Risk Analytics & Models

Valuation with Liquidity Risk

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Motivation

Three nagging problems

- Difference between markets valuation (OIS discounting) and Banking book management (based on FTP).
- Funding Valuation Adjustment is an adjustment on the value of derivatives to include funding costs. However, one (main) reason for funding longer term is liquidity risk.
- Liquidity Risk is a risk! Hence if valuation should include all risks, how to include liquidity risk?

The Question

- **How can a bank include liquidity risk in the valuation of its assets?**

- Consistent valuation of
  - Loans/mortgages vs bonds
  - OTC derivatives vs collateralized derivatives

Literature on Liquidity Risk

Regulatory/Risk Management

- Principles for Sound Liquidity Risk Management and Supervision, BCBS 2008
- Internal Liquidity Adequacy Assessment Process (ILAAP)
- LCR and NSFR in Basel 3.

CAPM extensions

- Main conclusion of this branch of research: investors require compensation for investing in less liquid assets (Acharya Pedersen,…)

Practical approaches

- Increase the discount rate by a liquidity premium for illiquidity of the asset. The liquidity premium could be chosen based on own risk-appetite or benchmarked in the market.

What is Liquidity Risk?

Definition in BCBS (Principles for Sound Liquidity Risk Management and Supervision (2008).)**:**

Funding liquidity risk is the risk that the firm will not be able to meet efficiently both expected and unexpected current and future cashflow and collateral needs without affecting either daily operations or the financial condition of the firm. Market liquidity risk is the risk that a firm cannot easily offset or eliminate a position at the market price because of inadequate market depth or market disruption.

Definition I will use for valuation

Liquidity risk is the risk for an event to occur that forces a bank to liquidate some of its assets.

We will call such an event a Liquidity Stress Event (LSE)

Liquidity Risk Model - approach

1. When a LSE hits the bank, no (new) funding can be obtained during the duration of the LSE

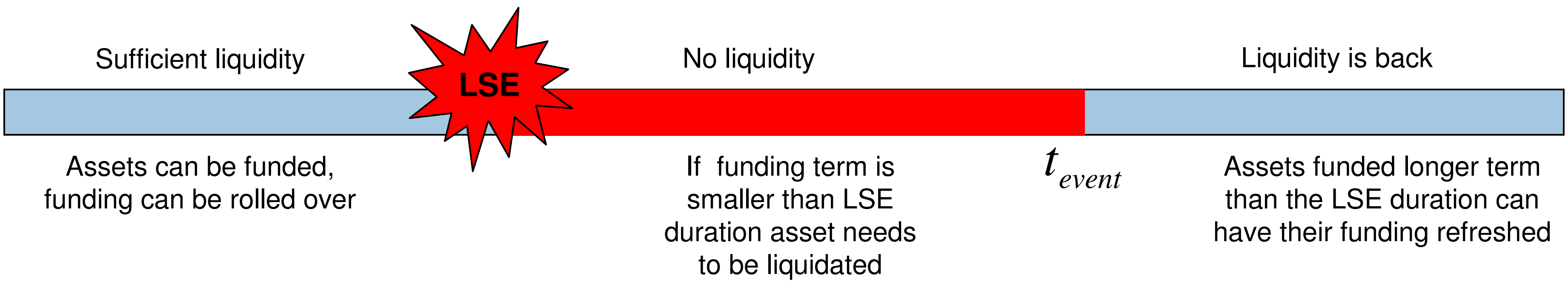
2. An asset whose funding expires during an LSE needs to be liquidated

3. The liquidation of an asset results in a loss depending on the liquidity of the asset

4. Determine the expected loss depending on the asset’s liquidity and funding term

5. Determine the optimal funding term and value of the asset can be determined

Liquidity Risk Model – Liquidity Stress Event



1. Consider a Poisson process for the occurrence of LSEs with intensity λ

the duration of an LSE is random. We choose a lognormal distribution

$$\boxed{t_{\text{event}} \sim \rho_{LN}(\mu, \sigma)}$$

In the examples we will use
$$\mu = \ln(0.5)$$
$$\sigma = 0.5$$

If the funding term is insufficient for the asset to 'survive' the event the asset will be liquidated resulting in a loss.

