Liquidity Risk in the Sovereign Credit Default Swap Market

Rob Sperna Weiland

University of Amsterdam

Rabobank
Model Validation

18-11-2015


IRLAND
GREECE
PORTUGAL
SPAIN

Problem Description

Problem:

- Estimate sovereign default probabilities.

Proposed solution:

- Extract market-implied default probabilities from Credit Default Swap (CDS) data.

Problem Description

Problem:

- Estimate sovereign default probabilities.

Proposed solution:

- Extract market-implied default probabilities from Credit Default Swap (CDS) data.

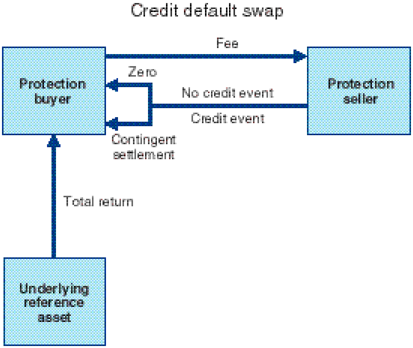


Figure: Schematic representation CDS contract

Problem Description

CDS premia may be "contaminated" by price-distorting factors such as *liquidity risk*:

- Not many market participants.
- Over-the-counter market $\Rightarrow$ search costs.
- Large observed bid-ask spreads.

Problem Description

CDS premia may be “contaminated” by price-distorting factors such as *liquidity risk*:

- Not many market participants.
- Over-the-counter market $\Rightarrow$ search costs.
- Large observed bid-ask spreads.

Research Goals

1. Quantify the effect of liquidity risk on sovereign CDS premia.
2. Extract implied PD’s (and account for the liquidity risk component).

General Methodology

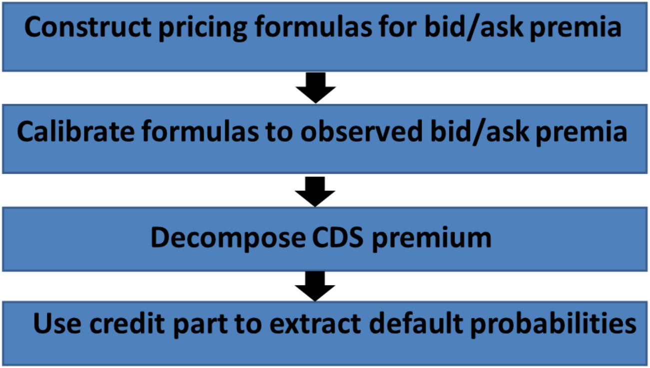


Figure: General methodology

Table of Contents

How to model default risk?

Intensity-based model:

- Default time is the first jump time of some jump process (e.g. Poisson process).
- Default probability is driven by an intensity process.
- Closely related to short-rate interest rate models.

How to capture liquidity effects?

Separate formulas for bid and ask premia with

- the same default components
- and different *liquidity discount factors*.

Some Notation

We have the following *stochastic* processes:

- $r(t)$: risk-free interest rate.
- $\lambda(t)$: default intensity process.
- $\gamma^{bid/ask}(t)$: bid/ask liquidity intensity processes.

They appear as the following 'discount' factors:

- $\bar{D}(t, T) = e^{-\int_t^T r(s)\mathrm{d}s}$.
- $\bar{L}^{bid/ask}(t, T) = e^{-\int_t^T \gamma^{bid/ask}(s)\mathrm{d}s}$.
- $\bar{P}(t, T) = e^{-\int_t^T \lambda(s)\mathrm{d}s}$.

Model-Implied Bid and Ask Premia

We obtain the following model prices:

$$s^{bid}(t,T) = \frac{(1-R)\cdot\sum_{i=1}^{n}\mathbb{E}^{\mathbb{Q}}\left[\left(\bar{P}(t,T_{i-1})-\bar{P}(t,T_i)\right)\bar{D}(t,T_i)\middle|\mathcal{F}_t\right]}{\sum_{i=1}^{n}\delta_i\mathbb{E}^{\mathbb{Q}}\left[\bar{D}(t,T_i)\bar{P}(t,T_i)\bar{L}^{bid}(t,T_i)\middle|\mathcal{F}_t\right]}$$

