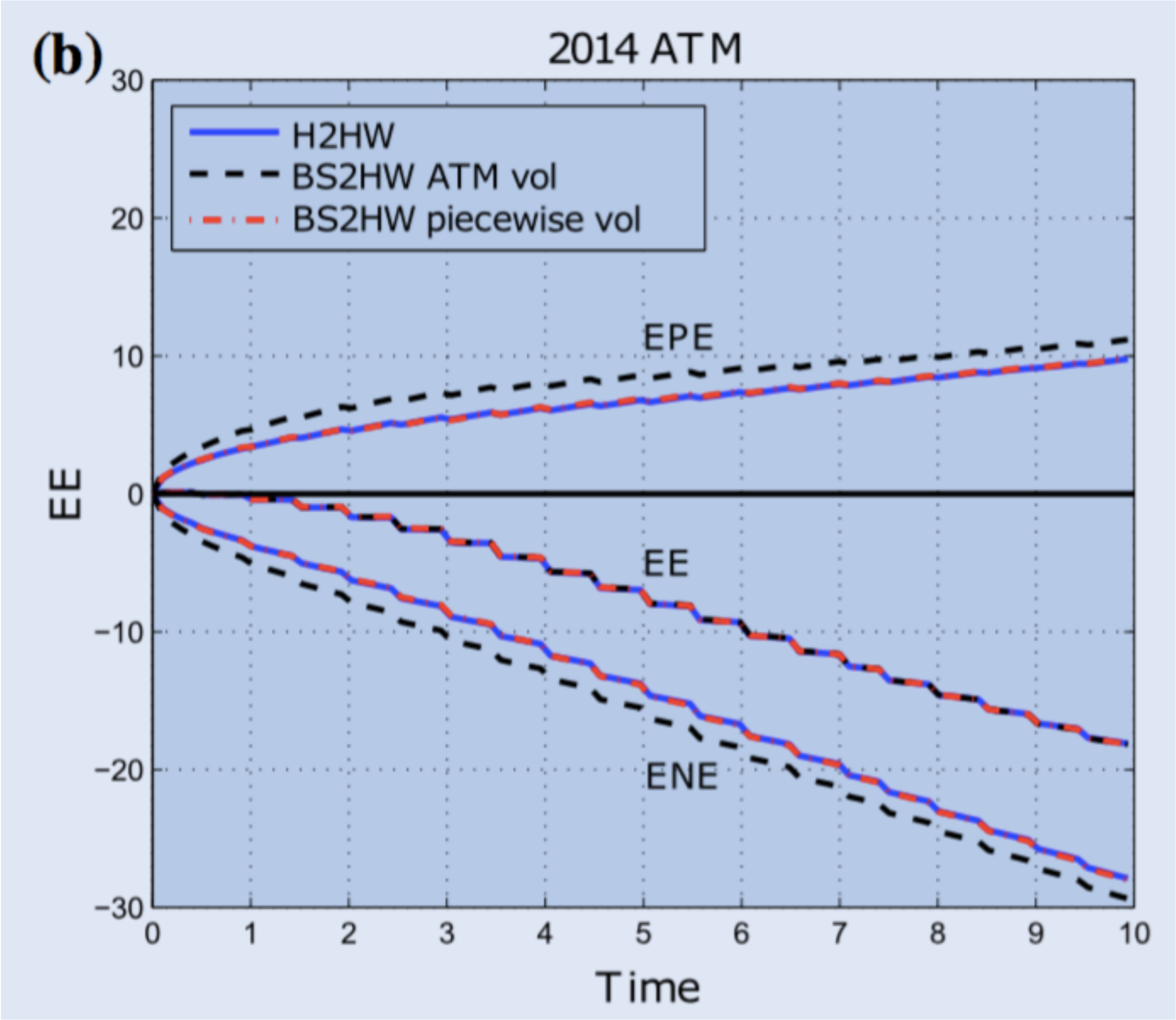
- How does stochastic volatility impact the computation and volume of IM and MVA?

Option prices:



Source: S. L. Heston, *A closed-form solution for options with stochastic volatility with applications to bond and currency options*, The review of financial studies, 1993

Exposure profiles:



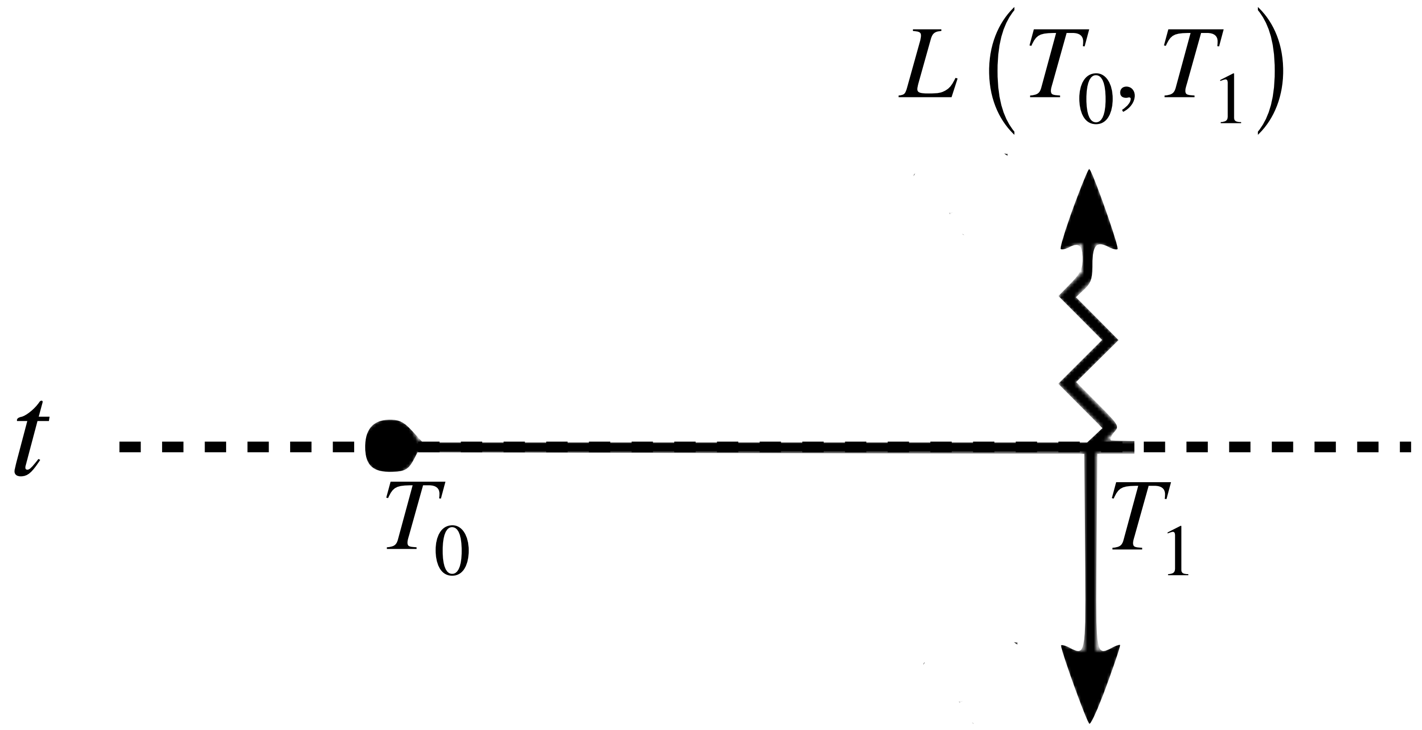
Source: S. Simaitis, C. de Graaf, N. Hari, and D. Kandhai, *Smile and default: the role of stochastic volatility and interest rates in counterparty credit risk*, Quantitative Finance, 2016

IM and MVA?

Research question 2

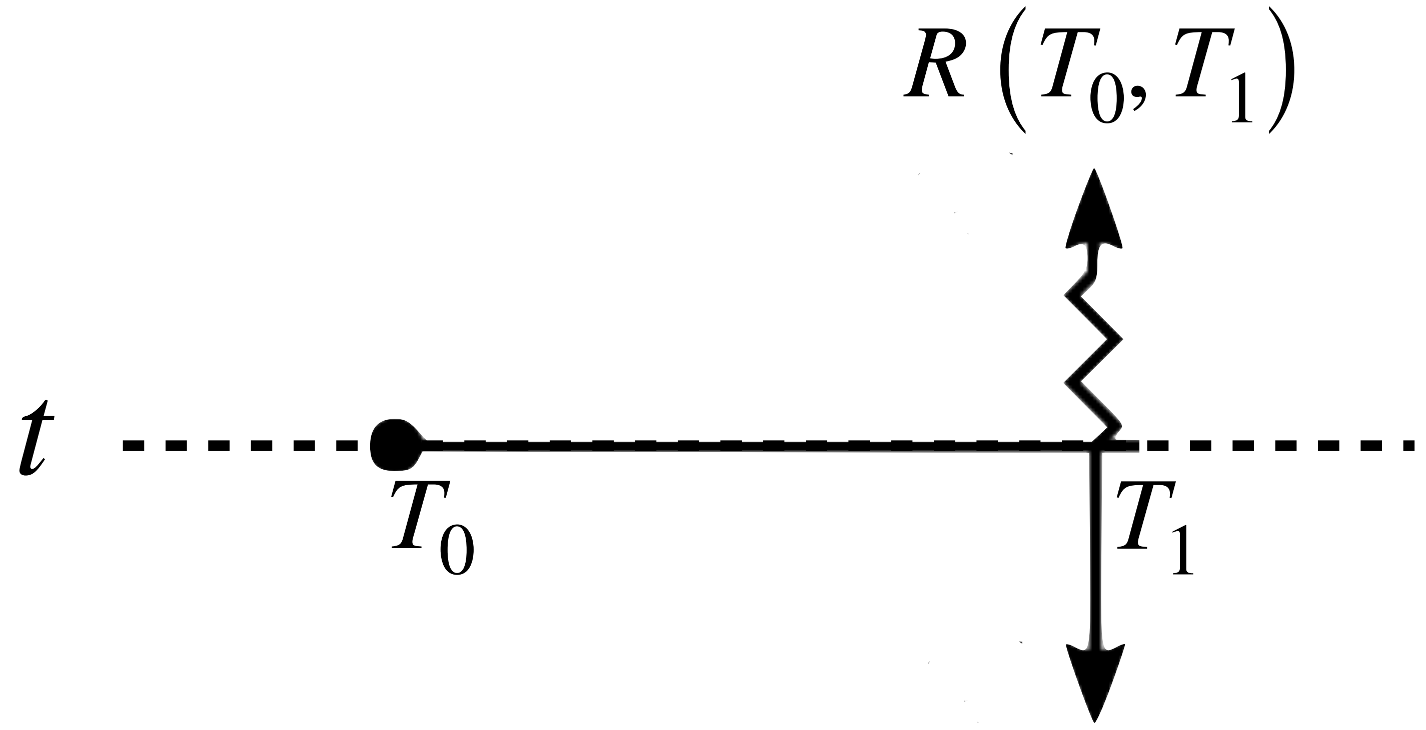
- What is the impact of incorporating the new risk-free rate (RFR) benchmark?

Forward-looking term rate



$$L\left(T_0, T_1\right) := \frac{1}{T_1 - T_0}\left(\frac{1}{P(T_0, T_1)} - 1\right)$$

Backward-looking term rate



$$R\left(T_0, T_1\right) := \frac{1}{T_1 - T_0}\left(\frac{1}{e^{\int_{T_0}^{T_1} r(u)du}} - 1\right)$$

Basic set-up

- We consider a continuous-time financial market.
- We consider an instantaneous risk-free rate r_t, which is used for discounting *and* indexing IR derivatives.
- We consider a money-market account associated to r_t, which compounds continuous interest, $M_t := e^{\int_0^t r_u du}$.
- We assume there exists a risk-neutral measure $\mathbb{Q}$ associated to the numeraire M_t.