Liquidity Risk Model – Liquidation Value

Mathematical formulation

2. Next we need the liquidation value for an asset depending on funding term t_{liq} We choose

$$\boxed{LV(t_{\text{event}}, t_{\text{liq}}) = \max(1 - c(t_{\text{event}} - t_{\text{liq}})I_{t_{\text{event}} > t_{\text{liq}}}, LV_{\text{min}})}$$

Note that in the limit c to infinity:

$$\begin{aligned} LV = 1 &\qquad \text{if } t_{\text{event}} \leq t_{\text{liq}} \\ LV = LV_{\text{min}} &\qquad \text{if } t_{\text{event}} > t_{\text{liq}} \end{aligned}$$

The expected liquidation value is

$$\begin{aligned} \mathbb{E}[LV(t_{\text{event}}, t_{\text{liq}})] &= N(\mu, \sigma; \log(t_{\text{liq}})) \\ &+ (1 + ct_{\text{liq}})N(\mu, \sigma; \log(t_{\text{liq}}), \log(t_m)) \\ &- ce^{\mu+\sigma^2/2}N(\mu + \sigma^2, \sigma; \log(t_{\text{liq}}), \log(t_m)) \\ &+ LV_{\text{min}}(1 - N(\mu, \sigma; \log(t_m))) \,, \end{aligned}$$

with

$$\boxed{t_m = t_{\text{liq}} + (1 - LV_{\text{min}})/c}$$

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Liquidity Risk Model – Liquidity Costs

3. Liquidity spread and expected liquidity costs

The value of a bullet loan at time 0 is given by

$$\boxed{\begin{aligned} V_0 &= e^{-rT}\mathbb{P}(\tau + t_{\text{liq}} \geq T) + e^{-rT}\mathbb{E}[LV]\mathbb{P}(\tau + t_{\text{liq}} < T) \\ &= e^{-rT}e^{-\lambda(T-t_{\text{liq}})} + e^{-rT}\mathbb{E}[LV](1 - e^{-\lambda(T-t_{\text{liq}})}) \end{aligned}}$$

For a small intensity this may be approximated by

$$\boxed{V_0 \approx e^{-rT}\left\{1 - \lambda(1 - \mathbb{E}[LV])(T - t_{\text{liq}})\right\}}$$

The liquidity spread may be defined as

$$\boxed{l(t_{\text{liq}}) = \lambda(1 - \mathbb{E}[LV])}$$

The liquidity costs (expected loss due to liquidation) are then

$$\boxed{LC = \lambda(1 - \mathbb{E}[LV])\frac{(T - t_{\text{liq}})}{T}}$$

Liquidity Risk Model – Funding

3. Multiple curves and funding costs

For derivatives pricing usually OIS is used as discount Rate. However now we use a funding term that may be larger than overnight (ON). Therefore

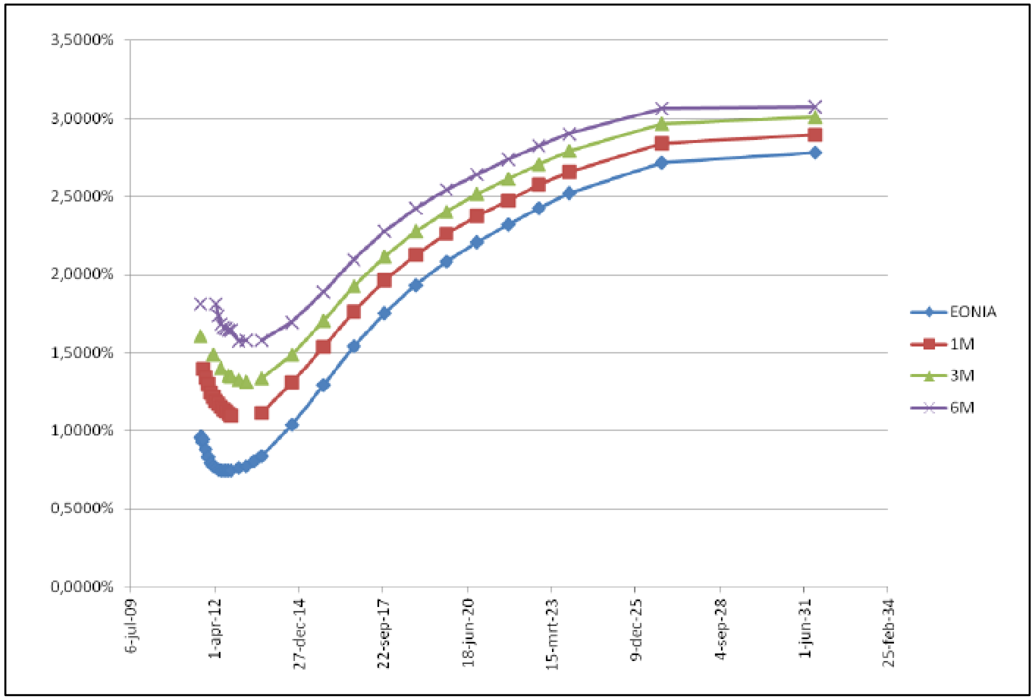
$$\boxed{r \approx L_{t_{\text{liq}}}(0,0,T)}$$

