Self-Exciting Point Processes in Expected Shortfall Backtesting

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Top Quants Lustrum Autumn 2023

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Agenda

1. Today's relevance of Expected Shortfall
2. Principles of VaR and ES backtesting
3. Motivating example
4. Current state of VaR and ES backtesting
5. The importance of having (and not having) assumptions
6. What are Self-exciting Point Processes?
7. How to apply SEPP in ES backtesting?
8. How does the SEPP ES backtest compare to its peers?
9. A quick digression: assuming a return distribution
10. Empirical example: the 2020 stock market crash
11. Wrap-up

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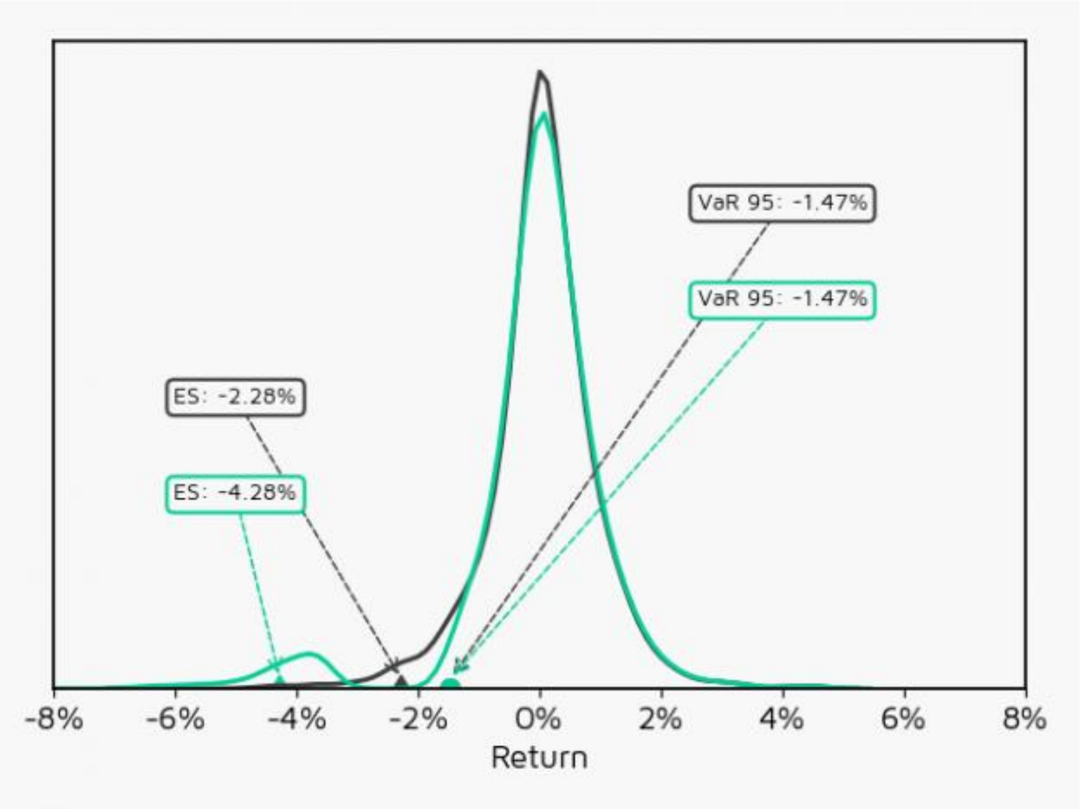
11. Wrap-up

Today's relevance of Expected Shortfall

- Capital requirements are broadly based on VaR
- VaR-based capital requirements suffer from a large deficiency: they do not capture tail risk
- Expected Shortfall (ES): the expected return in the worst α% of cases*
- FRTB presents a move from VaR to ES due to VaR's inability to capture tail risk
- 97.5% ES used to determine FRTB IMA capital requirements
- Same approach used to calibrate FRTB SA
- Regulatory backtesting still uses VaR

... or does it?



*ES is also known as CVaR, AVaR, expected tail loss and superquantile

Today's relevance of Expected Shortfall

- 24 March 2023: EBA publishes draft rules for local supervisors
- Methodology for assessing internal models under FRTB
- Includes the requirement to conduct backtesting ES
- Banks see issues with:
  - Additional operational costs
  - No prescribed backtest
  - Lack of added value

Risk.net

EU banks balk at new market risk models back test

EBA proposals introduce additional expected shortfall back test for market capital risk models under FRTB

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11. Wrap-up

Principles of VaR and ES backtesting

Value-at-Risk:
Denote L_t as the trading desk's loss at time *t* and $\alpha \in (0,1)$, then, VaR equals:

$$\mathbb{P}(L_t > \mathrm{VaR}_\alpha^t | \mathcal{F}_t) = \alpha$$

Two main assumptions:

1. The unconditional assumption – the expected value of number of VaR outliers equals αT

2. The conditional assumption – at any time *t*, the probability of having a VaR outlier that day equals α (not α_t!)

Expected Shortfall:
Given cdf F_L of losses L_t, the ES is given by

$$ES_\alpha^t = \frac{1}{\alpha}\int_{1-\alpha}^{1} q_u(F_L)\,du$$

Leading to the following assumptions:

3. $\mathbb{E}[\mathrm{ES}_\alpha^t - \mathrm{L_t} \mid \mathrm{L_t} > \mathrm{VaR}_\alpha^t] = 0$
4. The VaR is correct