Model framework

- Cheyette model, within HJM class, with stochastic volatility:

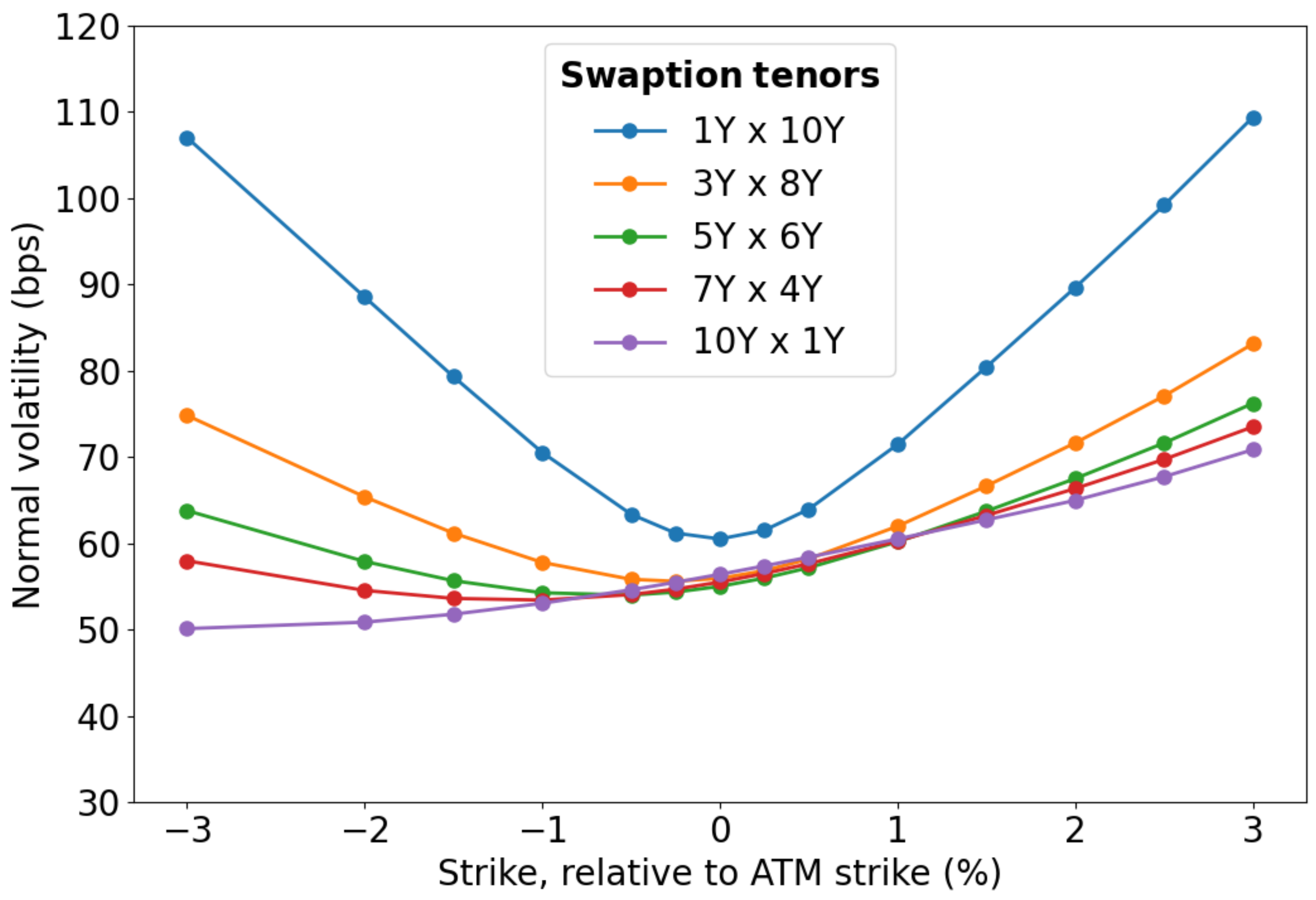
$$r_t := f(t,t) = x_t + f(0,t)$$
$$dx_t = \left(\phi_t - a x_t\right) dt + \sqrt{v_t} dW_t \qquad x_0 = 0$$
$$d\phi_t = \left(v_t - 2a\phi_t\right) dt \qquad \phi_0 = 0$$
$$dv_t = \kappa\left(\bar{v} - v_t\right) dt + \varepsilon \sqrt{v_t} dZ_t \qquad v_0 = v$$
$$d\left\langle W, Z \right\rangle_t = \rho dt$$

- Constant volatility benchmark: *Hull-White model* $\left(v_0 := \sigma^2,\ \kappa = \varepsilon := 0\right)$

- A zero-coupon bond price $P(t,T)$, for $t < T$ is given by

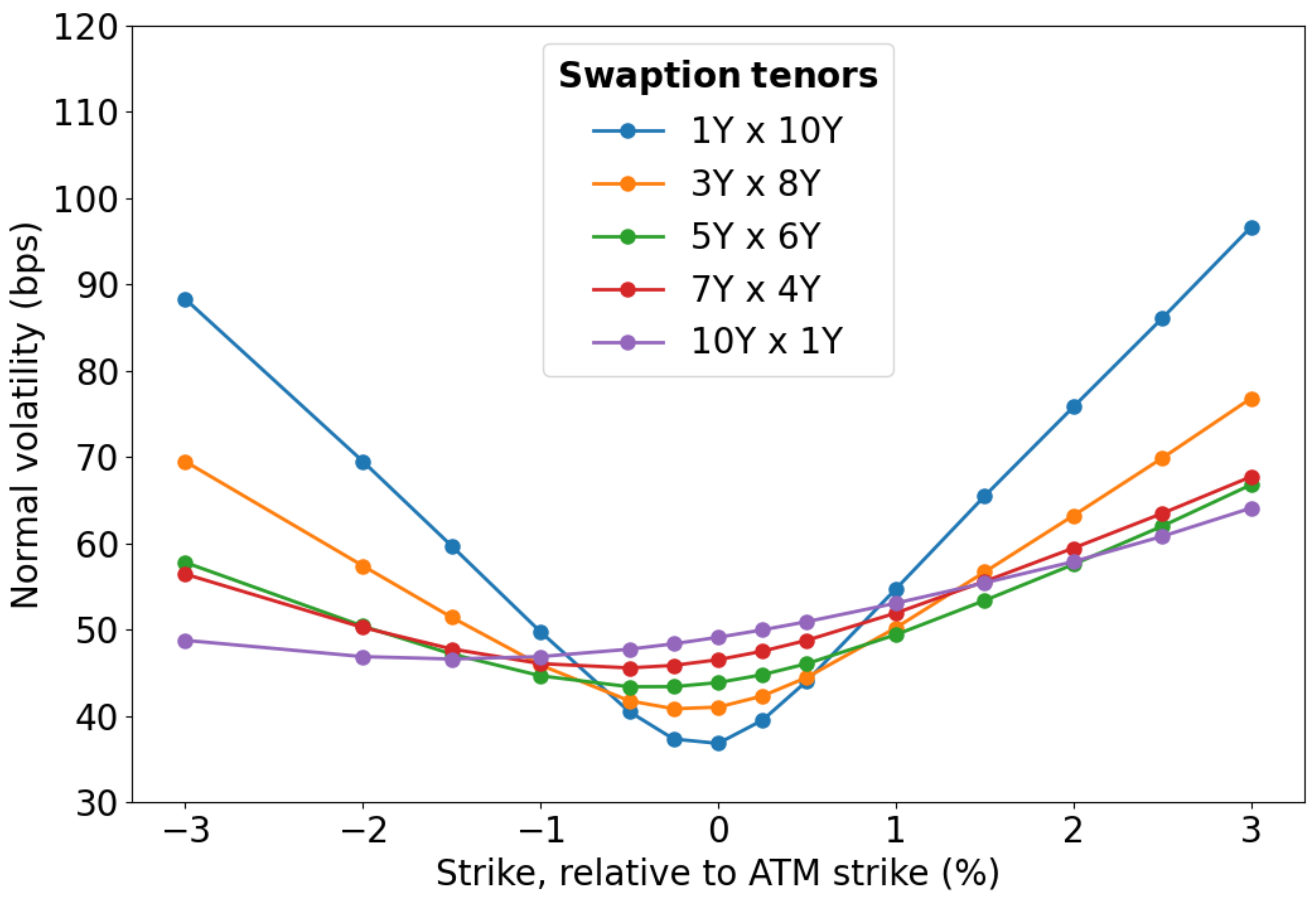
$$P(t,T) = \frac{P(0,T)}{P(0,t)} \exp\left\{ -\frac{1}{2} B^2(t,T)\phi_t - B(t,T)x_t \right\}$$