Using simple compounding:

$$\boxed{e^{-rT} = 1 - L_{t_{\text{liq}}}(0,0,T)T}$$

Incremental funding costs may be defined as

$$\boxed{FC = \left[L_{t_{\text{liq}}}(0,0,T) - L_{ON}(0,0,T)\right]}$$



Source: T. Broekhuizen – Multiple discount and forward curves, Topquants presentation nov 20

Liquidity Risk Model – Valuation

3. Liquidity spread and expected liquidity costs

The minimum of the funding plus liquidity costs, corresponds to the optimum value:

$$\boxed{V = \sup_{0 < t_{\mathrm{liq}} \leq T} V_0(t_{\mathrm{liq}})}$$

This supremum may be found by minimizing the sum of liquidity costs and funding costs

$$\boxed{\min_{t_{\mathrm{liq}}}(FC + LC)}$$

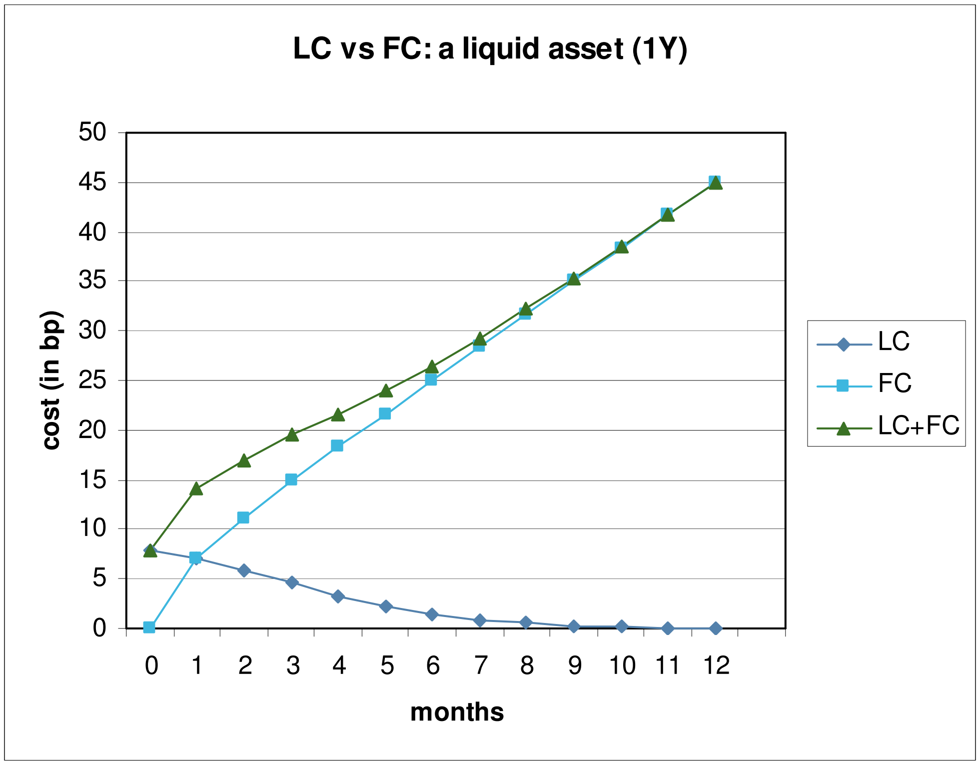
Results – a liquid asset (1Y)

Liquid asset with maturity = 1Y

C=0.5

LV_min =0.9

p=80bp



Results:

Optimal Funding Term = ON

FC = 0bp

LC =8bp

Total = 8bp

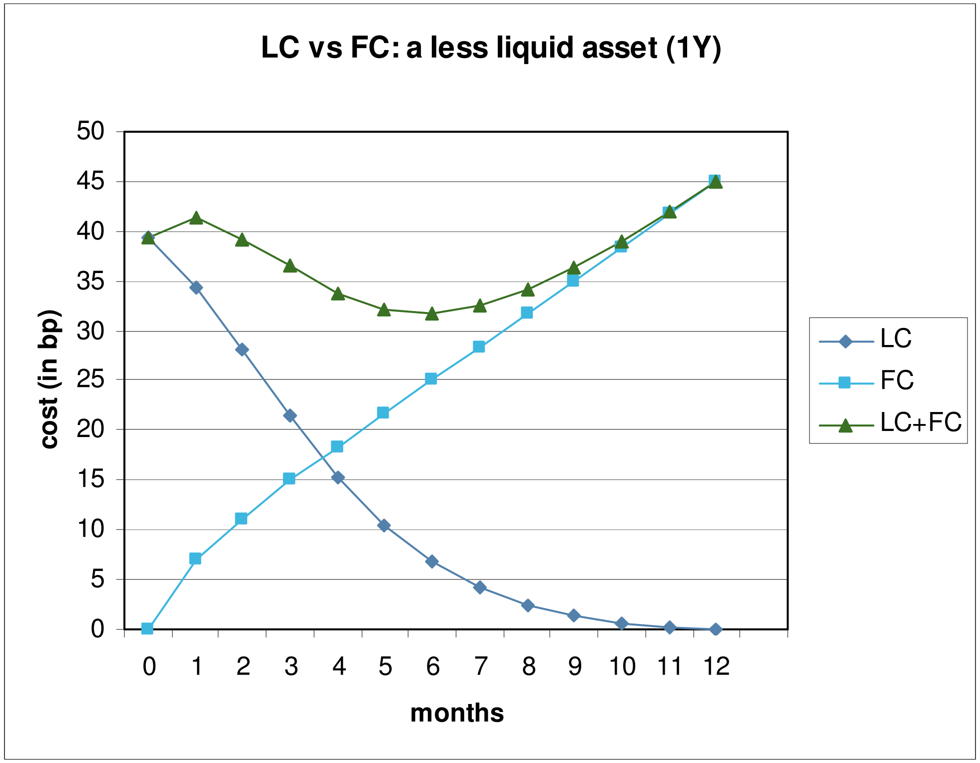
Results – a less liquid asset (1Y)

Less liquid asset with maturity = 1Y

C=2

LV_min =0.5

p=80bp



Results:

Optimal Funding Term = 6 months

FC = 25bp

LC =7bp

Total = 32bp

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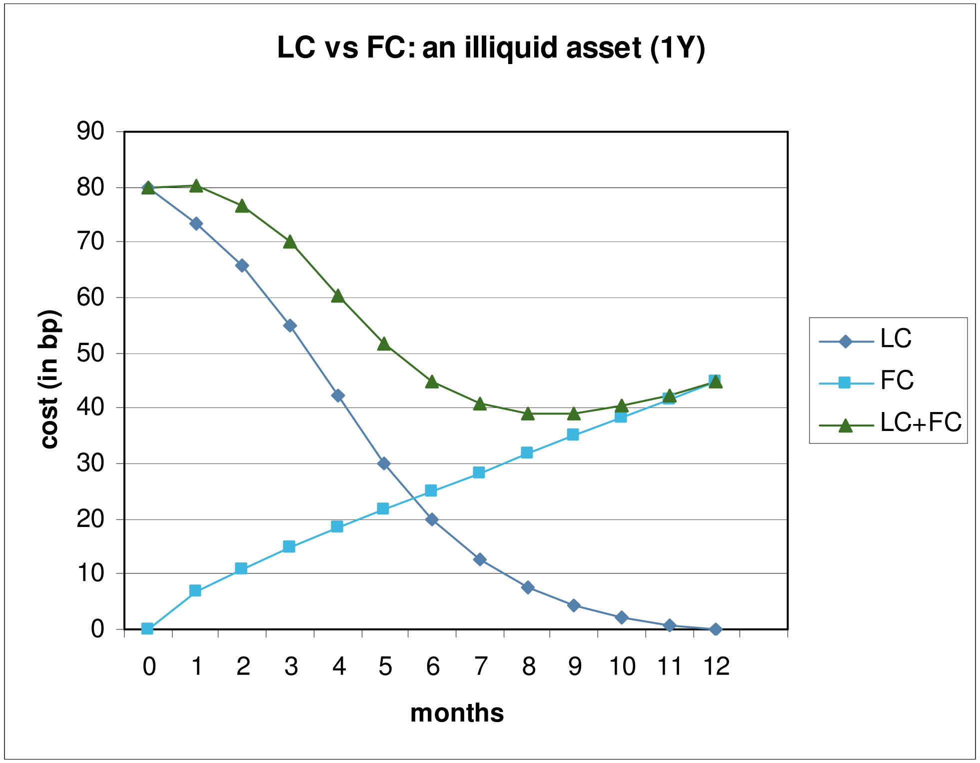
Results – an illiquid asset (1Y)