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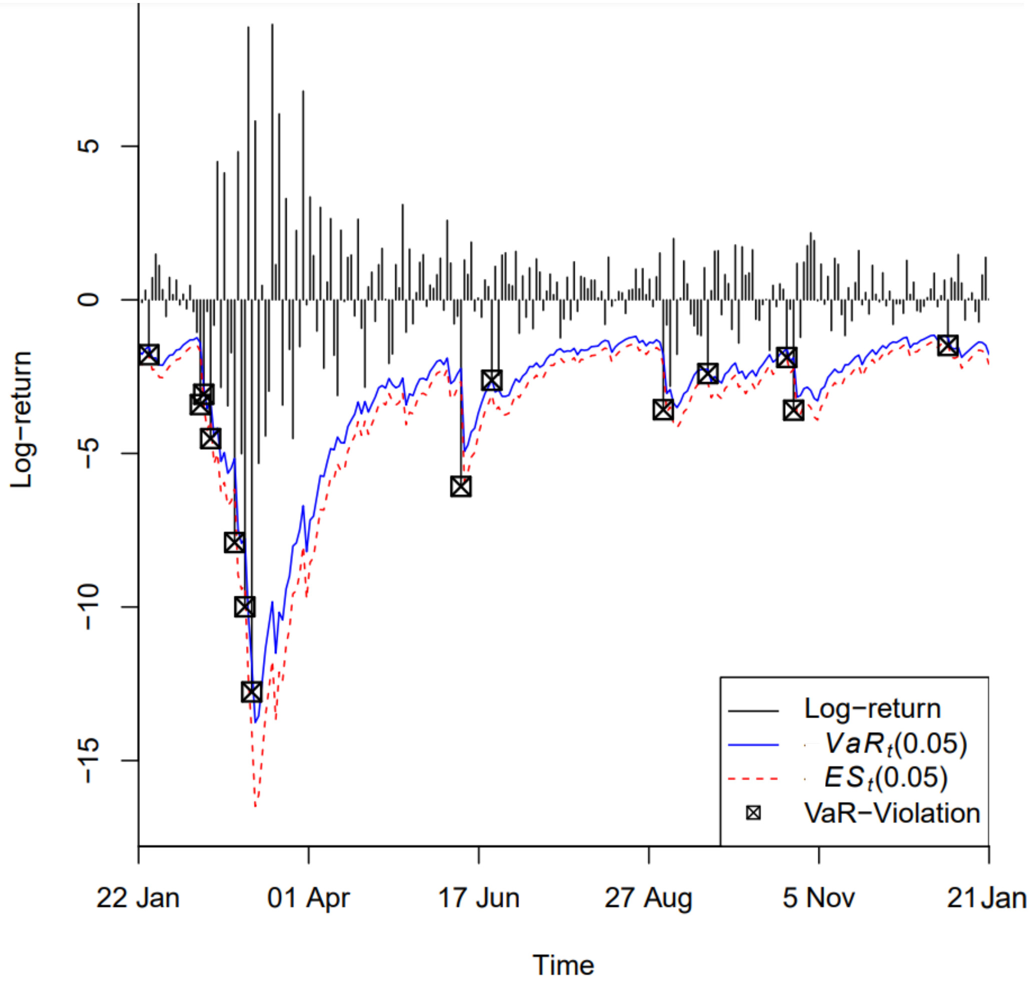
11. Wrap-up

Motivating example

- S&P500 daily log-returns during the 2020 stock market crash
- 5%-VaR outliers: 14 (5.6%)
- Clear cluster of VaR outliers: 6 outliers in 15 days (40% >> 5%)
- Existing VaR and ES backtests (both unconditional and conditional) accept the risk model
- Having an incorrect VaR model implies an incorrect ES model
- Led me to look for and ultimately develop a backtesting technique that outperforms existing backtests



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11. Wrap-up

Current state of VaR and ES backtesting

Development of ES and VaR backtesting:

- VaR backtesting is a mature research area
- Unconditional (traffic light) and conditional (e.g., Christoffersen's independence test)
- ES backtesting is not so mature:
  - Took off in 2014
  - Next to a few unconditional backtests, only one (underperforming) conditional backtest

Literature studies and my research show that:

- Current conditional VaR and ES backtests capture time dependence poorly
- VaR and ES backtests have low power in rejecting wrong risk models, especially when:
  - Sample size is low (< 500 observations)
  - High VaR levels are considered (> 97.5%)
- In other words: **when it matters**

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11. Wrap-up

The importance of having (and not having) assumptions

- To assess the bank's market risk model, it is important to test **all four assumptions** underlying VaR and ES
- Current ES and joint VaR-ES backtests require the risk manager to **assume a return distribution**

  → Pandora's assumption box

  → Estimation error
- Sparked a move to assumption-free backtesting:
  - Only requires returns and reported VaR & ES
  - Two assumption-free unconditional backtests exist
- How can we obtain an assumption-free conditional backtest of VaR/ES?
- Current conditional backtests capture time dependence poorly…

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11. Wrap-up

What are Self-exciting Point Processes?

- Originally developed for earthquake analysis
  - An event (earthquake/aftershock) excites more events soon after
  - The longer the wait, the lower the probability: back to steady state probability
- Involves modelling of the hit sequence (equals 1 if outlier, 0 if not)
- Arrival rate of a non-homogenous Poisson process modelled by

$$\lambda(t) = \tau + \psi \sum_{j:t_j<t} e^{-\gamma(t-t_j)}$$

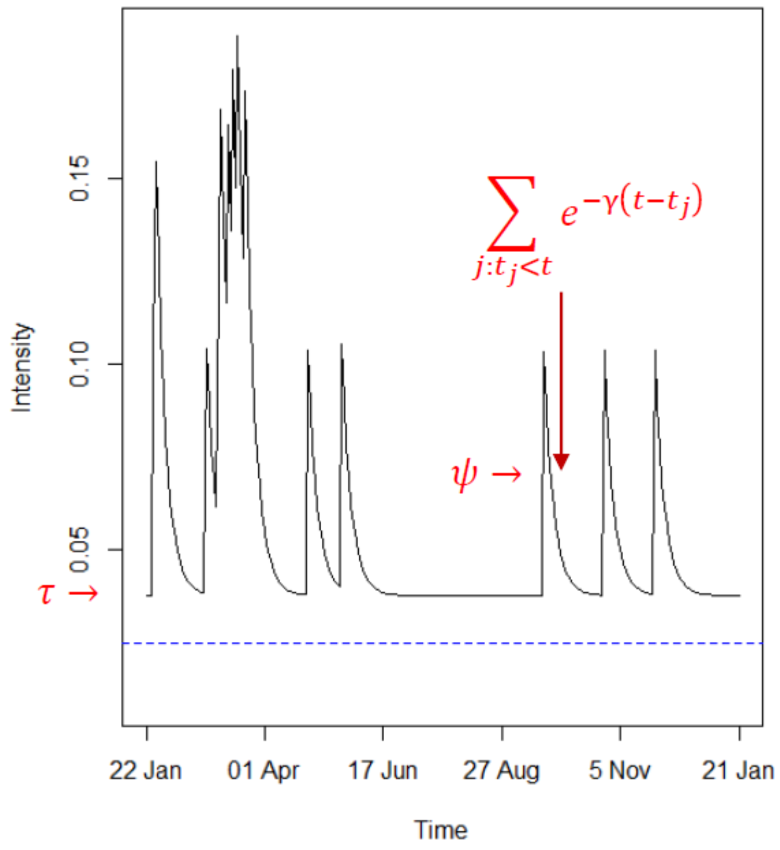
with $\tau, \gamma > 0$ and $\psi \geq 0$.