and

$$s^{ask}(t,T) = \frac{(1-R)\cdot\sum_{i=1}^{n}\mathbb{E}^{\mathbb{Q}}\left[\left(\bar{P}(t,T_{i-1})-\bar{P}(t,T_i)\right)\bar{D}(t,T_i)\middle|\mathcal{F}_t\right]}{\sum_{i=1}^{n}\delta_i\mathbb{E}^{\mathbb{Q}}\left[\bar{D}(t,T_i)\bar{P}(t,T_i)\bar{L}^{ask}(t,T_i)\middle|\mathcal{F}_t\right]},$$

where $\delta_i = T_i - T_{i-1}$ and $\delta_1 = T_1 - t$.

Stochastic Set-Up (1)

- The risk-free short-rate process is independent from the liquidity and default intensity processes.
- The risk-free discount curve is given by the (implied) swap rate curve.

$$s^{ask}(t,T) = \frac{(1-R)\cdot\sum_{i=1}^{n} D(t,T_i)\mathbb{E}^{\mathbb{Q}}\left[\left(\bar{P}(t,T_{i-1}) - \bar{P}(t,T_i)\right)\middle|\,\mathcal{F}_t\right]}{\sum_{i=1}^{n}\delta_i D(t,T_i)\mathbb{E}^{\mathbb{Q}}\left[\bar{P}(t,T_i)\bar{L}^{ask}(t,T_i)\middle|\,\mathcal{F}_t\right]}$$

and

$$s^{bid}(t,T) = \frac{(1-R)\cdot\sum_{i=1}^{n} D(t,T_i)\mathbb{E}^{\mathbb{Q}}\left[\left(\bar{P}(t,T_{i-1}) - \bar{P}(t,T_i)\right)\middle|\,\mathcal{F}_t\right]}{\sum_{i=1}^{n}\delta_i D(t,T_i)\mathbb{E}^{\mathbb{Q}}\left[\bar{P}(t,T_i)\bar{L}^{bid}(t,T_i)\middle|\,\mathcal{F}_t\right]},$$

Stochastic Set-Up (2)

$$\begin{pmatrix} \mathrm{d}\lambda(t) \\ \mathrm{d}\gamma^{bid}(t) \\ \mathrm{d}\gamma^{ask}(t) \end{pmatrix} = \begin{pmatrix} 1 & g_{bid} & g_{ask} \\ f_{bid} & 1 & \omega_{ask,bid} \\ f_{ask} & \omega_{bid,ask} & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}x(t) \\ \mathrm{d}y^{bid}(t) \\ \mathrm{d}y^{ask}(t) \end{pmatrix}.$$

- $x(t), y^{bid}(t)$ and $y^{ask}(t)$ are *pure* default and liquidity intensities and are *independent*.

Example of affine process dynamics:

$$\begin{aligned} \mathrm{d}x(t) &= (\alpha - \beta x(t))\mathrm{d}t + \sigma\sqrt{x(t)}\mathrm{d}W_x^{\mathbb{Q}}(t) \\ \mathrm{d}y^l(t) &= \sigma^l \mathrm{d}W_{y^l}^{\mathbb{Q}}(t), \qquad l \in \{bid, ask\}. \end{aligned}$$

Analytical Solution to Discount Factors

Recall bid price formula:

$$s^{bid}(t,T) = \frac{(1-R)\cdot\sum_{i=1}^{n} D(t,T_i)\mathbb{E}^{\mathbb{Q}}\left[\left(\bar{P}(t,T_{i-1})-\bar{P}(t,T_i)\right)\middle|\mathcal{F}_t\right]}{\sum_{i=1}^{n}\delta_i D(t,T_i)\mathbb{E}^{\mathbb{Q}}\left[\bar{P}(t,T_i)\bar{L}^{bid}(t,T_i)\middle|\mathcal{F}_t\right]},$$

By our set-up:

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}}\left[\bar{P}(t,T_i)\bar{L}^{bid}(t,T_i)\middle|\mathcal{F}_t\right] &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{T_i}\lambda(s)\mathrm{d}s}e^{-\int_t^{T_i}\gamma^{bid}(s)\mathrm{d}s}\middle|\mathcal{F}_t\right] \\
&= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{T_i}(1+f_{bid})x(s)\mathrm{d}s}\middle|\mathcal{F}_t\right] \\
&\quad\times\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{T_i}(1+g_{bid})y^{bid}(s)\mathrm{d}s}\middle|\mathcal{F}_t\right] \\
&\quad\times\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{T_i}(\omega_{ask,bid}+g_{ask})y^{ask}(s)\mathrm{d}s}\middle|\mathcal{F}_t\right].
\end{aligned}$$

Pricing formulas are reduced to bond price formulas!