Market smiles



March 31, 2020

Peak of the Covid-19 crisis —> "stressed" market

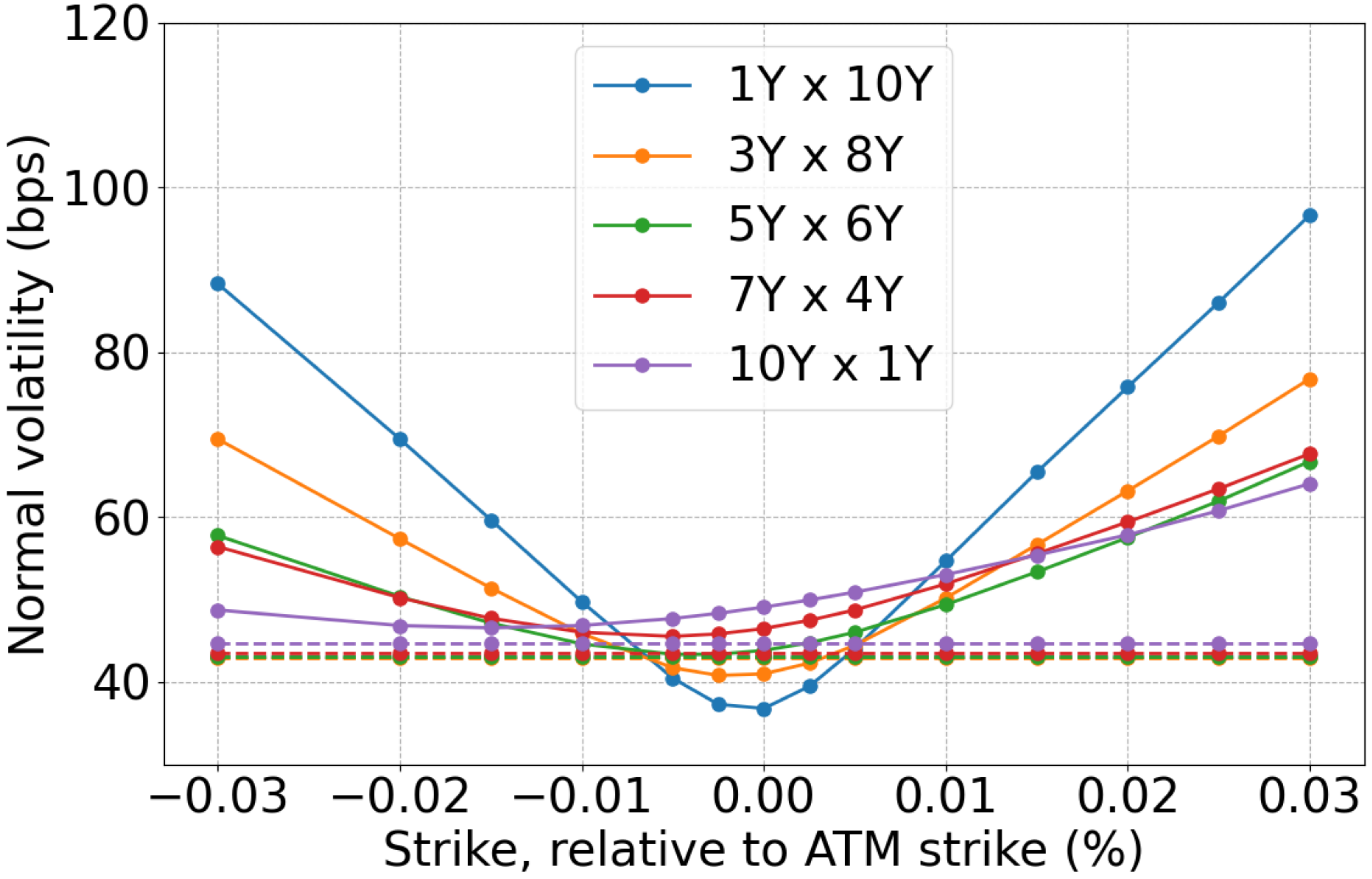


December 31, 2020

"relaxed" market

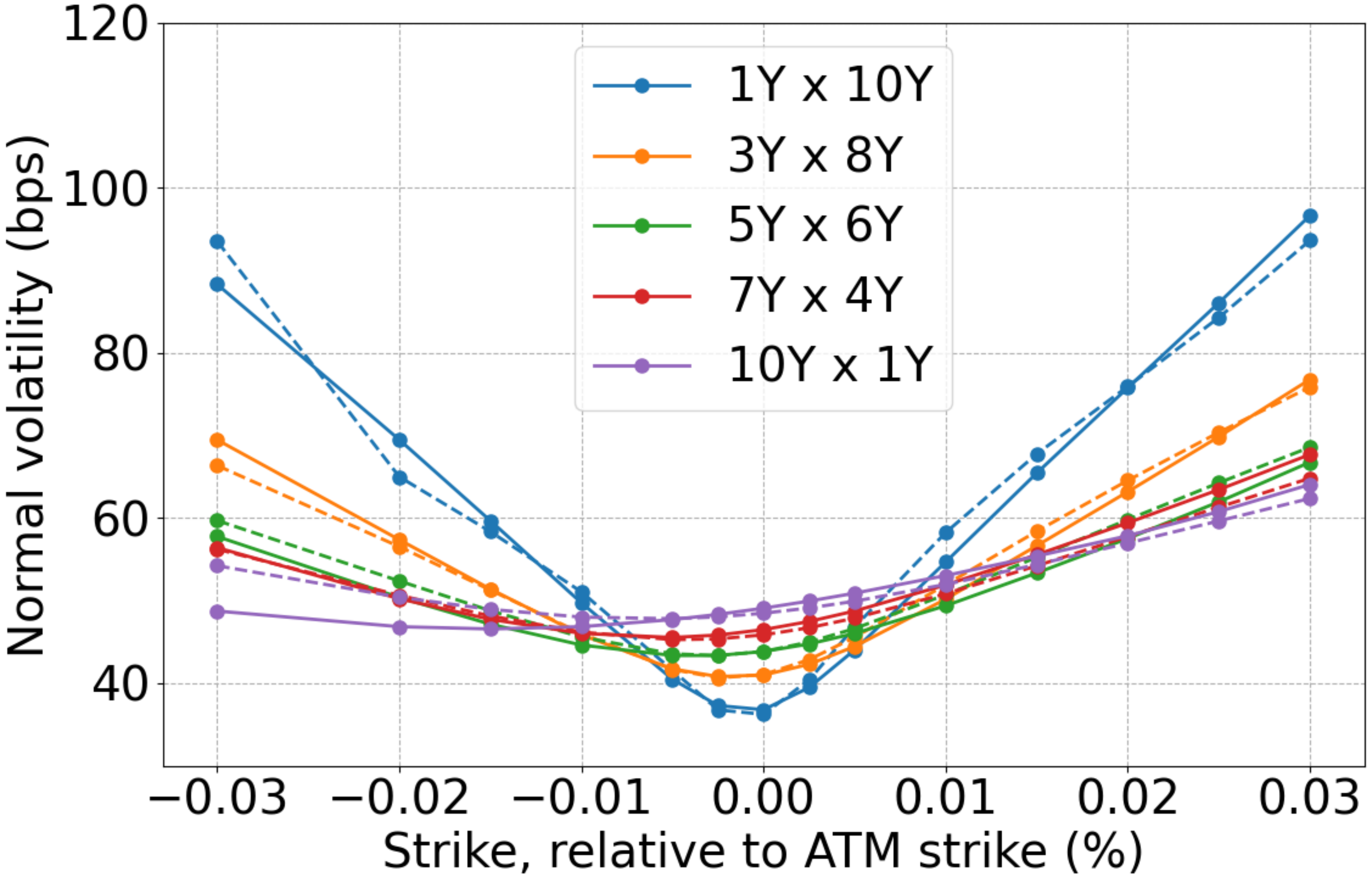
Model calibration

Constant volatility: — market - - model



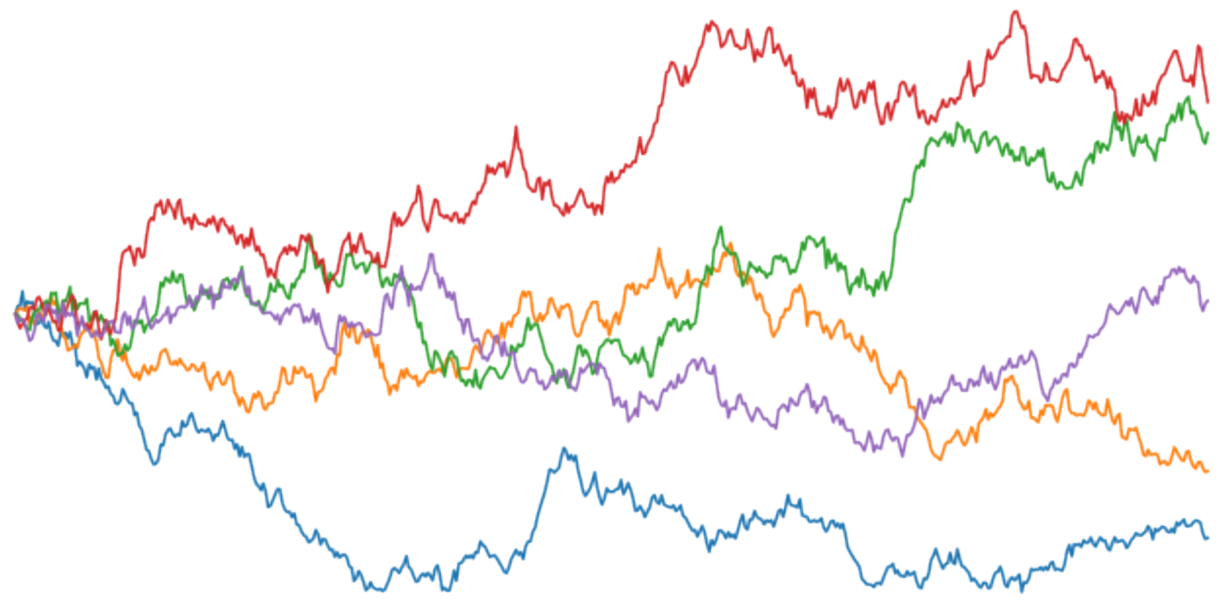
$$\sigma = 0.006690$$

Stochastic volatility: — market - - model

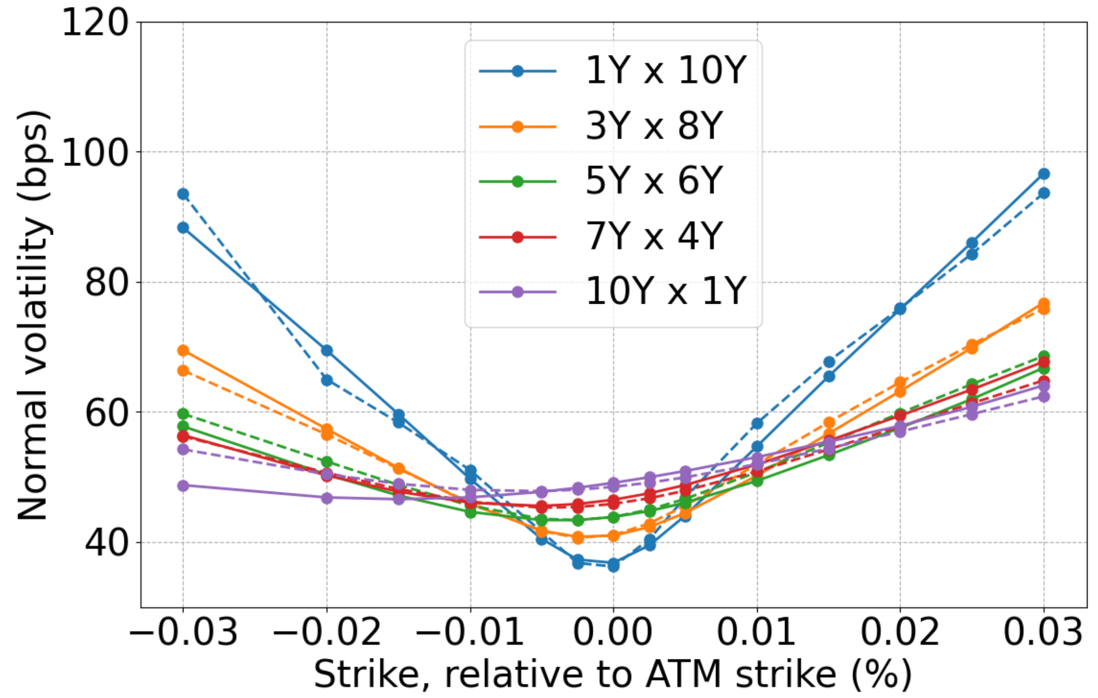


$$\kappa = 0.9952, \quad \bar{v} = 6.4443 \times 10^{-5}, \quad \epsilon = 2.3687 \times 10^{-2}$$
$$v_0 = 3.2292 \times 10^{-5}, \quad \rho = 0.1714$$

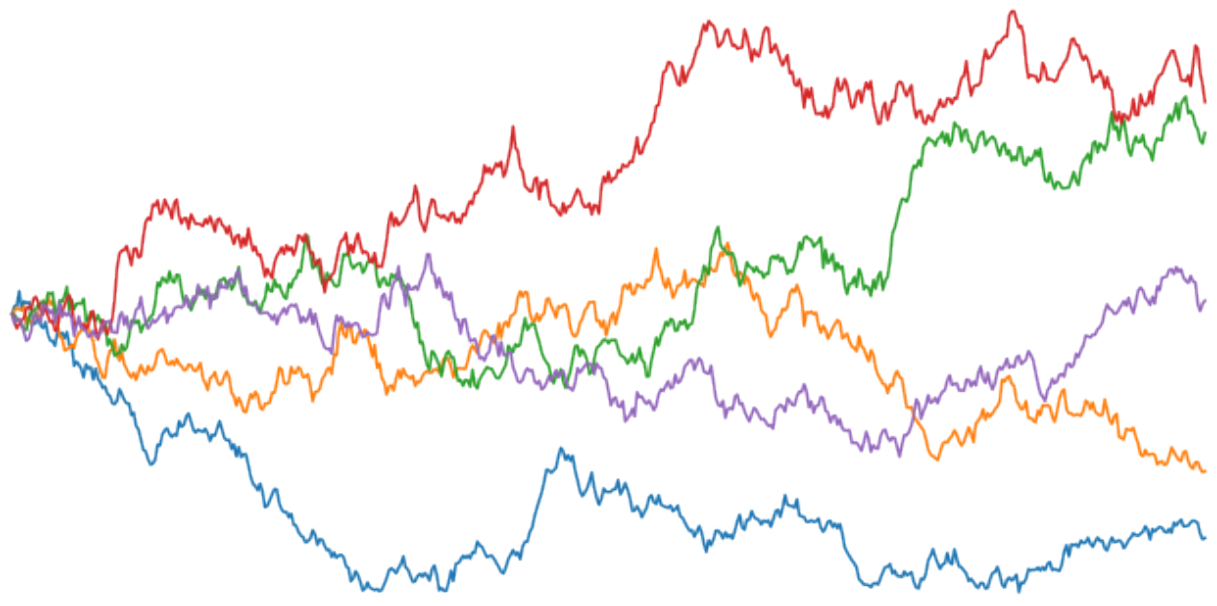
Sensitivity computation: bump-and-reval



Compute derivative price



- **Bump a market rate:** $R\left(T_k\right) + \epsilon$
- **Recalibrate the model**



- **Recompute derivative price**
- **Sensitivity:** $V\left(R_k + \epsilon\right) - V\left(R_k\right)$

Must be repeated for each:

- MC scenario
- Time-step
- Sensitivity bucket

Delta computation: closed-form

Transform model- to market sensitivities

Chain rule + inverse function theorem:

$$\frac{\partial V}{\partial \mathbf{S}} = \frac{\partial V}{\partial \mathbf{R}} \cdot \frac{\partial \mathbf{R}}{\partial \mathbf{S}} = \frac{\partial V}{\partial \mathbf{R}} \cdot \left(\frac{\partial \mathbf{S}}{\partial \mathbf{R}}\right)^{-1}$$