- **Many** applications: earthquakes, social media, crimes, deaths, etc.
- First financial application: Chavez-Demoulin (2005) in VaR modelling
  - SEPP's efficient modelling of tail behavior → Better VaR model

What are Self-exciting Point Processes?

$$\lambda(t) = \tau + \psi \sum_{j:t_j<t} e^{-\gamma(t-t_j)}$$

- τ is the base intensity (arrival rate)
- ψ is the immediate jump in the intensity
- $\sum_{j:t_j<t} e^{-\gamma(t-t_j)}$ is the decay factor. As $t - t_j$, the distance between current time t and the time of the last violation t_j, gets larger, the intensity jump decays



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11. Wrap-up

How to apply SEPP in ES backtesting?

- Input: returns and VaR/ES predictions
- Calculate hit sequence and store the values $A_{t_j} = L_{t_j} - ES_{t_j}$ (note $\mathbb{E}[A_{t_j}] = 0$)

Part 1:

- Use the hit sequence to fit a non-homogenous point process with intensity:

$$\lambda(t) = \tau + \psi \sum_{j:t_j<t} e^{\delta A_{t_j} - \gamma(t-t_j)}$$

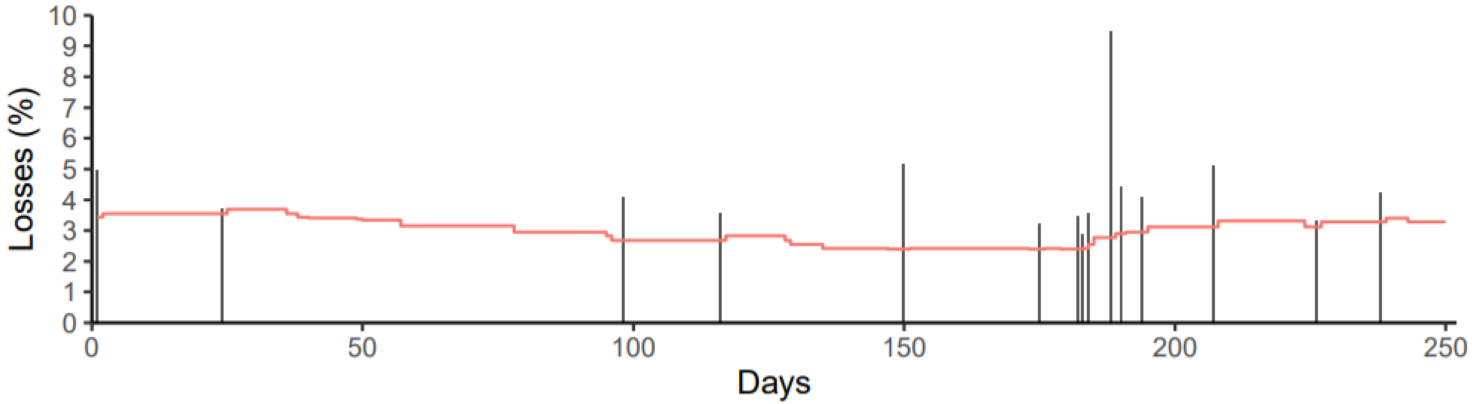
- Note the addition of A_{t_j} (although ES_{t_j} could also be used)
- Under the null:
  - $\tau = \alpha$ → Assumption 1
  - $\psi = 0$ → Assumption 2
  - $\gamma, \delta \in \mathbb{R}^+$

Tests an extended version of Assumption 2 by incorporating that high ES values should not affect the outlier intensity → important in the aggregation of Part 1 and 2

How to apply SEPP in ES backtesting?

- Part 1 formed the basis of SEPP VaR backtests
- SEPP VaR outperformed all other VaR backtests, how?

Toy example: HVaR vs GARCH(1,1) model:

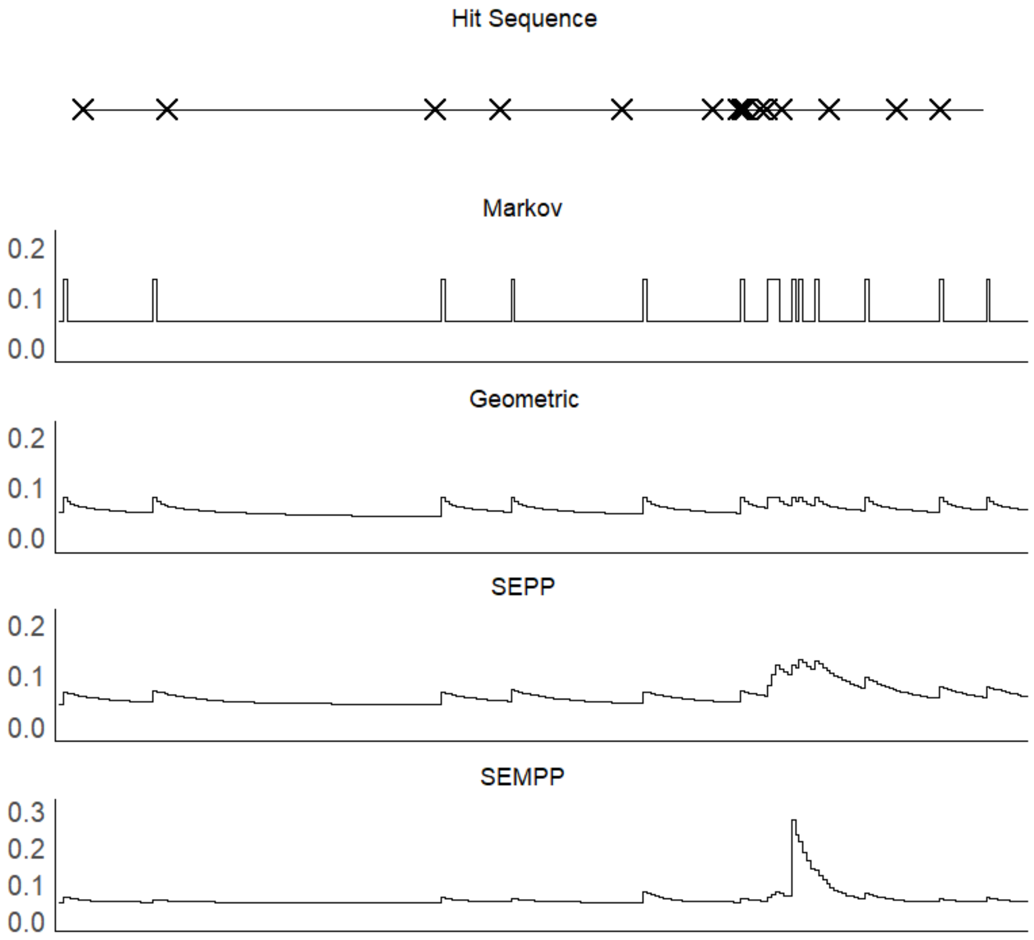


How to apply SEPP in ES backtesting?

Crux lies in the modelling of the alternative

Under the alternative:

- Christoffersen's (Markov) test does not fit clusters well
- Geometric test is bounded, requiring to balance in the parameters
- SEPP test is more flexible during the clustering period
- SEMPP accounts for the large violation and models the cluster well



How to apply SEPP in ES backtesting?

Part 2:

Due to the specification (especially the inclusion of $A_{t_j} = L_{t_j} - ES_{t_j}$) :

$$\lambda(t) = \tau + \psi \sum_{j:t_j<t} e^{\delta A_{t_j} - \gamma(t-t_j)}$$

we can make use of a common assumption in Extreme Value Theorem:

the conditional independence of times and marks

- Given all the information put into the outlier intensity function, realized A_{t_j} values are independent
- Given the realized A_{t_j} values, the intensity function does not further depend on realized A_{t_j} values

→ Perform a t-test on the values A_{t_j}

How to apply SEPP in ES backtesting?

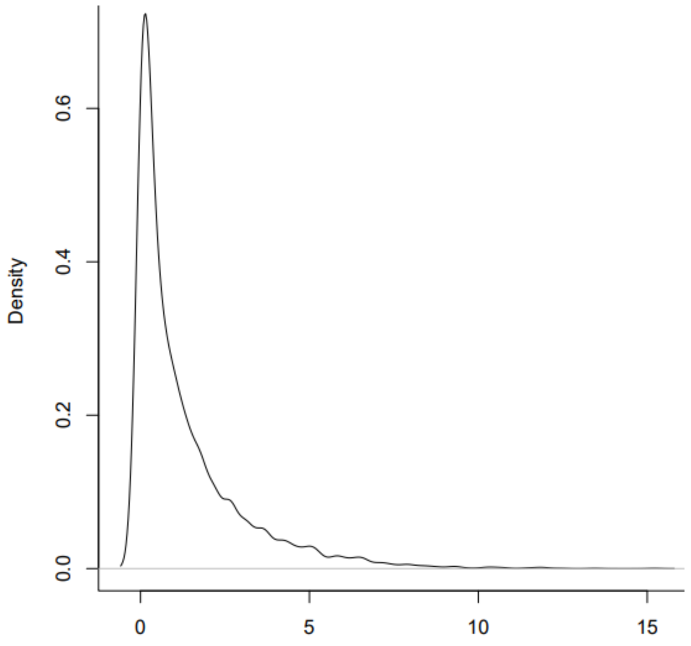
- Compute the estimated log-likelihood of a non-homogenous point process and the corresponding likelihood-ratio test statistic: $\mathcal{L}_1 \sim \chi^2_n$
- Transform the t-test to a χ^2-distributed test statistic: $\mathcal{L}_2 \sim \chi^2_1$
- Under the assumption of conditional independence, we have:

$$\mathcal{L}_1 + \mathcal{L}_2 \equiv \mathcal{L}^* \sim \chi^2_{n+1}$$

n is complicated due to two abnormalities:

- γ, δ are not identified under the null
- ψ is on the boundary of the parameter space under the null

Complicates the asymptotic distribution, but solutions exist



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11. Wrap-up

How does the SEPP ES backtest compare to its peers?

Monte Carlo experiment (B = 1000) to compare existing ‘assumption-free’ backtests:

- H0: giving size, namely AR(1)-GARCH(1,1)-skewed student t distribution
- H1: Reported VaR is 10% underestimated
- H2: Reported ES is 10% underestimated
- H3: Reported VaR and ES are 10% underestimated
- H4: Risk manager estimates normal innovations instead of the skewed student t
- H5: Risk manager estimates t-distribution instead of skewed student t distribution
- H6: estimating a GARCH(1,1)-skewed t, hence ignoring the AR(1) part

$\alpha = 5\%, n = 500$	SEPP ES	ESR1	ESR2	ESR3	ESR4	E-Backtest
H0	0.069	0.075	0.085	0.037	0.003	0.003
H1	**0.192**	0.074	0.084	0.037	0.003	0.006
H2	**0.163**	0.057	0.055	0.030	0.053	0.073
H3	**0.237**	0.057	0.055	0.030	0.053	0.076
H4	**0.920**	0.645	0.651	0.613	0.762	0.908
H5	**0.864**	0.242	0.231	0.218	0.386	0.556
H6	**0.115**	0.085	0.078	0.037	0.007	0.005

Preliminary results show outperformance across scenarios

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11. Wrap-up

A quick digression: assuming a return distribution

- The SEPP ES test was first designed with return distribution assumptions, using a Probability Integral Transform as in Du & Escanciano (2017)
- Let us create fair comparison between the SEPP ES test and the remainder of ES backtests
- Monte Carlo analysis was performed:
  - Null model: AR(1)-GARCH(1,1)-n
  - Alternatives:
    - AR(1)-GARCH(1,1)-t: Tails are thicker → more violations/clusters
    - AR(1)-GARCH(1,1)-M: returns are higher when volatility higher → fewer clusters in the lower tail of the distribution
    - AR(1)-EGARCH(1,1)-n: asymmetric response to large losses → more clusters
    - GARCH-HS: persistent returns → more clusters
    - Markov process with two volatility regimes → more violations/clusters