Analytical Solution to Discount Factors (2)

Here we have

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{T_i}(1+f_{bid})x(s)\mathrm{d}s} \middle| \mathcal{F}_t \right] = A_1(t, T_i, 1+f_{bid}) e^{-A_2(t,T_i,1+f_{bid})(1+f_{bid})x(t)},$$

where

$$\begin{aligned} A_1(t, T_i, 1+f_{bid}) &= \frac{2\alpha}{\sigma^2} \log\left(\frac{2\phi(1+f_{bid}) e^{\phi(1+f_b)+\beta)(T_i-t)/2}}{(\phi(1+f_{bid})+\beta)(e^{\phi(1+f_{bid})(T_i-t)}-1)+2\phi(1+f_{bid})} \right), \\ \phi(1+f_{bid}) &= \sqrt{\beta^2+2(1+f_{bid})\sigma^2}, \\ A_2(t, T_i, 1+f_{bid}) &= \frac{2\left(e^{\phi(1+f_{bid})(T_i-t)}-1\right)}{(\phi(1+f_{bid})+\beta)(e^{\phi(1+f_{bid})(T_i-t)}-1)+2\phi(1+f_{bid})}. \end{aligned}$$

Cox, Ingersoll, Ross (CIR) bond price formula.

Parameters to be Calibrated

We have the following parameters we need to calibrate:

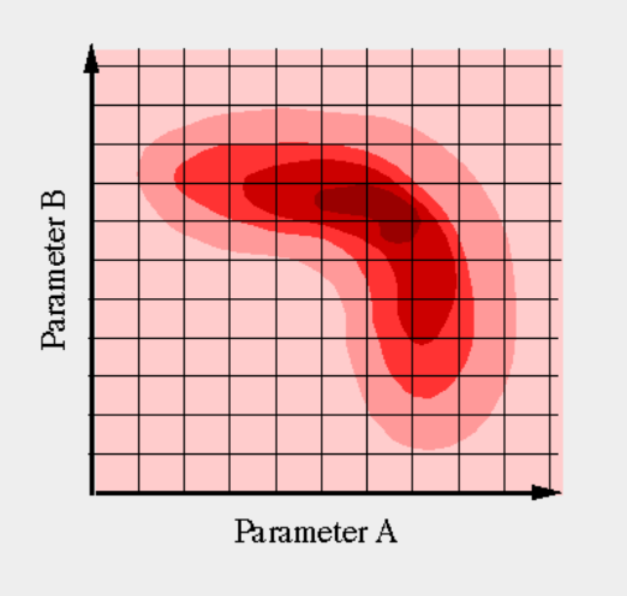
- 5 process dynamics parameters $(\alpha, \beta, \sigma, \sigma^{bid}, \sigma^{ask})$.
- 6 correlation factor parameters $(f_{bid}, f_{ask}, g_{bid}, g_{ask}, \omega_{ask,bid}, \omega_{bid,ask})$.
- The pure intensity values $(x(t), y^{bid}(t), y^{ask}(t))$ for every day t in our sample (challenge).

Table of Contents

Proposed Calibration Procedure

- Test our model on Brazil and Turkey.
- Bid and ask premia of 2, 3, 5 and 10 year CDSs (so 8 quotes per day).
- 1200 days of data (period 01-06-2009 until 28-02-2014).

Proposed Calibration Procedure (2)



Proposed Calibration Procedure (3)

In each grid point, we

1. fix the values of the process parameters (here $\alpha, \beta, \sigma, \sigma^{ask}, \sigma^{bid}$), and
2. calibrate the other parameters assuming the fixed process parameters.

The "best" grid point gives the parameter values.