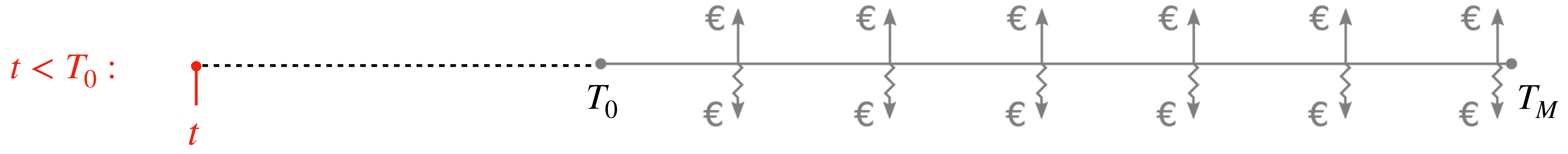
Sensitivity w.r.t. the model parameters

$$\begin{bmatrix} \frac{\partial V(t)}{\partial S_1(t)} & \cdots & \frac{\partial V(t)}{\partial S_{12}(t)} \end{bmatrix} = \begin{bmatrix} \frac{\partial V(t)}{\partial R_{\tau_1}} & \cdots & \frac{\partial V(t)}{\partial R_{\tau_{12}}} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial S_1(t)}{\partial R_{\tau_1}} & \cdots & \frac{\partial S_1(t)}{\partial R_{\tau_{12}}} \\ \vdots & \ddots & \vdots \\ \frac{\partial S_{12}(t)}{\partial R_{\tau_1}} & \cdots & \frac{\partial S_{12}(t)}{\partial R_{\tau_{12}}} \end{bmatrix}^{-1}$$

Jacobian transformation

- S_k = **Market rate** (e.g. swap rate)
- R_{τ_k} = **Model rate** (e.g. zero-rate)

Vanilla swaption



$$V(t) = \sum_{m=1}^{M'} \tau'_m P(t, T'_m) \mathbb{E}^{0,M} \left[\left(S_{0,M}(T_0) - K \right)^+ \middle| \mathcal{F}_t \right]$$

The swap rate process

- Short rate process: $dr(t) = \left(\theta(t) - ar(t)\right) dt + \sqrt{v(t)}dW(t)$
- Swap-rate, apply Ito's lemma: $dS_{0,M}(t) = \frac{\partial S_{0,M}(t)}{\partial r(t)}\sqrt{v(t)}dW^{A_{0,M}}(t)$
- Quasi-Gaussian: $dS_{0,M}(t) \approx \left(\sum_{m=1}^{M'} \bar{\zeta}_m B(t, T'_m)\right)\sqrt{v(t)}dW^{A_{0,M}}(t)$

The ‘Bachelier’-version of Heston

- For $\begin{cases} dS(t) = \xi(t)\sqrt{v(t)}dW_1(t) \\ dv(t) = \kappa(\bar{v} - v(t))dt + \epsilon\sqrt{v(t)}dW_2(t) \end{cases}$
- Let $\phi(u,t) = \mathbb{E}\left[e^{iuS(T)}\middle|\mathcal{F}_t\right]$, the characteristic function of $S(t)$.
- Then $\mathbb{E}\left[\left(S(T)-K\right)^+\middle|\mathcal{F}_t\right] = \frac{e^{-\alpha K}}{\pi}\int_0^\infty \text{Re}\left(e^{-iuK}\frac{\phi(u-i\alpha,t)}{(iu+\alpha)^2}\right)du$

Sensitivities along the path

- Consider the *path-wise derivative method*:

$$\frac{\partial}{\partial R_k}\mathbb{E}\left[\left(S(T)-K\right)^+\Big|\mathcal{F}_t\right] = \mathbb{E}\left[\frac{\partial}{\partial R_k}\left(S(T)-K\right)^+\Big|\mathcal{F}_t\right]$$

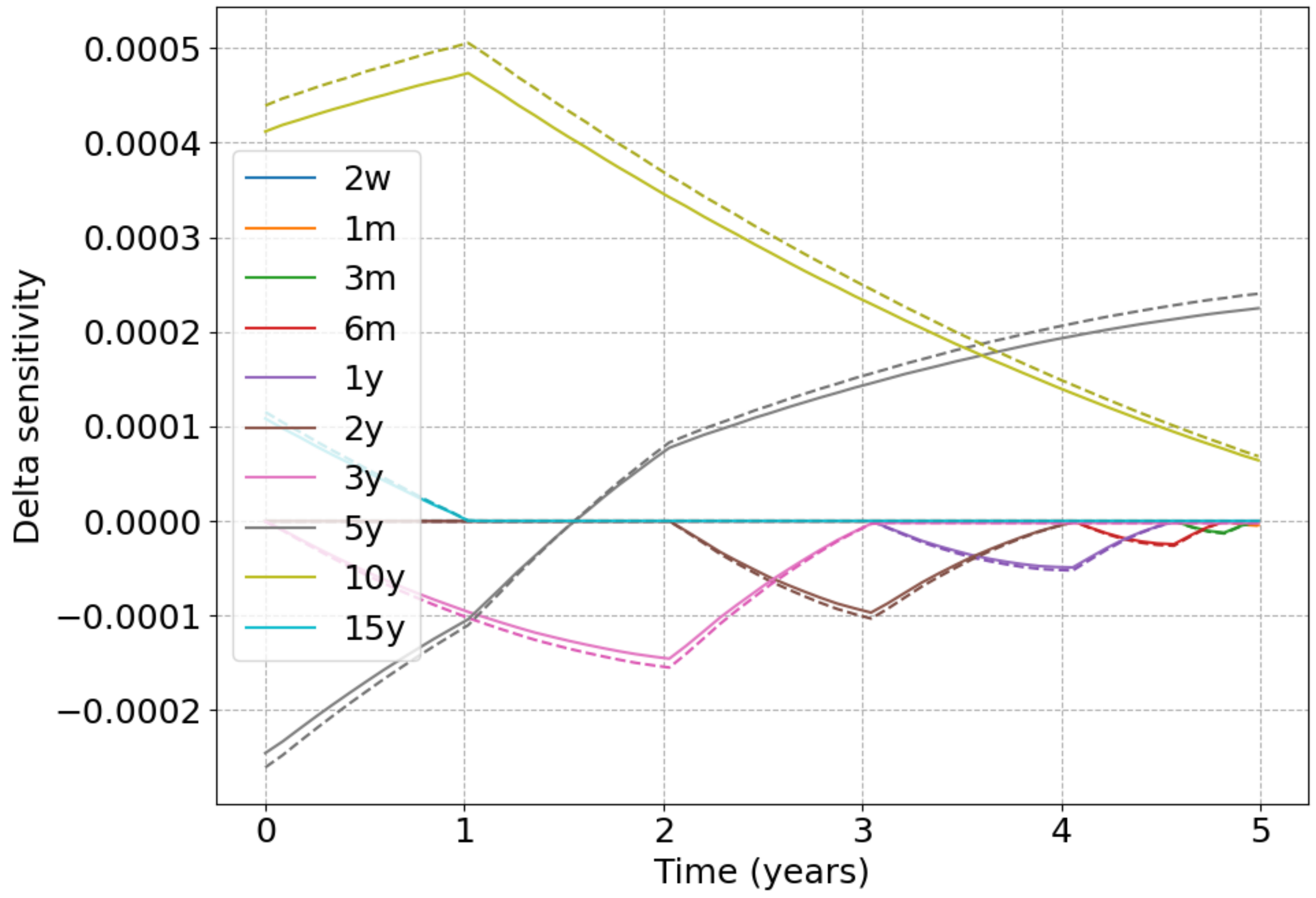
- Then it follows:

$$\frac{\partial V(t)}{\partial R_k} = \left(\alpha_0 P(t,T_0) - \alpha_M P(t,T_M)\right)\Psi_0(t,K) + \sum_{m=1}^{M}\tau_m\alpha_m P(t,T_m)\left(\Psi(t,K) - S(t)\Psi_0(t,K)\right)$$

- where: $\Psi_0(t,K) := \mathbb{E}\left[1_{\{S(T)>K\}}\Big|\mathcal{F}_t\right] = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\text{Im}\left(e^{-iuK}\phi(u,t)\right)}{u}du$