ES backtesting – Monte Carlo simulations

- SEPP ES backtest significantly outperforms all existing tests in most cases
- All models perform dissatisfactory for M2 (less clustering than anticipated by the null) and M3 (asymmetric responses)
- The SEPP ES especially outperforms under the sudden volatility shocks in the M5 specification
- Similar conclusions for T = 1000 and n = 250

Table 4-1. Size and power comparison (T=2500)

α	DGP	McNeil	Z_1	Z_2	Z_{ES}	Kratz	SEPP	BSEPP	U	MU	C	MC
						n=500						
0.025	H0	0.055	0.052	0.058	0.062	0.051	0.055	0.062	0.061	0.052	0.088	0.074
	M1	0.337	0.571	0.155	0.469	0.190	0.617	**0.634**	0.041	0.032	0.076	0.062
	M2	0.056	0.059	0.066	0.065	0.031	0.060	0.067	**0.094**	0.083	**0.094**	0.081
	M3	0.172	0.292	0.084	0.296	0.123	0.335	**0.351**	0.016	0.011	0.022	0.019
	M4	0.733	0.759	0.581	0.863	0.666	0.866	**0.878**	0.344	0.123	0.070	0.049
	M5	0.536	0.628	0.388	0.720	0.470	0.775	**0.781**	0.174	0.143	0.072	0.055
0.05	H0	0.052	0.048	0.066	0.065	0.053	0.058	0.063	0.079	0.067	0.082	0.073
	M1	0.390	0.611	0.087	0.316	0.124	0.625	**0.630**	0.064	0.052	0.076	0.065
	M2	0.052	0.059	0.079	0.075	0.037	0.061	0.067	**0.118**	0.103	0.093	0.081
	M3	0.251	**0.368**	0.023	0.183	0.081	0.331	0.338	0.020	0.013	0.017	0.014
	M4	0.844	**0.917**	0.254	0.759	0.603	0.911	0.914	0.218	0.086	0.072	0.051
	M5	0.638	0.759	0.207	0.587	0.358	0.809	**0.815**	0.168	0.141	0.119	0.097

Note: In-sample size equals T=2500. The tests are performed at the 5%-significance level. The best power per DGP is highlighted.

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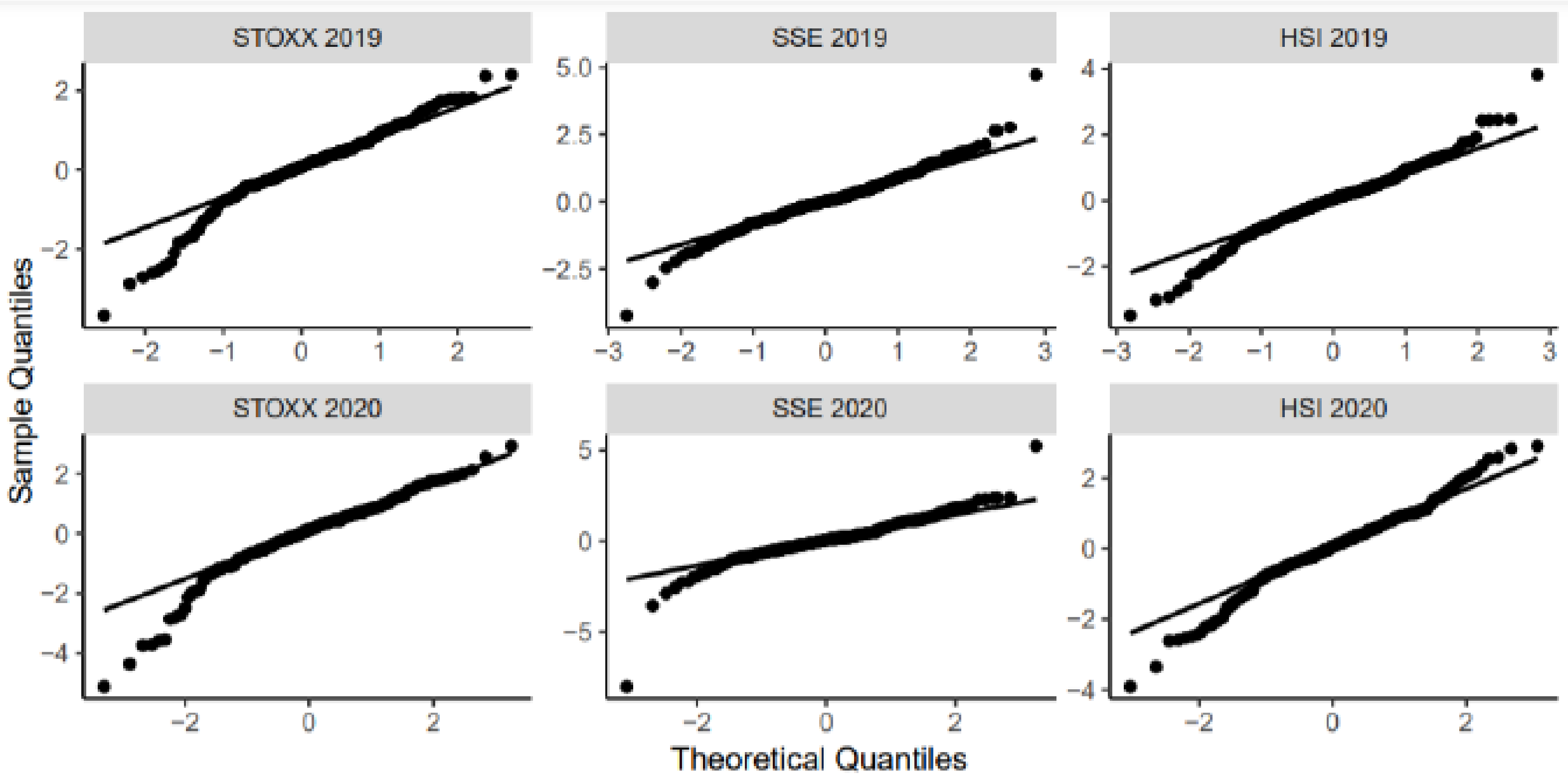
11. Wrap-up