Illiquid asset with maturity = 1Y

C=1000

LV_min =0

p=80bp



Results:

Optimal Funding Term = 9 months

FC = 35bp

LC =4bp

Total = 39bp

Valuation of a bullet loan/ZC bond

Discount factor of a single payment is

$$DF(t,T) = \mathbb{E}[e^{-[L_{t^*_{\text{liq}}}(t,t,T)+l(t_{\text{event}},t^*_{\text{liq}})](T-t)}].$$

A useful approximation is given by

$$r_{\text{discount}} = L_{t^*_{\text{liq}}}(t,t,T) + l(t^*_{\text{liq}}).$$

Summary so far

- Liquidity Risk modelled as external events, that depending on the funding term may lead to a liquidation loss.
- By minimizing the sum of funding costs and expected liquidation losses the optimal funding term can be determined.
- The discount rate references the optimal funding term
- The discount rate depends on the liquidity of the asset

Derivatives valuation

ON funding

The liquidity costs when the funding term is ON are given by

$$\boxed{LC = \mathbb{E}\left[\int_0^\infty dt DF_{\mathrm{ON}}(0,t)(MtM(t))^+ \rho_{\exp}(\lambda;t)(1 - LV(t_{\mathrm{event}}, t_{\mathrm{ON}}))\right]}$$

Assuming independence of the MtM and duration of the LSE this becomes

$$\boxed{LC = \int_0^\infty dt DF_{\mathrm{ON}}(0,t) EE(t) \rho_{\exp}(\lambda;t)(1 - \mathbb{E}\left[LV(t_{\mathrm{event}}, t_{\mathrm{ON}})\right])}$$

where

$$\boxed{EE(t) = \mathbb{E}\left[(MtM(t))^+\right]}$$

Derivatives valuation

Similarity to CVA

The expression for the liquidity costs (based on ON funding) is similar to CVA

LC	CVA
$\rho_{\mathrm{exp}}(\lambda; t)dt$	$PD(t, t + dt)$
$(1 - \mathbb{E}\left[LV(t_{\mathrm{event}}, t_{\mathrm{ON}})\right])$	LGD
Funding set	Netting set

Derivatives valuation

Optimal funding

Define collateralized derivatives value relative to ideal (no IM) collateralized derivatives

$$\boxed{\mathrm{Uncoll} = \mathrm{Coll}^{\mathrm{ideal}} - \mathrm{FC} - \mathrm{LC}}$$

Besides a funding term a funding buffer is required to avoid liquidation when MtM increases during an LSE:

$$\boxed{FC = \mathbb{E}\left[\int_0^\infty dt DF_{\mathrm{ON}}(0,t)(MtM(t) + FB_{t_{\mathrm{liq}}}(t))^+ e^{-\lambda t}[L_{t_{\mathrm{liq}}}(0,t,t+t_{\mathrm{liq}}) - L_{t_{ON}}(0,t,t+t_{\mathrm{liq}})]\right]}$$

$$\boxed{LC = \mathbb{E}\left[\int_0^\infty dt DF_{\mathrm{ON}}(0,t+\tilde{t}_{\mathrm{liq}})(MtM(t+\tilde{t}_{\mathrm{liq}}))^+ \rho_{\mathrm{exp}}(\lambda;t)(1 - LV(t_{\mathrm{event}},\tilde{t}_{\mathrm{liq}}))\right]}$$

Where the funding term is replaced by the minimum of the funding term and the time of depletion of the funding buffer:

$$\boxed{\tilde{t}_{\mathrm{liq}} = \min(t_{\mathrm{liq}}, t_{\mathrm{hit}})}$$

$$\boxed{MtM(\tau + t_{\mathrm{hit}}) = MtM(\tau) + FB_{\mathrm{liq}}(\tau)}$$

Derivatives valuation

Estimating the funding buffer

Define collateralized derivatives value relative to ideal (no IM) collateralized derivatives

$$dMtM = \mathrm{vol} \times MtM_0 \times dW$$

Besides a funding term a funding buffer is required to avoid liquidation when MtM increases during an LSE:

$$\rho(t_{\mathrm{hit}}) = \frac{a}{\sqrt{2\pi} t_{\mathrm{hit}}^{3/2}} e^{-a^2/2t_{\mathrm{hit}}}$$

$$a = \frac{FB}{\mathrm{vol} \times MtM_0}$$

Minimization of the costs determines the size of the funding buffer and funding term

$$\mathrm{costs} = (MtM + FB)[L_{t_{\mathrm{liq}}}(0, t, t + t_{\mathrm{liq}}) - L_{ON}(0, t, t + t_{\mathrm{liq}})] + MtM \lambda \mathbb{E}\left[(1 - LV(t_{\mathrm{event}}, \tilde{t}_{\mathrm{liq}}))\right]$$