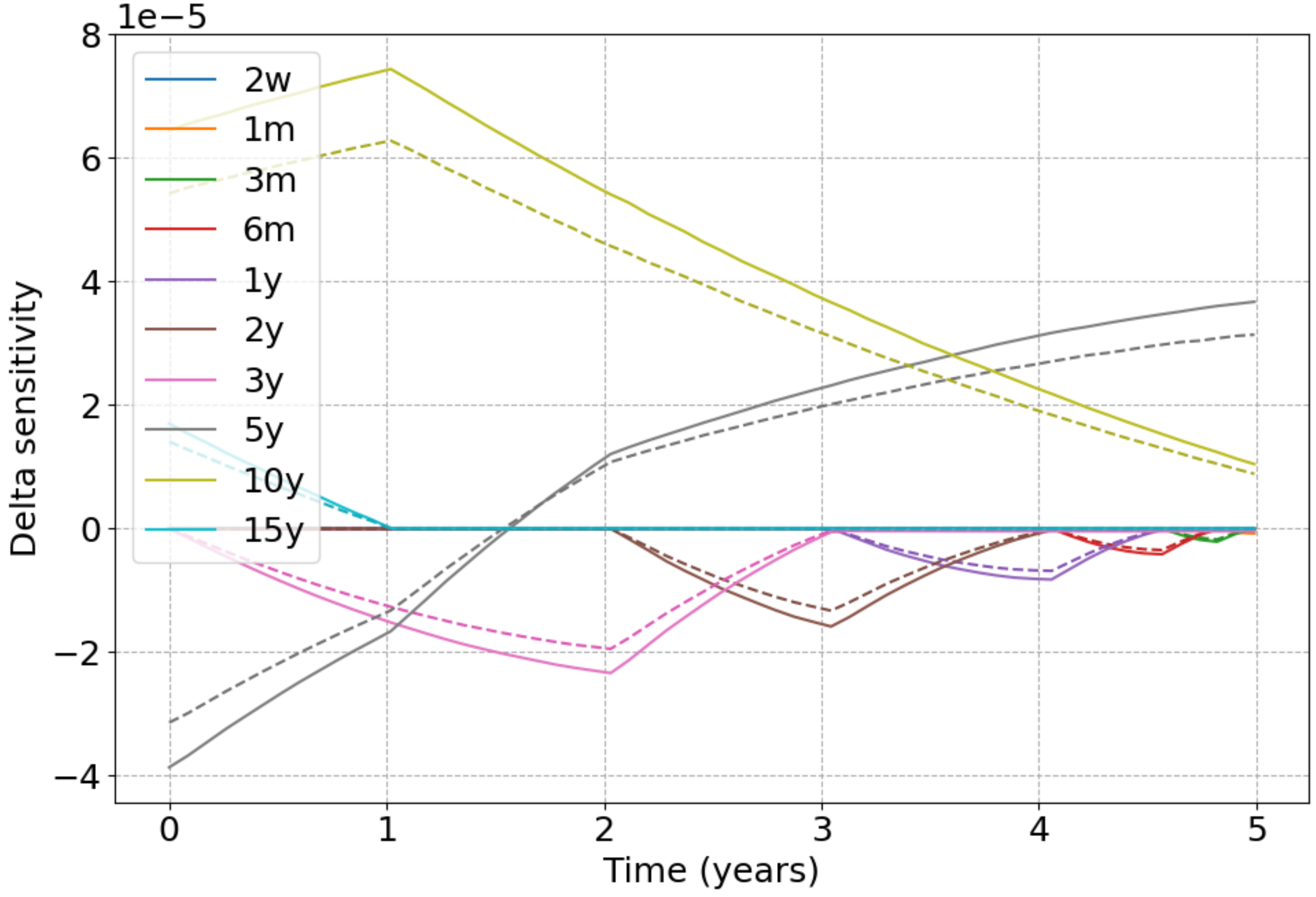
Delta profile - 5Yx6Y swaption

ATM $\left(K = S_{0,M}(0)\right)$



— stochastic vol - - constant vol

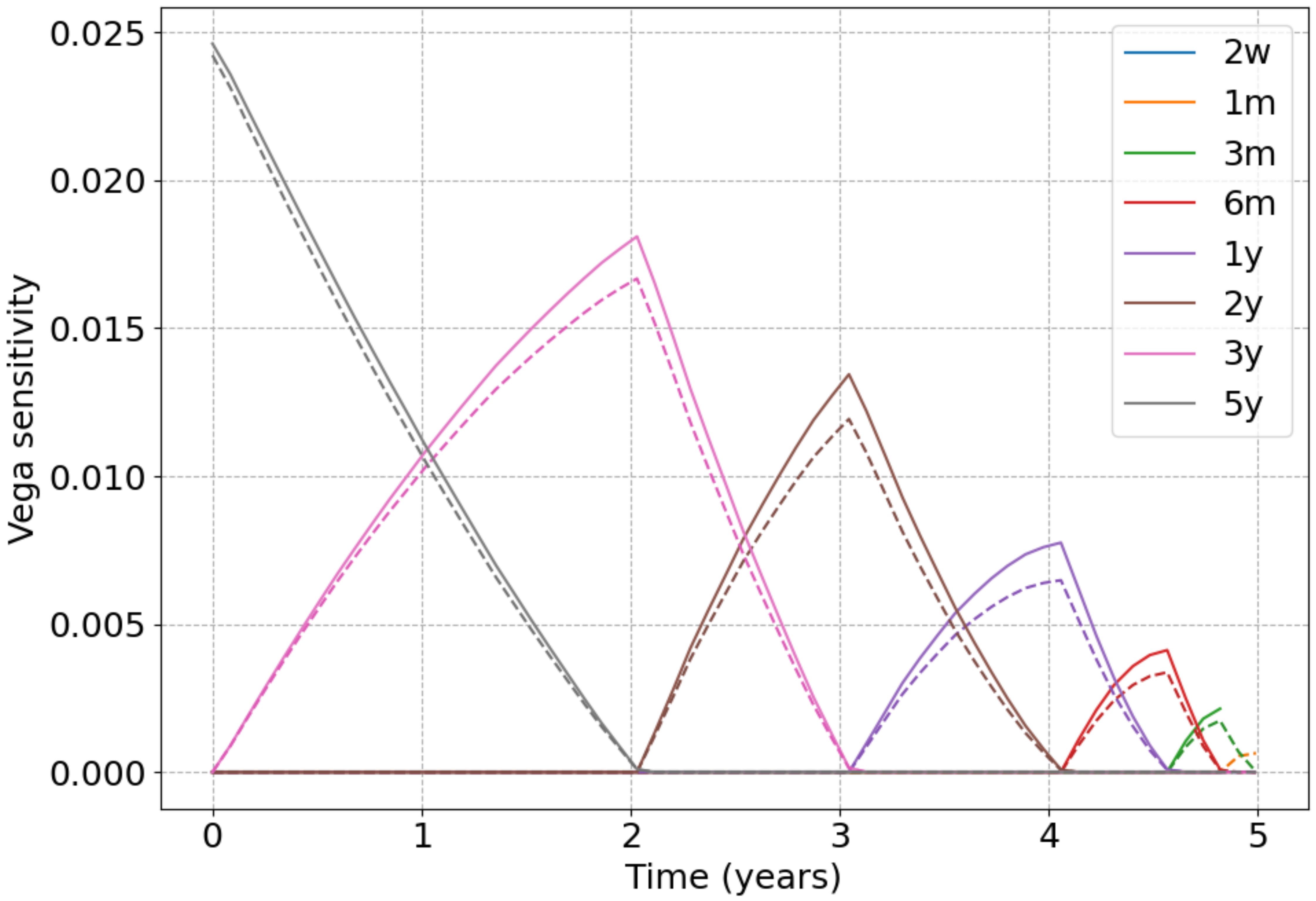
OTM $\left(K = K_{ATM} + 1.5\,\%\right)$