Empirical example: the 2020 stock market crash

- STOXX 600, Shanghai Composite, and Hang Seng
- AR(1)-GARCH(1,1) with Gaussian innovations is fitted
- Q-Q plots and normality tests reject Gaussian distribution of residuals



Empirical example: the 2020 stock market crash

- During 2020:

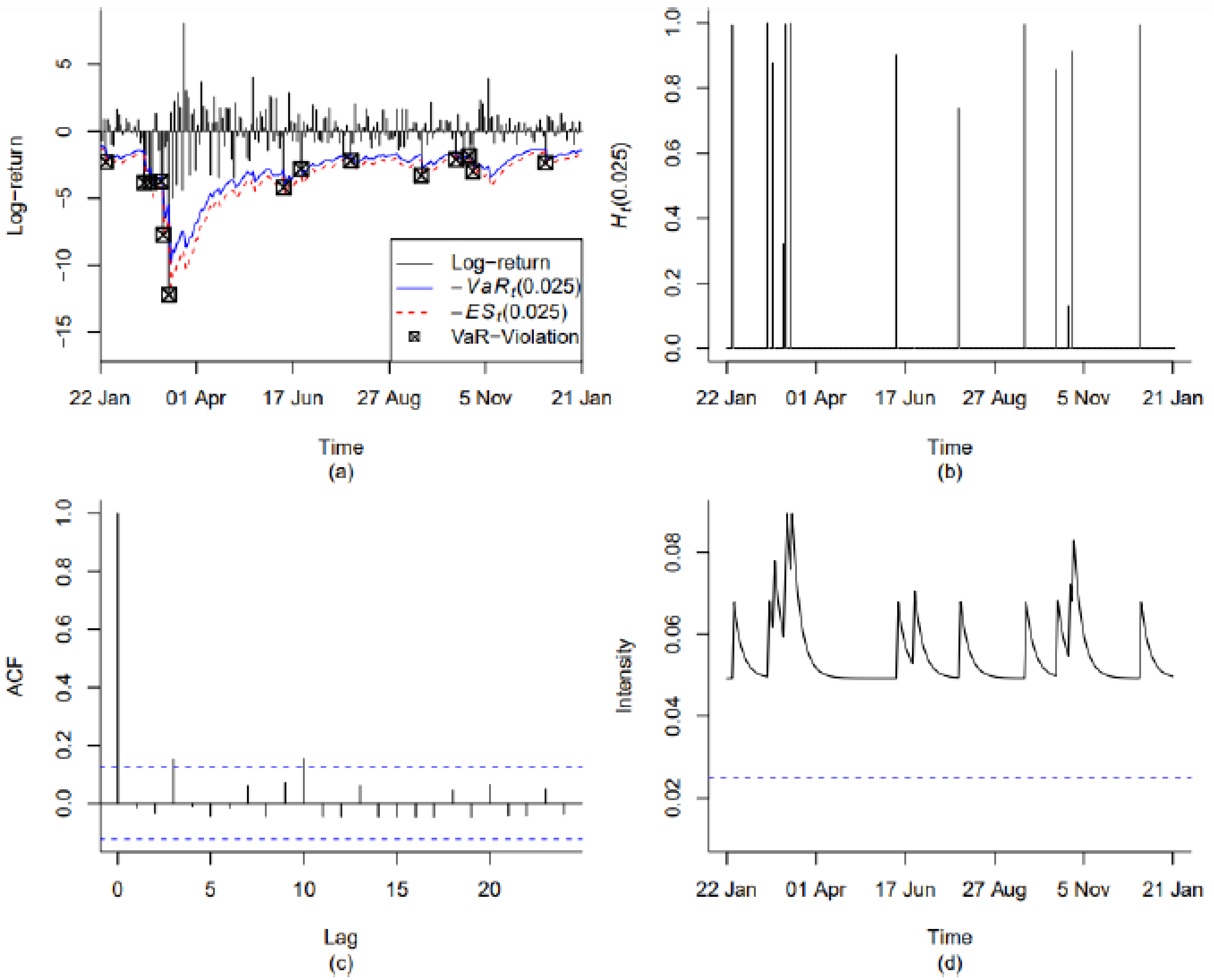


Figure 5-2. STOXX: VaR & ES (a), cumulative violations (b), ACF of cumulative violations (c), and estimated intensity process (d)

Empirical example: the 2020 stock market crash

- During 2019:

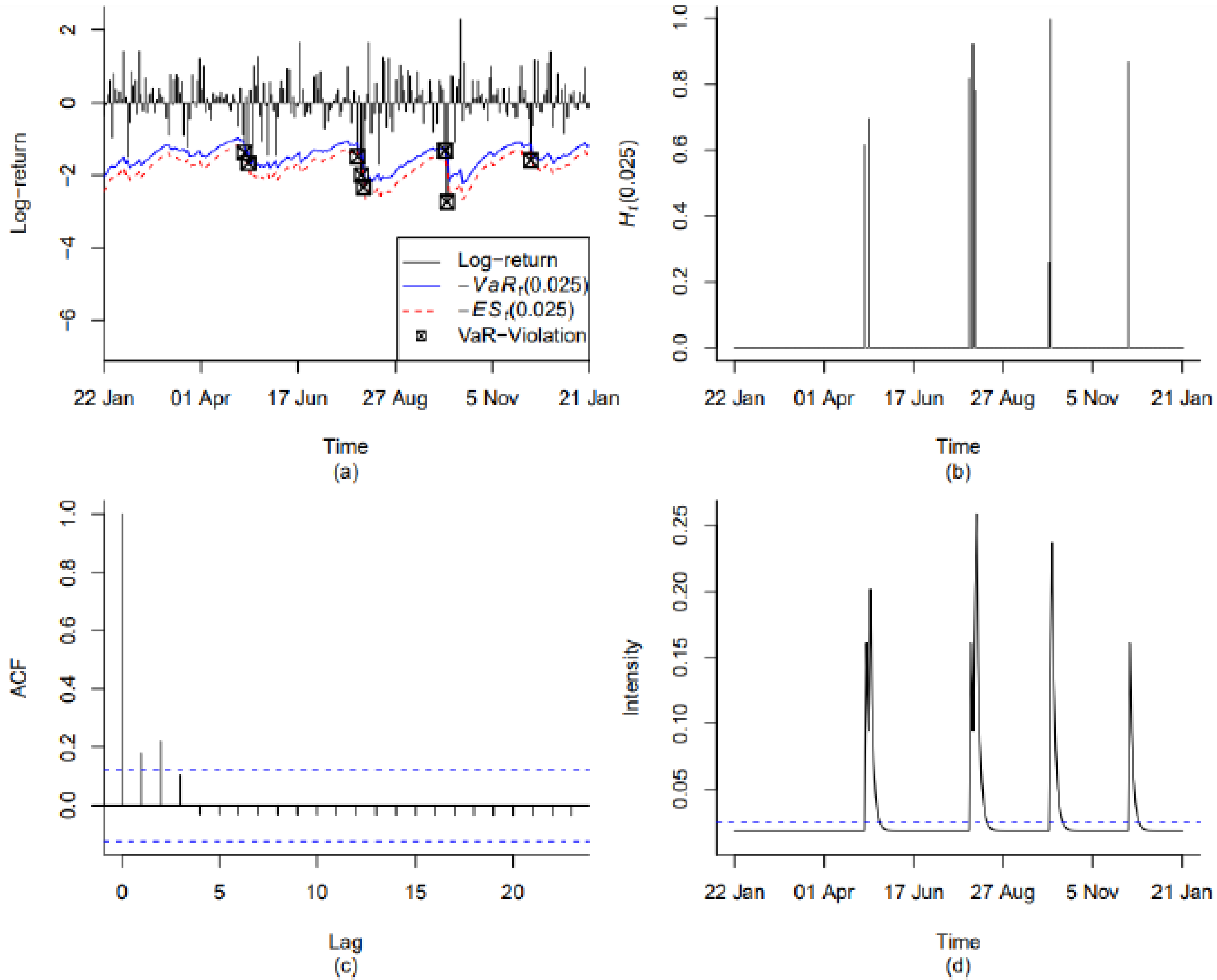


Figure 5-5. STOXX: VaR & ES (a), cumulative violations (b), ACF of cumulative violations (c), and estimated intensity process (d)

Empirical example: the 2020 stock market crash

- During 2020:

Table 5-4. Backtesting p-values

α	Index	McNeil	Z_1	Z_2	Z_{ES}	Kratz	SEPP	BSEPP	U	MU	C	MC
0.025	STOXX	**0.003**	**0.006**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.014**	**0.021**	0.149	0.172
	SSE	0.112	**0.004**	0.121	**0.000**	0.218	**0.000**	**0.000**	0.370	0.959	**0.021**	**0.035**
	HSI	0.187	0.271	**0.002**	**0.002**	**0.027**	**0.001**	**0.002**	**0.033**	0.696	**0.000**	0.351
0.05	STOXX	**0.000**	**0.000**	**0.004**	**0.000**	**0.000**	**0.000**	**0.000**	**0.030**	**0.034**	0.223	0.324
	SSE	0.058	**0.000**	0.558	**0.004**	0.218	**0.000**	**0.000**	0.718	0.978	0.278	0.281
	HSI	**0.006**	**0.019**	**0.050**	**0.003**	**0.027**	**0.000**	**0.000**	0.056	0.564	**0.001**	0.611

- During 2019:

Table 5-6. Backtesting p-values

α	Index	McNeil	Z_1	Z_2	Z_{ES}	Kratz	SEPP	BSEPP	U	MU	C	MC
0.025	STOXX	**0.034**	0.064	0.254	**0.028**	0.066	**0.002**	**0.002**	0.190	0.760	**0.000**	0.924
	SSE	0.182	**0.017**	0.934	0.136	0.684	**0.003**	**0.002**	0.844	0.992	0.996	1.000
	HSI	0.072	0.058	0.240	**0.028**	0.063	**0.032**	**0.021**	0.200	0.964	0.259	0.705
0.05	STOXX	0.157	0.085	0.400	0.096	0.066	**0.026**	**0.019**	0.445	0.891	**0.000**	0.016
	SSE	0.106	**0.005**	0.462	0.428	0.684	**0.002**	**0.002**	0.737	0.990	0.894	0.907
	HSI	0.059	**0.039**	0.333	**0.044**	0.063	**0.043**	**0.030**	0.265	0.977	0.552	0.614

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Wrap-up

- SEPP implementation in VaR and ES backtesting significantly boosts power by providing a more flexible outlier intensity specification
- SEPP ES backtest is the first powerful conditional ES backtest and more powerful than unconditional ES backtests
- Only requires PnL and reported VaR & ES
- Looking ahead:
  - ES backtesting will become relevant to banks with the introduction of FRTB (especially when EBA pushes through its RTS)
  - With a lack of powerful ES backtests and no guidance from the regulator, this will become a difficult task
  - A move towards joint VaR-ES backtesting is more attractive (you have the information anyway!)
  - The SEPP ES backtest is a powerful solution

Thank you!

Questions?

ES backtesting – Monte Carlo simulations

- SEPP-ES backtest significantly outperforms all existing tests in most cases
- All models perform bad for M2 (less clustering than anticipated by the null) and M3 (asymmetric responses)
- Apart from M2, best performers are SEPP-ES, Z_1, and Z_{ES}
- Size of SEPP-ES is among the most accurate (Bootstrap SEPP is slightly oversized)

Table 4-1. Size and power comparison (T=2500)