Implementation Issues

Problems:

- Dimension of the grid is equal to number of process parameters.
- Number of points blows up exponentially with more general processes (more parameters).

Implementation Issues

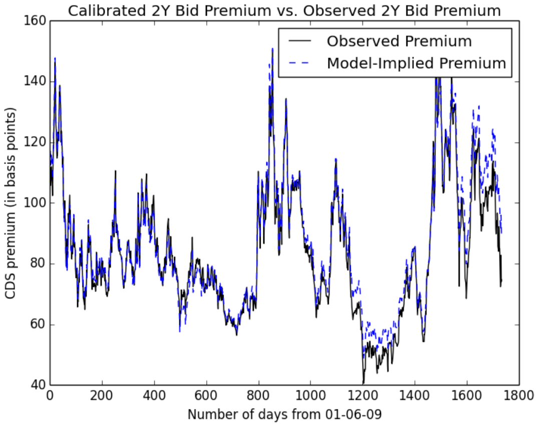
Problems:

- Dimension of the grid is equal to number of process parameters.
- Number of points blows up exponentially with more general processes (more parameters).

Tricks to speed up the calibration:

- Parallel programming.
- "Zooming into the grid" instead of dense grids.
- Use search algorithm over the grid.

Model Fit



- Brazil: average relative pricing error of 2.05%.
- Turkey: average relative pricing error of 2.44%.

Table of Contents

Decomposition of CDS Premium

- We define the pure credit risk premium s^{cred} by setting $\bar{L}^{bid/ask} = 1$ (i.e., $\gamma^{bid/ask} = 0$). In this case the (model) bid and ask premia are equal.
- We define the liquidity risk premium as $s^{liq} = s^{mid} - s^{cred}$, where s^{mid} is the model-implied mid-premium.

Decomposition of CDS Premium (2)

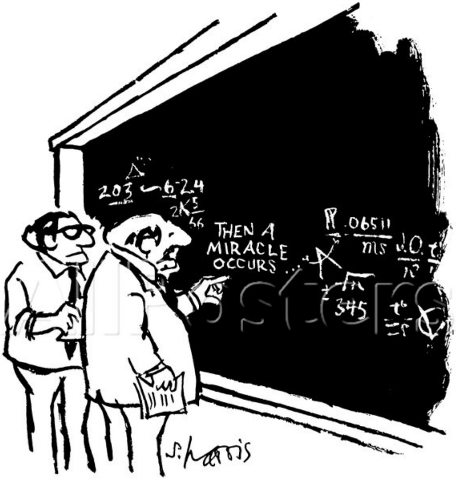
Decomposition 2 Year CDS Mid Premium					
Brazil			**Turkey**		
Credit Part	mean	0.6175	Credit Part	mean	0.5783
	Std. Dev.	0.0249		Std. Dev.	0.0277
	Max	0.6677		Max	0.6398
	Min	0.5494		Min	0.5181
Liquidity Part	mean	0.3825	Liquidity Part	mean	0.4217
	Std. Dev.	0.0227		Std. Dev.	0.0272
	Max	0.4417		Max	0.4802
	Min	0.3379		Min	0.3602

Table: Decomposition of 2 year CDS premia. Entries are denoted as a fraction of the mid-premium.

Table of Contents

Change of Probability Measure

- So far, we have worked under the risk-neutral probability measure $\mathbb{Q}$.
- We are interested in the real-world default probability and, therefore, we need do an analysis under the real-world probability measure $\mathbb{P}$.



"I think you should be more explicit here in step two."

Estimated Default Probabilities

Average One-Year Default Probabilities Brazil and Turkey			
Brazil		**Turkey**	
Our result	Rabobank	Our result	Rabobank
27.84	-	56.44	-

Table: Estimated one-year default probabilities Brazil and Turkey (in basis points) using average intensities.

Our estimates are very close to Rabobank's estimates!

Table of Contents

Conclusions

- We developed a methodology to extract default probabilities from CDS premia, while accounting for liquidity risk.
- Our model allows for an easy and natural decomposition of the CDS premia into credit and liquidity components.

Main findings:

- Liquidity risk has a large impact on sovereign CDS premia.
- Our PD estimates are close to Rabobank's estimates.

Questions?