— stochastic vol - - constant vol

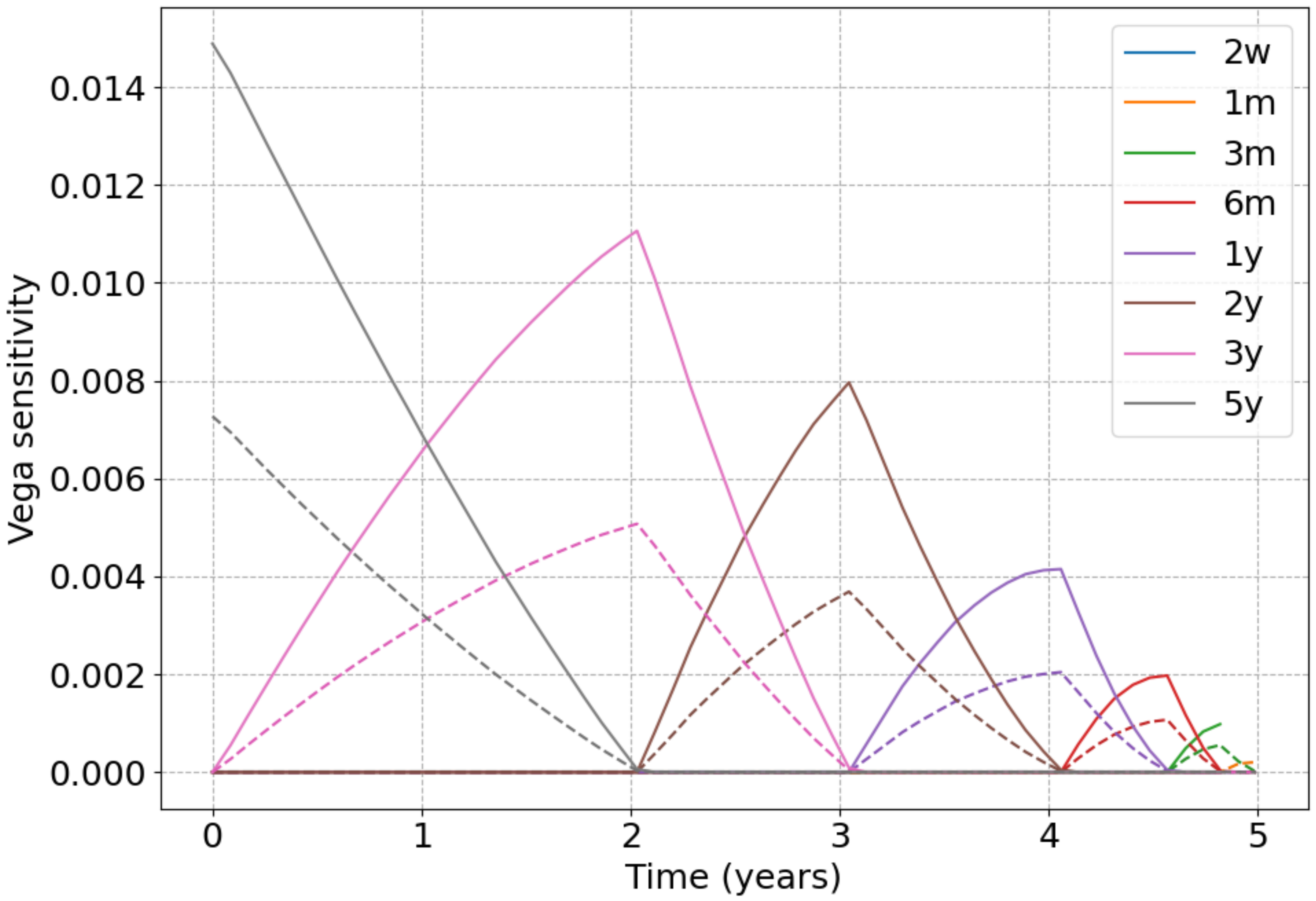
Vega profile - 5Yx6Y swaption

ATM $\left(K = S_{0,M}(0)\right)$



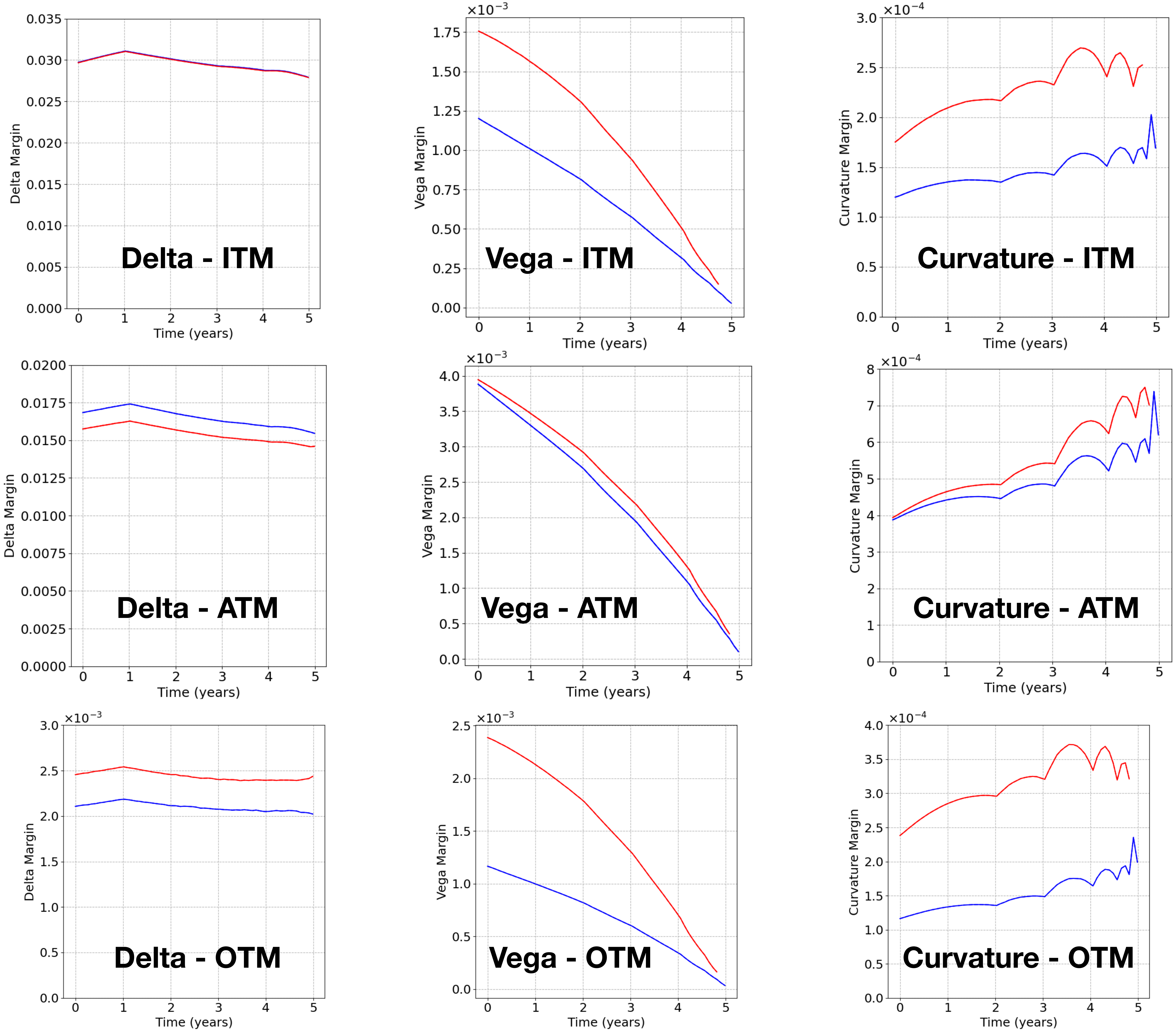
— stochastic vol - - constant vol

OTM $\left(K = K_{ATM} + 1.5\,\%\right)$



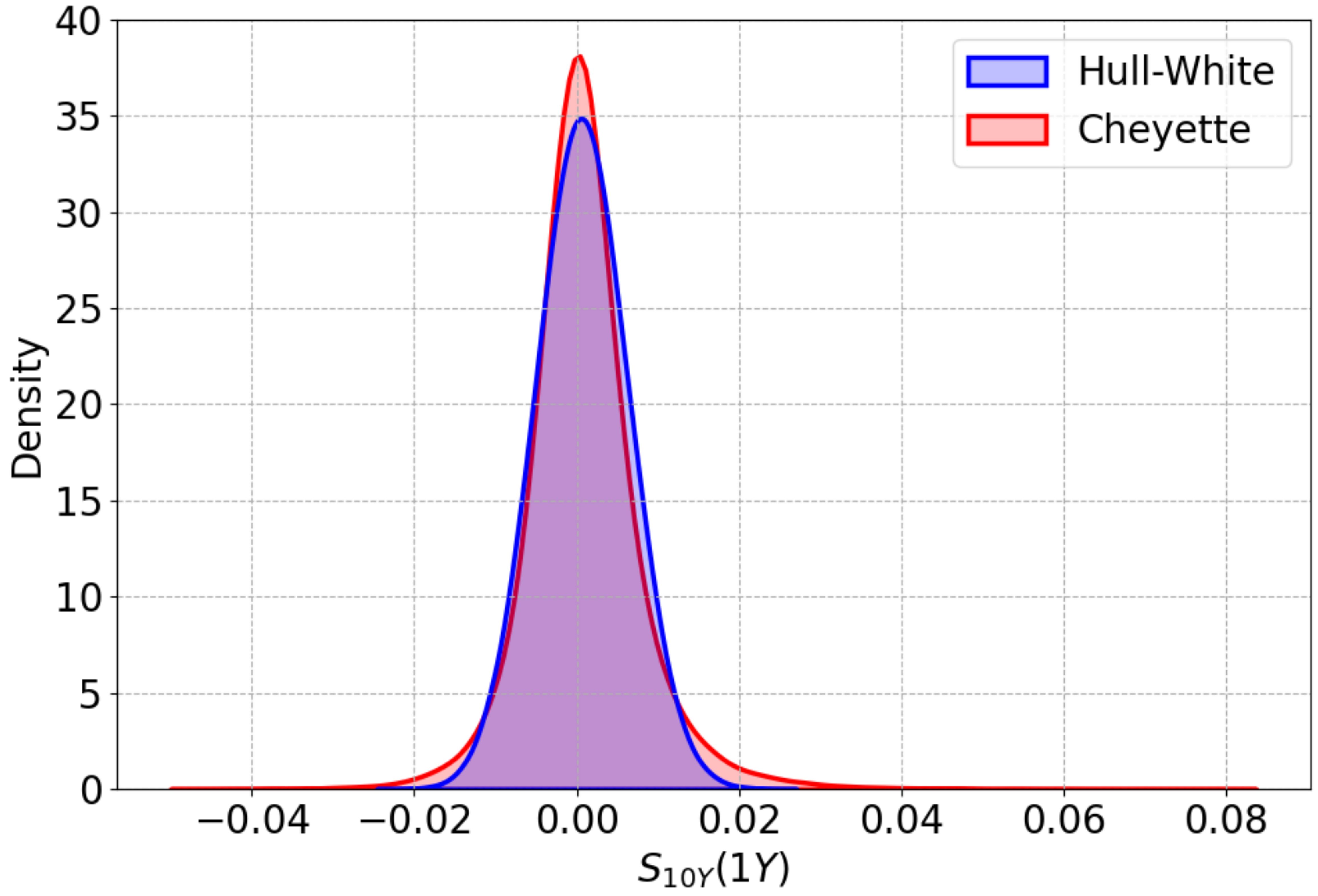
— stochastic vol - - constant vol

Delta-, vega- and curvature-margin profiles



— stochastic vol
— constant vol

Risk-neutral swap rate distribution



The swap rate distribution under stochastic volatility exhibits…

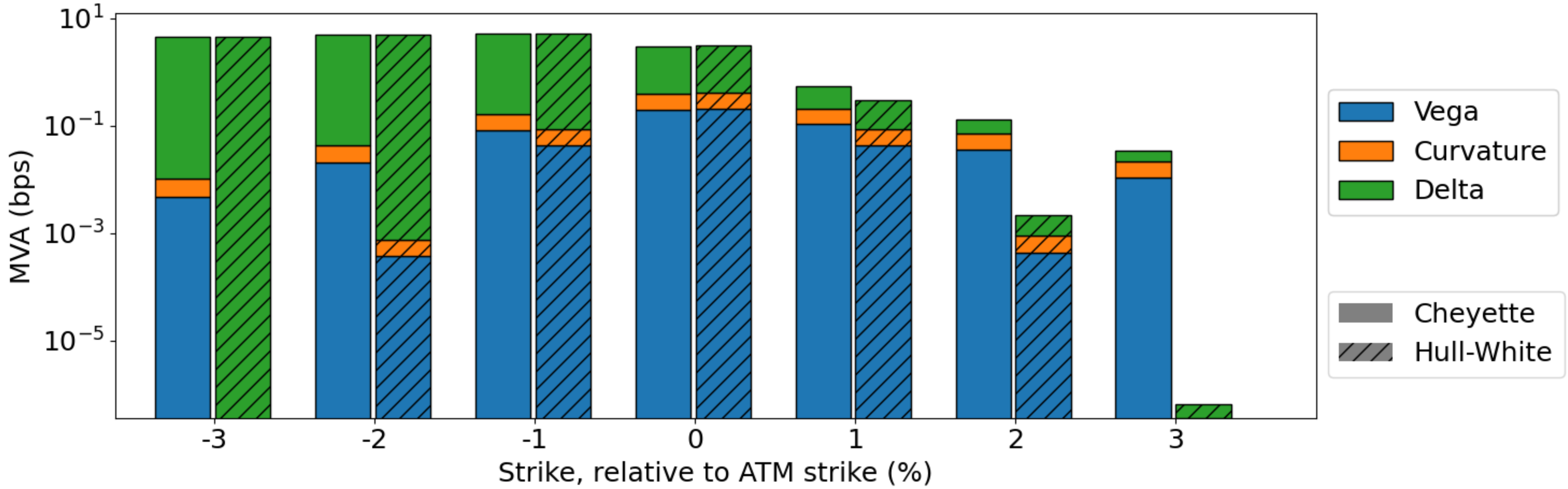
- …Fatter tails
- …Higher central peaks (convex smile)
- …Shifted to the left (skewed smile)

$$delta \propto N\left(d_1\right) \propto \mathbb{Q}\left(S_{0,M}(T_0) > K\right)$$

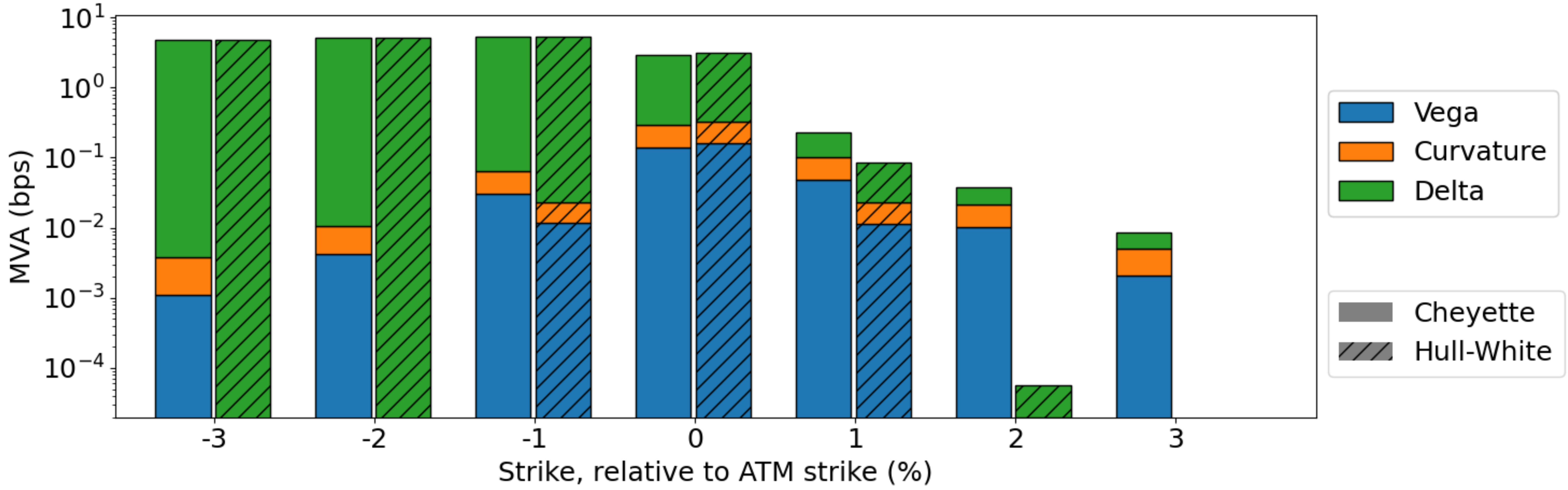
$$vega\ /\ gamma \propto N'\left(d_1\right) \propto f_{\mathbb{Q}}\left(S_{0,M}(T_0) = K\right)$$

MVA for 1Yx10Y swaptions

‘stressed’ market



‘relaxed’ market



Take-aways

- The Cheyette model extended with a stochastic volatility component can satisfactory fit multiple implied volatility swaption smiles simultaneously.

- Both forward- and backward-looking term rates can be accommodated.

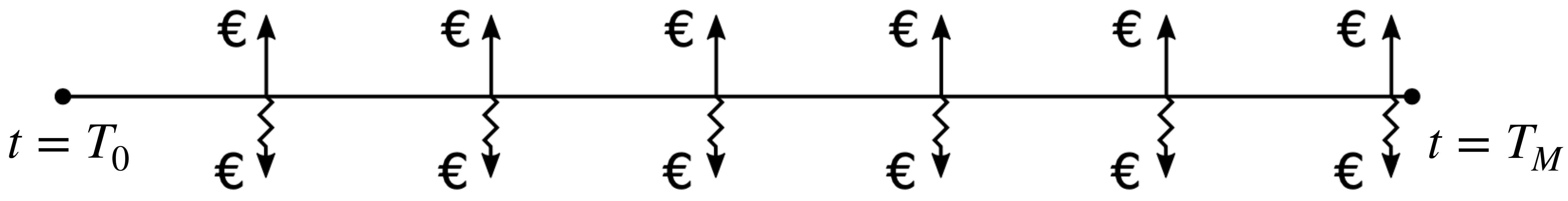
- Vanilla option prices and sensitivities can be efficiently computed along the MC paths.

- Margins for non-linear derivatives can be significantly mis-valued with a constant volatility model. Far OTM options in particular.

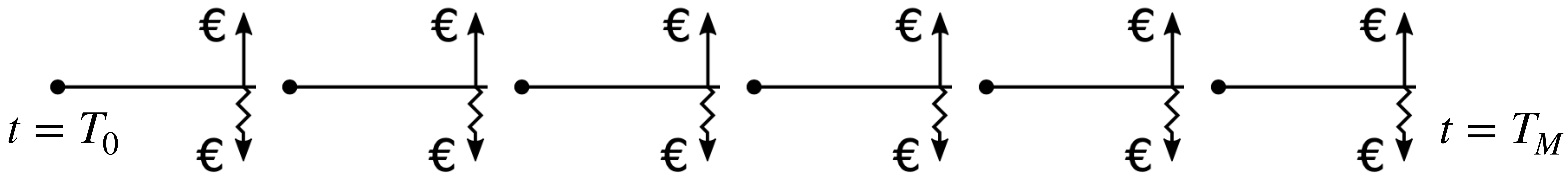
Thank you!

Vanilla IR derivatives

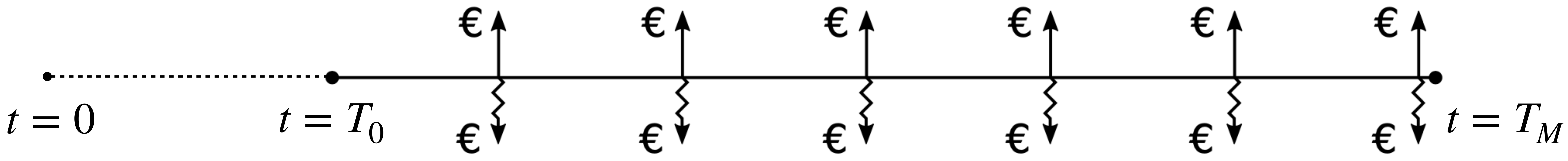
- **Fix-floating swap:** exchanges fixed vs. floating interest payment.



- **Cap / Floor:** exchanges fixed vs. floating interest payment, but only if the fixed rate is higher / lower than the floating rate.