α	DGP	McNeil	Z_1	Z_2	Z_{ES}	Kratz	SEPP	BSEPP	U	MU	C	MC
						n=250						
0.025	H0	0.064	0.053	0.063	0.059	0.057	0.049	0.059	0.086	0.078	0.096	0.081
	M1	0.128	0.356	0.128	0.340	0.137	0.432	**0.434**	0.049	0.042	0.090	0.075
	M2	0.070	0.063	0.059	0.059	0.040	0.049	0.053	**0.116**	0.108	0.105	0.091
	M3	0.063	0.162	0.058	0.189	0.090	0.218	**0.227**	0.023	0.020	0.034	0.028
	M4	0.328	0.429	0.374	0.646	0.428	0.638	**0.654**	0.103	0.043	0.077	0.056
	M5	0.187	0.334	0.247	0.494	0.287	0.532	**0.549**	0.060	0.051	0.080	0.062
0.05	H0	0.051	0.050	0.063	0.062	0.053	0.048	0.056	0.086	0.079	0.083	0.076
	M1	0.169	0.402	0.084	0.243	0.068	0.421	**0.439**	0.077	0.069	0.091	0.078
	M2	0.053	0.060	0.070	0.063	0.036	0.047	0.054	**0.118**	0.107	0.091	0.084
	M3	0.105	**0.239**	0.018	0.112	0.045	0.215	0.229	0.025	0.021	0.026	0.022
	M4	0.484	0.682	0.174	0.530	0.317	0.699	**0.711**	0.103	0.044	0.074	0.054
	M5	0.289	0.500	0.156	0.400	0.182	0.572	**0.596**	0.105	0.094	0.118	0.100
						n=500						
0.025	H0	0.055	0.052	0.058	0.062	0.051	0.055	0.062	0.061	0.052	0.088	0.074
	M1	0.337	0.571	0.155	0.469	0.190	0.617	**0.634**	0.041	0.032	0.076	0.062
	M2	0.056	0.059	0.066	0.065	0.031	0.060	0.067	**0.094**	0.083	**0.094**	0.081
	M3	0.172	0.292	0.084	0.296	0.123	0.335	**0.351**	0.016	0.011	0.022	0.019
	M4	0.733	0.759	0.581	0.863	0.666	0.866	**0.878**	0.344	0.123	0.070	0.049
	M5	0.536	0.628	0.388	0.720	0.470	0.775	**0.781**	0.174	0.143	0.072	0.055
0.05	H0	0.052	0.048	0.066	0.065	0.053	0.058	0.063	0.079	0.067	0.082	0.073
	M1	0.390	0.611	0.087	0.316	0.124	0.625	**0.630**	0.064	0.052	0.076	0.065
	M2	0.052	0.059	0.079	0.075	0.037	0.061	0.067	**0.118**	0.103	0.093	0.081
	M3	0.251	**0.368**	0.023	0.183	0.081	0.331	0.338	0.020	0.013	0.017	0.014
	M4	0.844	**0.917**	0.254	0.759	0.603	0.911	0.914	0.218	0.086	0.072	0.051
	M5	0.638	0.759	0.207	0.587	0.358	0.809	**0.815**	0.168	0.141	0.119	0.097

Note: In-sample size equals T=2500. The tests are performed at the 5%-significance level. The best power per DGP is highlighted.

ES backtesting – Monte Carlo simulations

- T=1000 reaches similar findings
- All tests become oversized, but not yet problematic
- The SEPP-ES especially outperforms under the sudden volatility shocks in the M5 specification
- For T=500 and T=250, similar findings:
  - Size becomes >0.10 (T=500) and >0.20 (T=250) for most tests, except for MU and MC
  - Power statistics remain comparable

Table 4-2. Size and power comparison (T=1000)

α	DGP	McNeil	Z_1	Z_2	Z_{ES}	Kratz	SEPP	BSEPP	U	MU	C	MC
						n=250						
0.025	H0	0.072	0.061	0.077	0.075	0.069	0.069	0.079	0.104	0.085	0.100	0.071
	M1	0.136	0.358	0.145	0.359	0.156	0.444	**0.458**	0.053	0.036	0.092	0.066
	M2	0.075	0.070	0.077	0.075	0.054	0.066	0.081	**0.127**	0.107	0.112	0.081
	M3	0.066	0.179	0.068	0.207	0.103	0.236	**0.250**	0.022	0.014	0.034	0.024
	M4	0.351	0.455	0.368	0.643	0.430	0.643	**0.660**	0.112	0.054	0.075	0.046
	M5	0.205	0.345	0.275	0.506	0.303	0.560	**0.579**	0.072	0.046	0.101	0.064
0.05	H0	0.056	0.059	0.085	0.087	0.063	0.067	0.074	0.112	0.087	0.087	0.068
	M1	0.171	0.403	0.113	0.262	0.079	0.436	**0.450**	0.090	0.068	0.092	0.071
	M2	0.059	0.068	0.089	0.089	0.044	0.070	0.076	**0.137**	0.116	0.099	0.079
	M3	0.106	0.241	0.038	0.131	0.054	0.234	**0.246**	0.035	0.020	0.024	0.018
	M4	0.482	0.674	0.193	0.529	0.317	0.691	**0.706**	0.114	0.058	0.078	0.050
	M5	0.313	0.512	0.181	0.418	0.205	0.592	**0.614**	0.115	0.083	0.138	0.101
						n=500						
0.025	H0	0.055	0.055	0.084	0.086	0.062	0.075	0.079	0.072	0.054	0.093	0.065
	M1	0.349	0.567	0.187	0.490	0.215	0.617	**0.620**	0.064	0.034	0.080	0.059
	M2	0.055	0.065	0.082	0.082	0.043	0.078	0.079	0.098	0.072	**0.104**	0.070
	M3	0.195	0.316	0.114	0.322	0.147	0.356	**0.366**	0.028	0.011	0.022	0.015
	M4	0.737	0.775	0.565	0.854	0.654	0.862	**0.868**	0.349	0.148	0.071	0.041
	M5	0.535	0.635	0.414	0.714	0.478	0.774	**0.783**	0.196	0.122	0.090	0.050
0.05	H0	0.058	0.059	0.091	0.094	0.070	0.084	0.091	0.100	0.065	0.084	0.061
	M1	0.406	0.618	0.133	0.359	0.160	0.646	**0.648**	0.098	0.062	0.089	0.062
	M2	0.057	0.065	0.100	0.096	0.058	0.082	0.089	**0.131**	0.096	0.093	0.070
	M3	0.259	0.379	0.056	0.210	0.101	0.358	**0.361**	0.038	0.021	0.017	0.012
	M4	0.832	**0.906**	0.277	0.745	0.583	0.902	0.903	0.233	0.111	0.078	0.045
	M5	0.639	0.766	0.242	0.596	0.366	0.812	**0.815**	0.197	0.134	0.015	0.096

Note: In-sample size equals T=1000. The tests are performed at the 5%-significance level. The best power per DGP is highlighted.