Derivatives valuation

Estimating the funding buffer

Assuming a vol of 10% the results are

$$\boxed{\begin{aligned} &t_{\text{liq}}^{\text{optimal}} = 0.77 \\ &FB = 16.9\% \times MtM \\ &costs = 57bp \times MtM \end{aligned}}$$

In terms of a VaR the funding buffer can be expressed as

$$\boxed{FB \sim \text{1-year } VaR_{95\%}}$$

Derivatives valuation

Estimating the funding buffer

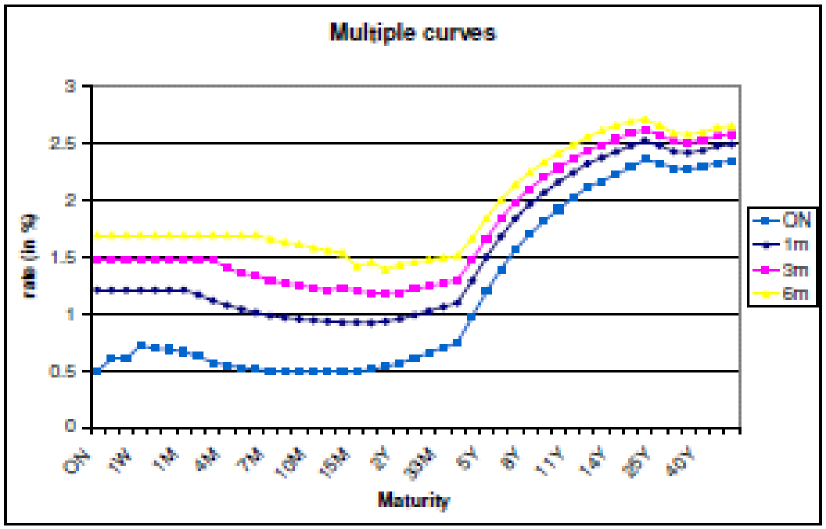
The optimal funding term and buffer for different vols:

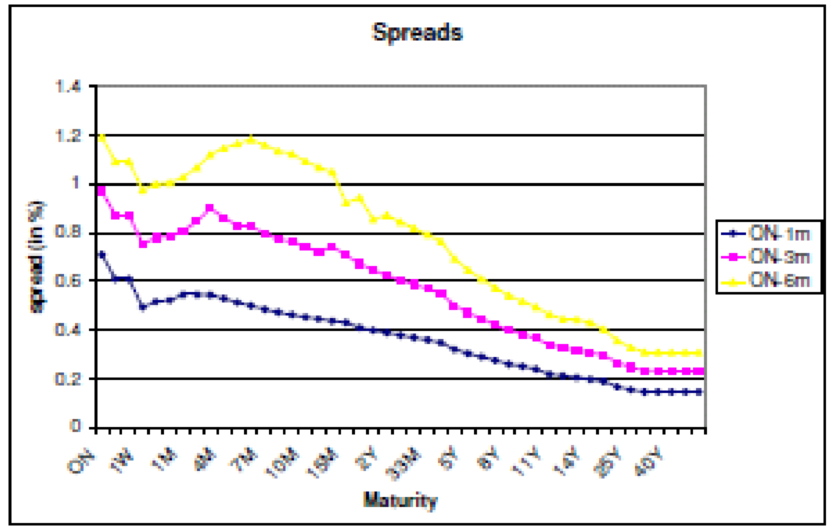
vol	$t_{\text{liq}}^{\text{optimal}}$	FB/MtM in %	costs in bp
0.01	0.82	2.3	51
0.02	0.81	4.3	52
0.03	0.8	6.1	52
0.04	0.8	7.9	53
0.05	0.79	9.5	54
0.06	0.79	11.1	55
0.07	0.78	12.6	55
0.08	0.78	14.1	56
0.09	0.77	15.5	56
0.1	0.77	16.9	57
0.11	0.76	18.2	58
0.12	0.76	19.5	58
0.13	0.75	20.8	59
0.14	0.75	22.1	59
0.15	0.75	23.3	60
0.16	0.74	24.5	60
0.17	0.74	25.7	61
0.18	0.74	26.9	61
0.19	0.73	28.0	62
0.2	0.73	29.1	62