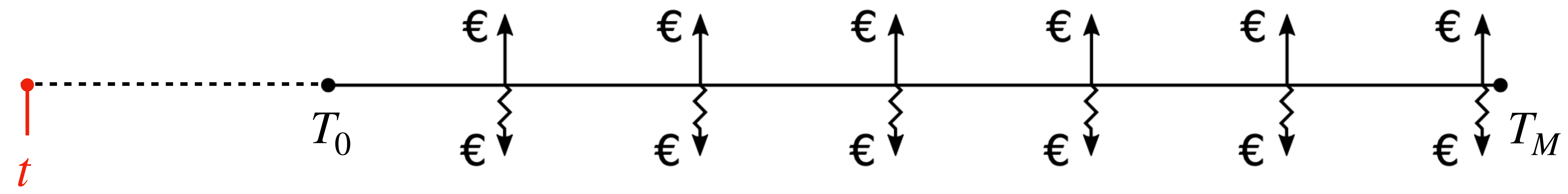


- **Swaption:** Option on a forward starting swap



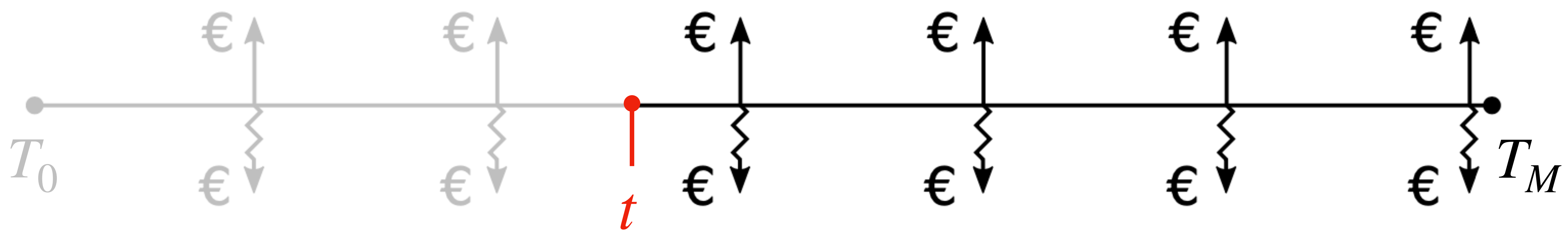
Interest rate swap

$t < T_0$:



$$V(t) = P\left(t, T_0\right) - P\left(t, T_M\right) - \sum_{m=1}^{M'} \tau_j' K P\left(t, T_m'\right)$$

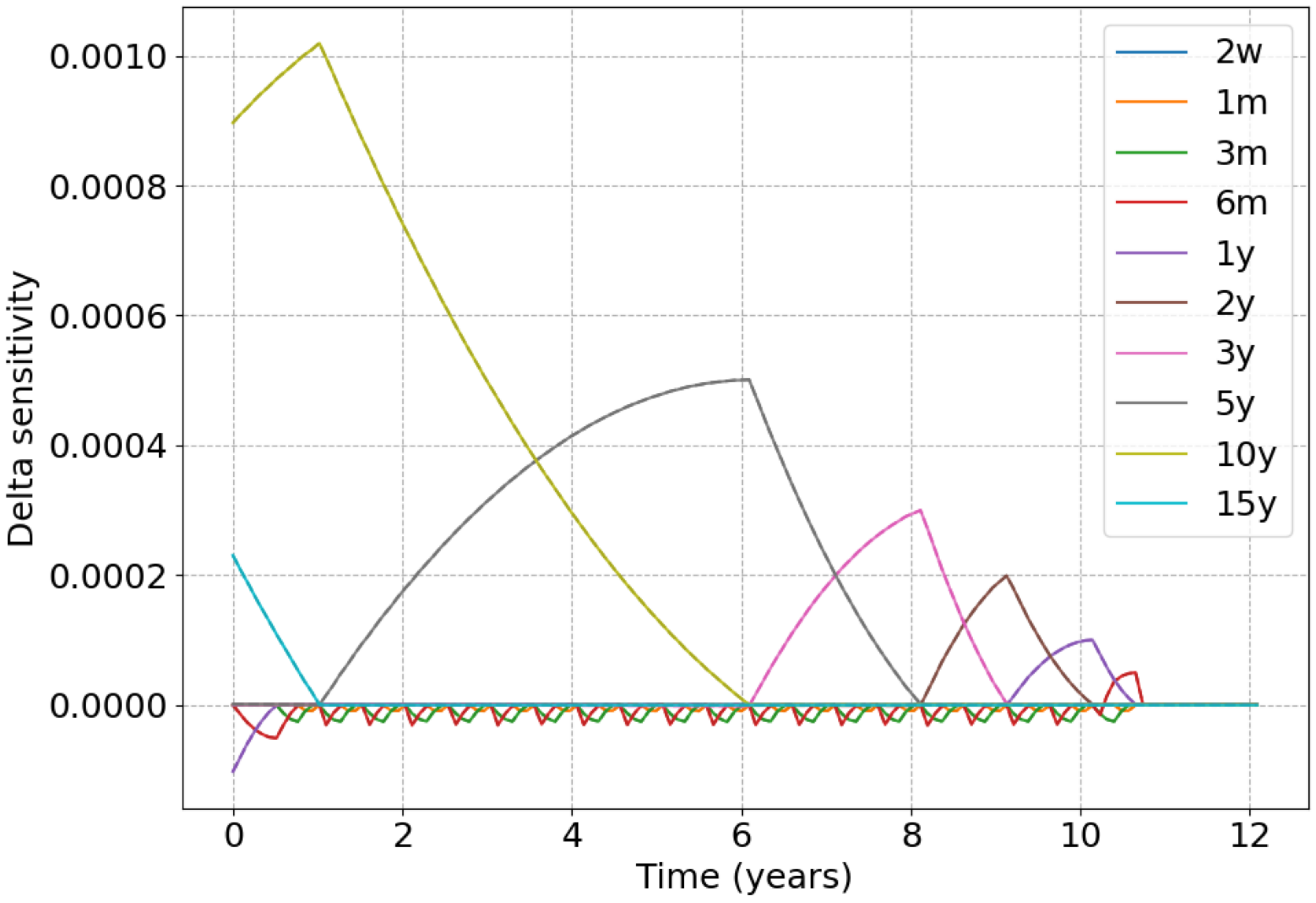
$T_0 < t < T_M$:



$$V(t) = \begin{cases} \dfrac{P(t, T_i)}{P(T_{i-1}, T_i)} - P\left(t, T_M\right) - \sum_{m=j}^{M'} \tau_j' K P\left(t, T_m'\right) & \textbf{LIBOR swap} \\ e^{\int_{T_{i-1}}^{t} r_u du} - P(t, T_M) - \sum_{m=j}^{M'} \tau_j' K P\left(t, T_m'\right) & \textbf{RFR swap} \end{cases}$$

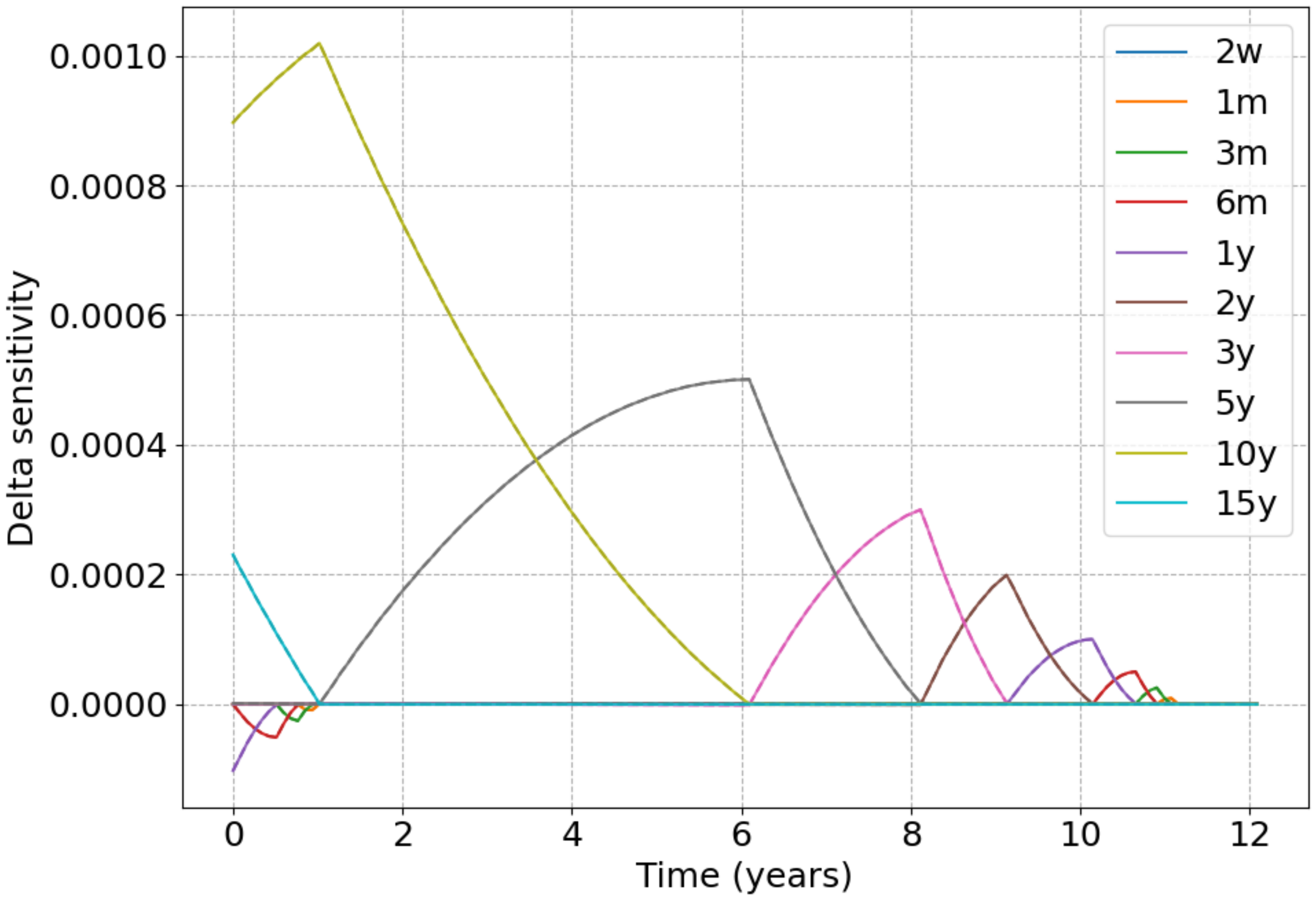
Delta profile - 1Y x10Y ATM swap

Forward-looking (LIBOR)



— stochastic vol - - constant vol

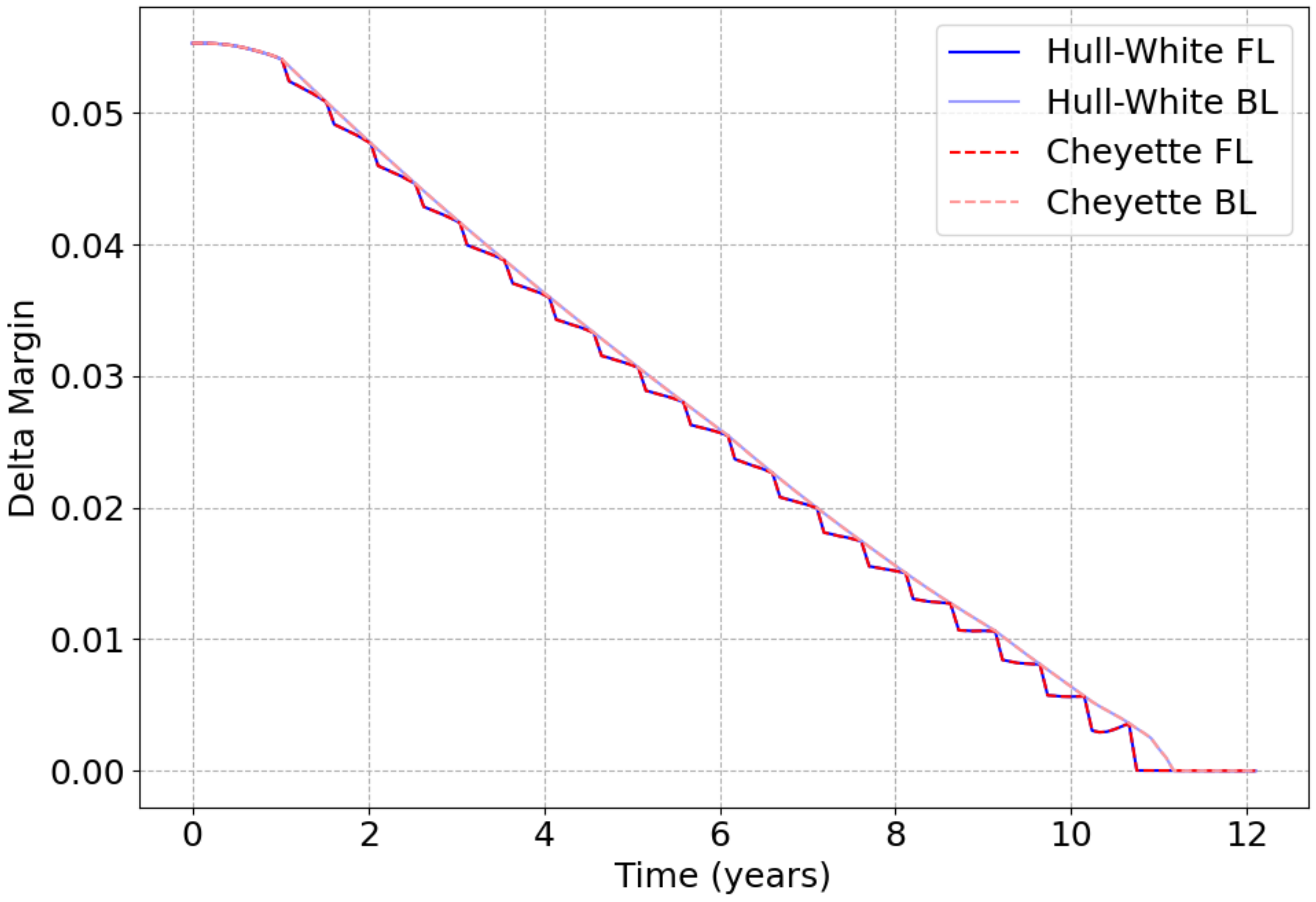
Backward-looking (RFR)



— stochastic vol - - constant vol

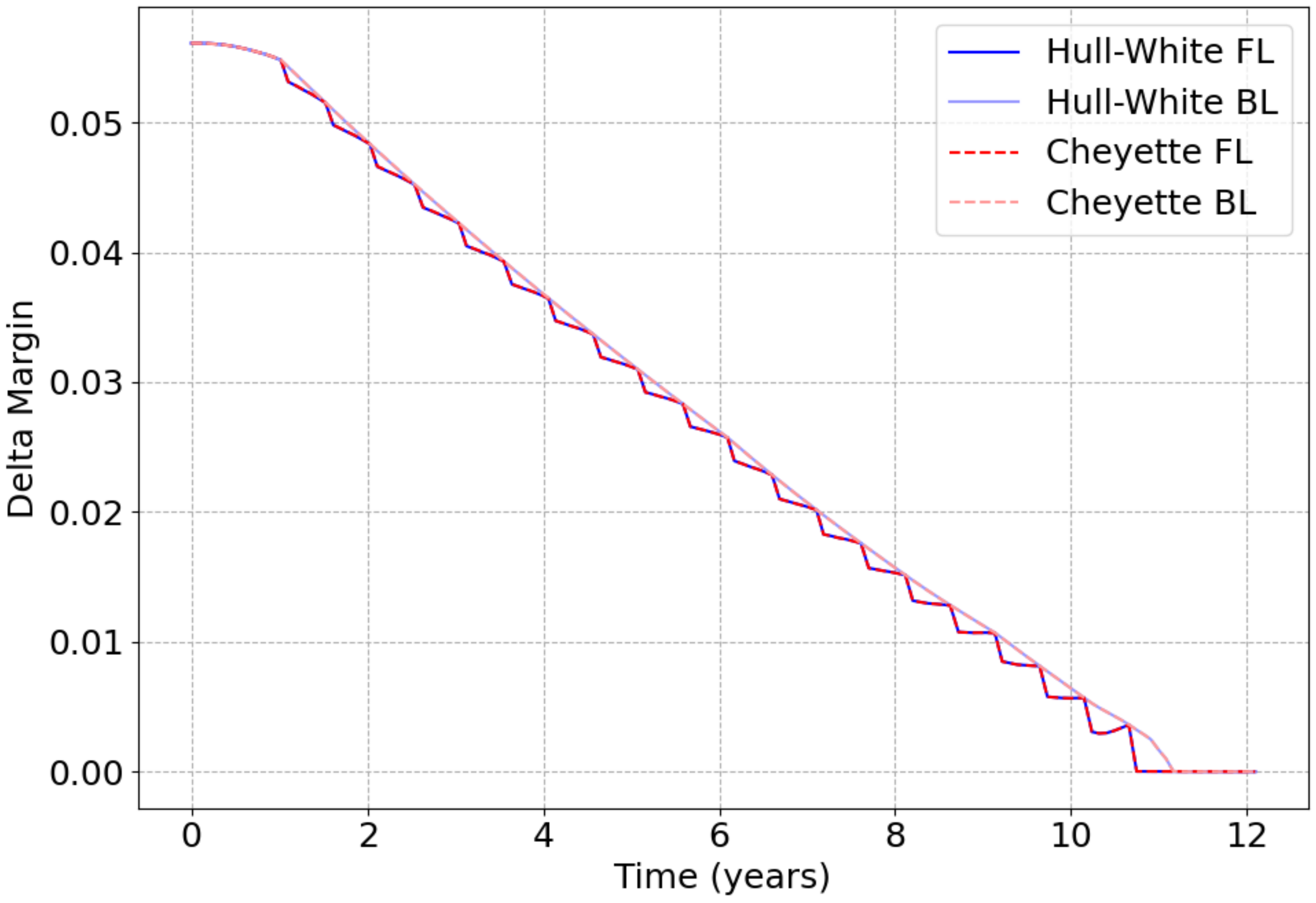
Delta margin - 1Yx10Y ATM swap

‘stressed’ market



- - stochastic vol — constant vol

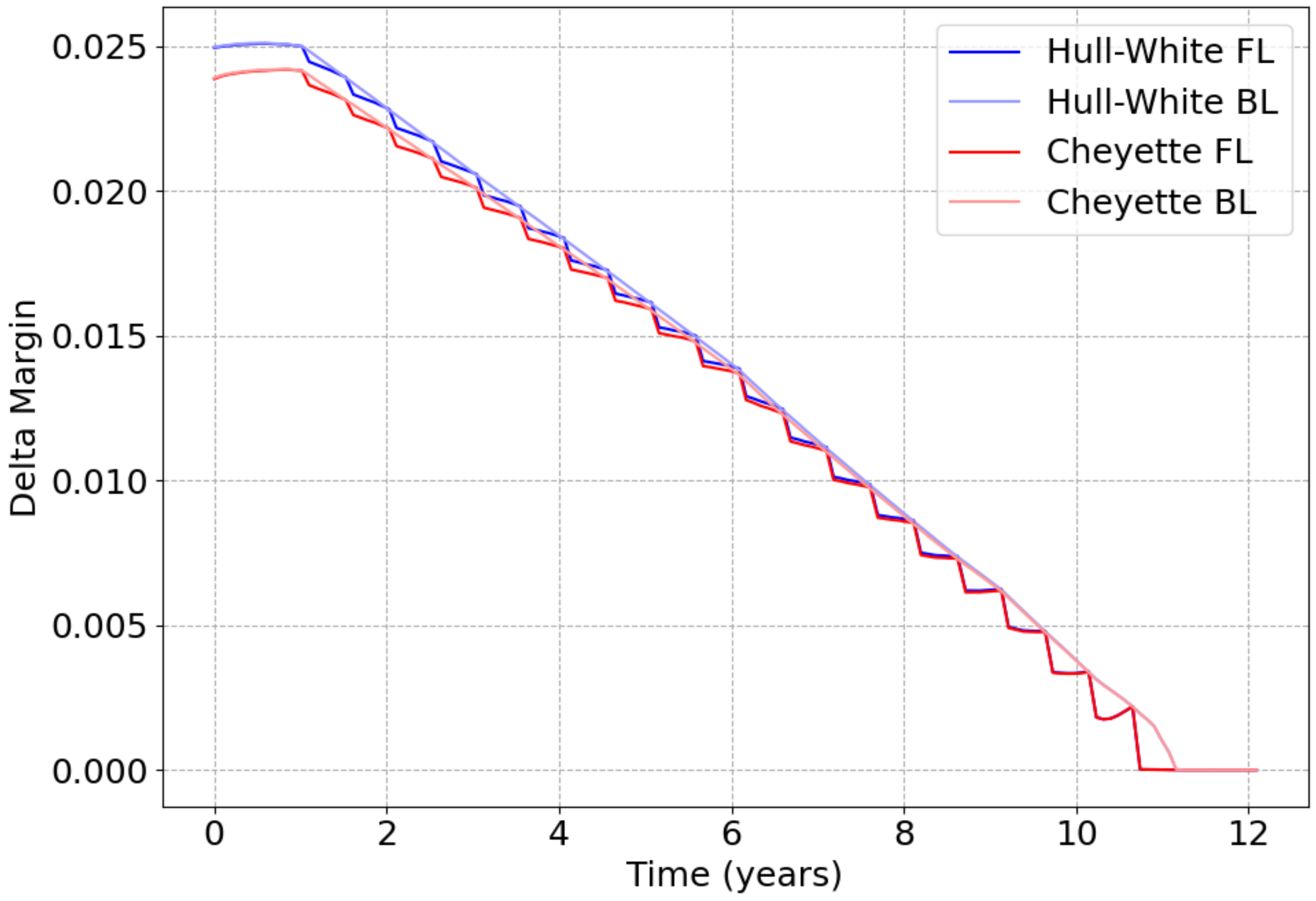
‘relaxed’ market



- - stochastic vol — constant vol

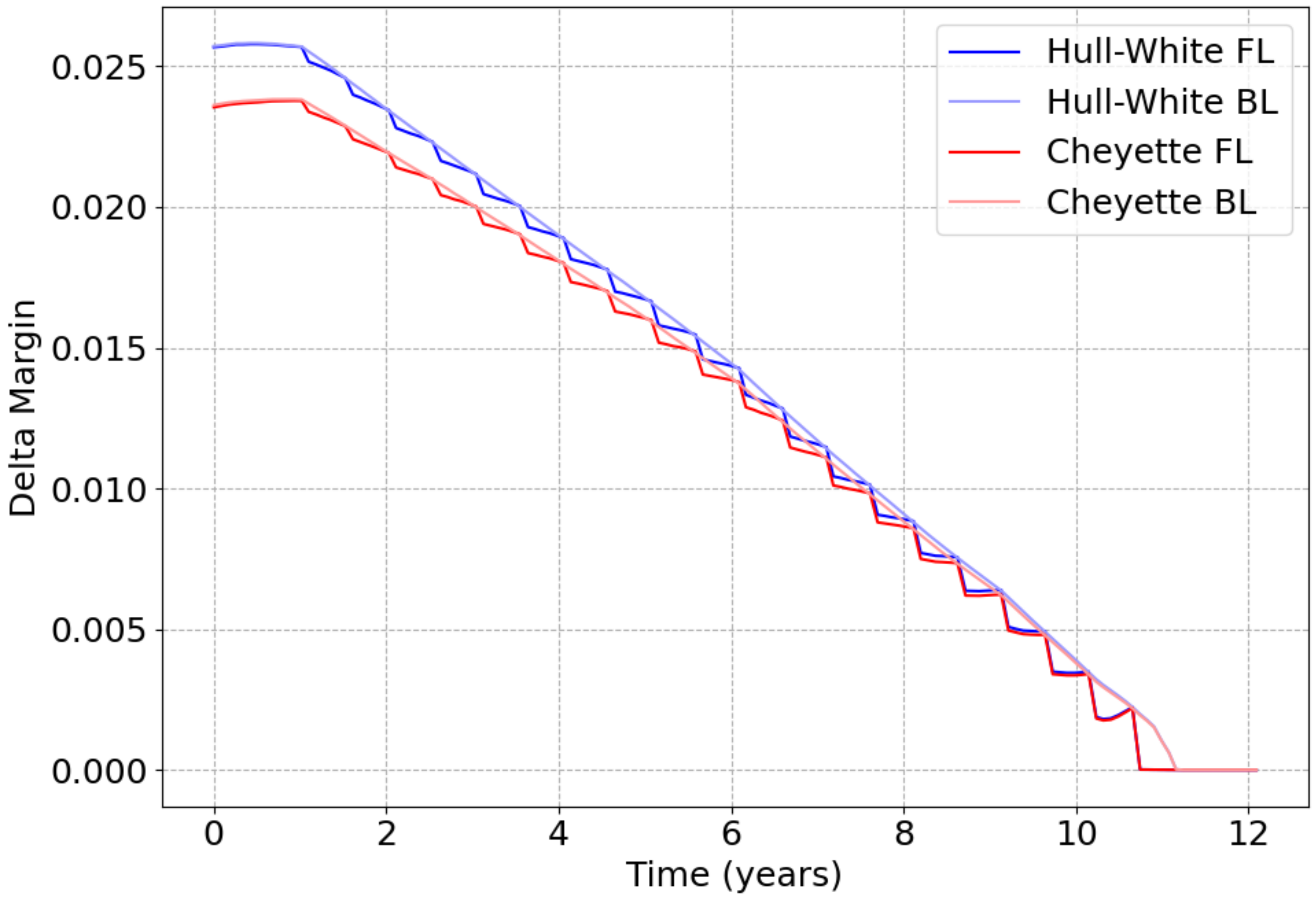
Delta margin - 1Yx10Y ATM cap

‘stressed’ market



- - stochastic vol — constant vol

‘relaxed’ market



- - stochastic vol — constant vol