Calibration to Libor spreads

Market data

ON, 1M,3M, 6M curves shown below





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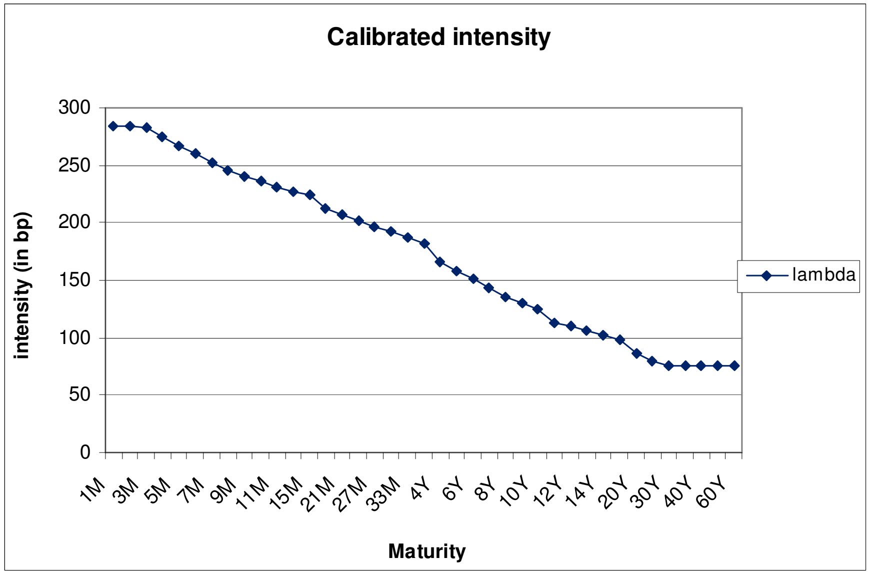
Calibration to Libor spreads

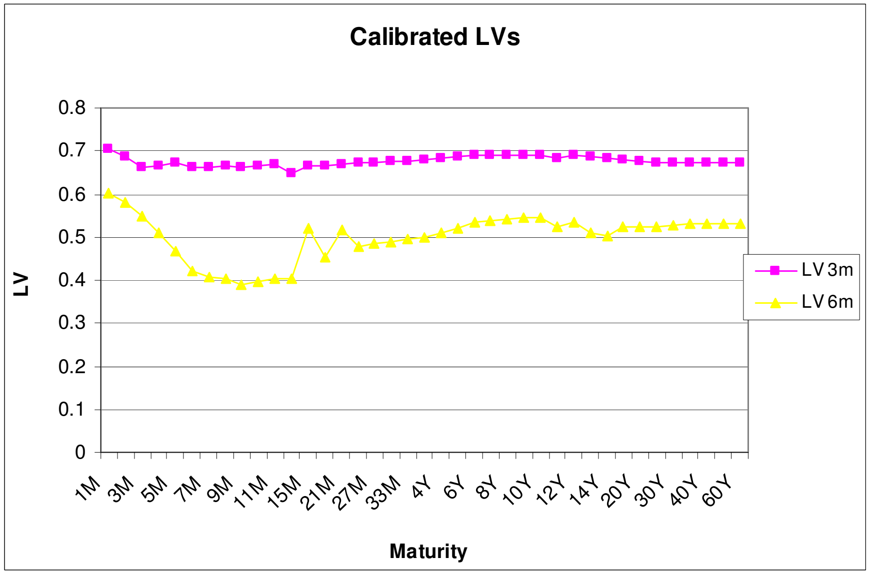
Calibration results

We have fixed the following parameters

parameter	value
μ	$\log(0.5)$
σ	0.5
c	1
$LV_{\min}$ for 1M	80%

And calibrate the intensity and liquidation values for 3M and 6M





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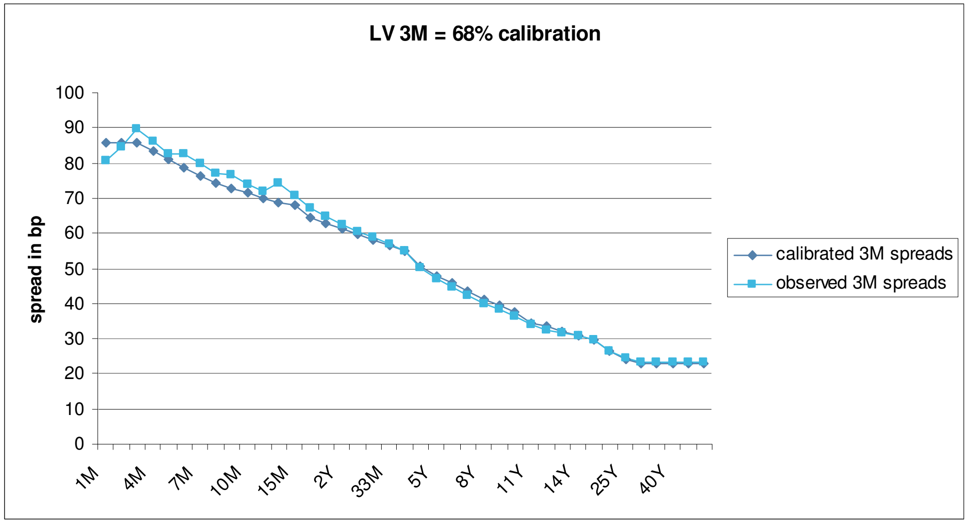
Calibration to Libor spreads

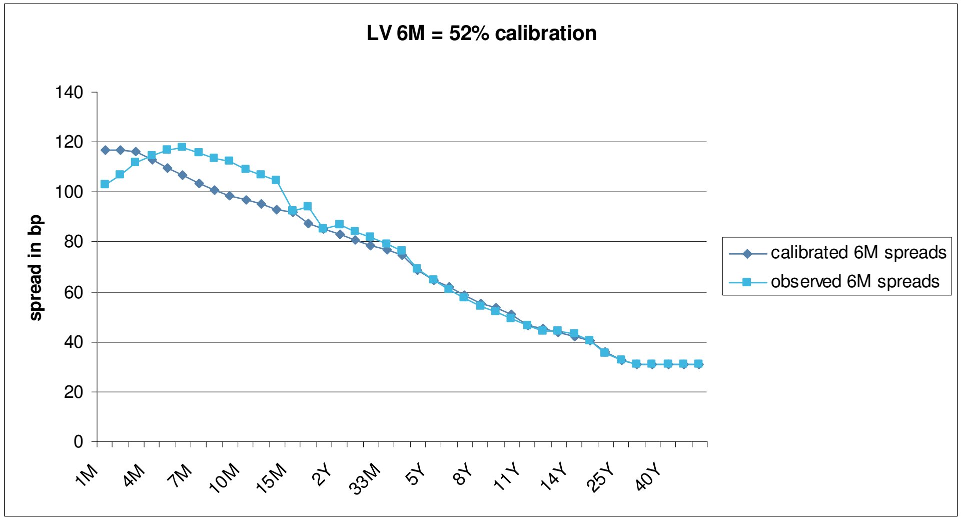
Calibration results fixed LV's

If we additional fix

LV 3M = 68%

LV 6M = 52%





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Summary

A simple model for liquidity risk was introduced. This model implies

Discount factor

- that the discount factor depends on the liquidity of the asset
- an optimal funding term per asset

Derivatives:

- the funding strategy consists of a funding term and funding buffer
- valuation requires determining the optimal funding term and funding buffer

Calibration:

- In principle, calibration is difficult, since by definition calibration instruments are not liquid. Nevertheless many choices are possible: bonds, loans, derivatives.
- An example, was shown where the model was calibrated to libor curves

References

Based on:

A Model for the Valuation of Assets with Liquidity Risk, B J Nauta, http://ssrn.com/abstract=2687936

FVA and Liquidity Risk, B J Nauta, available at SSRN: http://ssrn.com/abstract=2373147

Funding sets vs netting sets

Accounting for OTC Derivatives: Funding Adjustments and the Re-Hypothecation Opion, C Albanese and L. Andersen, available at SSRN: http://ssrn.com/abstract=2482955

Initial Margin

MVA: Initial Margin Valuation Adjustment by Replication and Regression, A. Green and C. Kenyon, available at SSRN: http://ssrn.com/abstract=2432281

Optimal Funding Term of Posted Collateral, B J Nauta, BTRM Working Paper Series # 2

Alternative Liquidity Risk approach

Liquidity risk and coherent measures of risk. C. Acerbi and G. Scandolo. Quantitative Finance, 8:681, 2008

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