Impact of negative rates on pricing models

Veronica Malafaia
ING Bank - FI/FM Quants, Credit & Trading Risk
Amsterdam, 18th November 2015

thinkforward

ING

Disclaimer: The views and opinions expressed in this presentation are those of the author and may not necessarily reflect the views and opinions of her employer.

Negative rates!

CNN Money — Business · Markets · Tech · Personal Finance · Small Business · Luxury

Very weird: Corporate bond rates go negative

By Matt Egan @mattmegan5

THE MARKET MOGUL — FINANCE FROM A FRESH PERSPECTIVE

The Strange Case of Negative Interest Rates

Rare ECB move signals strange economic times

By Andrew Walker
BBC World Service Economics correspondent

5 June 2014

The (Strange) New World of Negative Yields

BY CARLOS A. ALVARENGA on FEBRUARY 28, 2015 • (0)

The Strange World Of Negative Interest Rates

Lowell Yura , BMO Global Asset Management | FOLLOW+
March 02, 2015 4:42pm | Comments

The Strange Puzzle of Negative Interest Rates

By Nouriel Roubini / Feb 24, 2015

The Washington Post — Search — Sign In

Opinions

A new economic mystery: negative interest rates

Negative rates!

"If you haven't found something strange during the day, it hasn't been much of a day."

John Archibald Wheeler

Negative rates!

ft.com > markets >

Capital Markets

Home | World | Companies | Markets | Glo

fastFT | Alphaville | FTfm | Markets Data | Trading Room | Ec

February 15, 2015 1:46 pm

Negative rates to shake up financial system, say experts

Ralph Atkins and Elaine Moore in London

The Economist

World politics | Business & finance | Economics | Science & technology | Culture

The Economist explains

Why negative interest rates have arrived—and why they won't save the global economy

Feb 18th 2015, 23:50 BY C.W. | WASHINGTON, DC

Timekeeper | Like 2.1k | Tweet 48

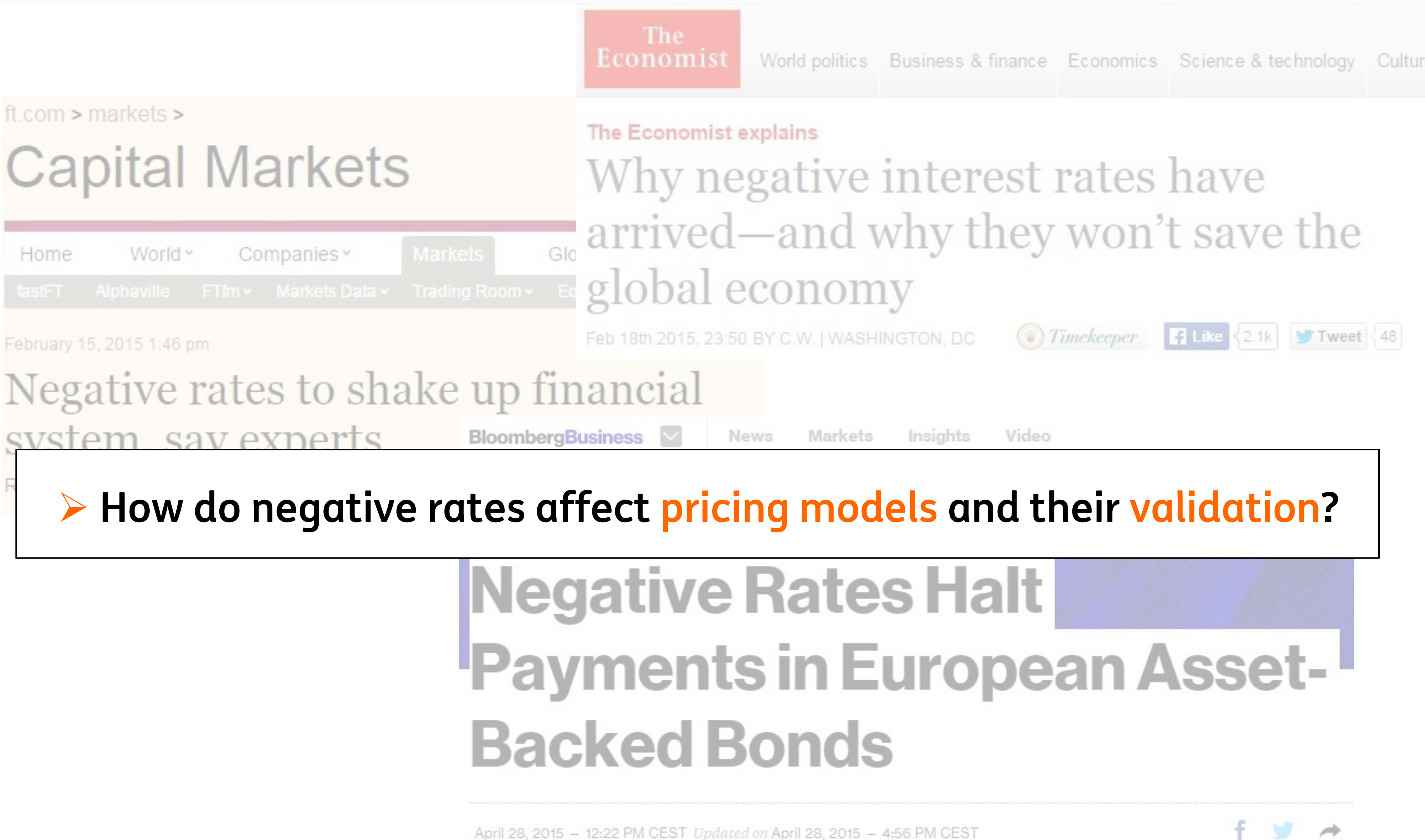
BloombergBusiness

News | Markets | Insights | Video

Negative Rates Halt Payments in European Asset-Backed Bonds

April 28, 2015 – 12:22 PM CEST *Updated on* April 28, 2015 – 4:56 PM CEST

Negative rates!



The Economist | World politics | Business & finance | Economics | Science & technology | Culture

The Economist explains

Why negative interest rates have arrived—and why they won't save the global economy

Feb 18th 2015, 23:50 BY C.W. | WASHINGTON, DC

ft.com > markets >

Capital Markets

Home | World | Companies | Markets

fastFT | Alphaville | FTfm | Markets Data | Trading Room

February 15, 2015 1:46 pm

Negative rates to shake up financial system, say experts

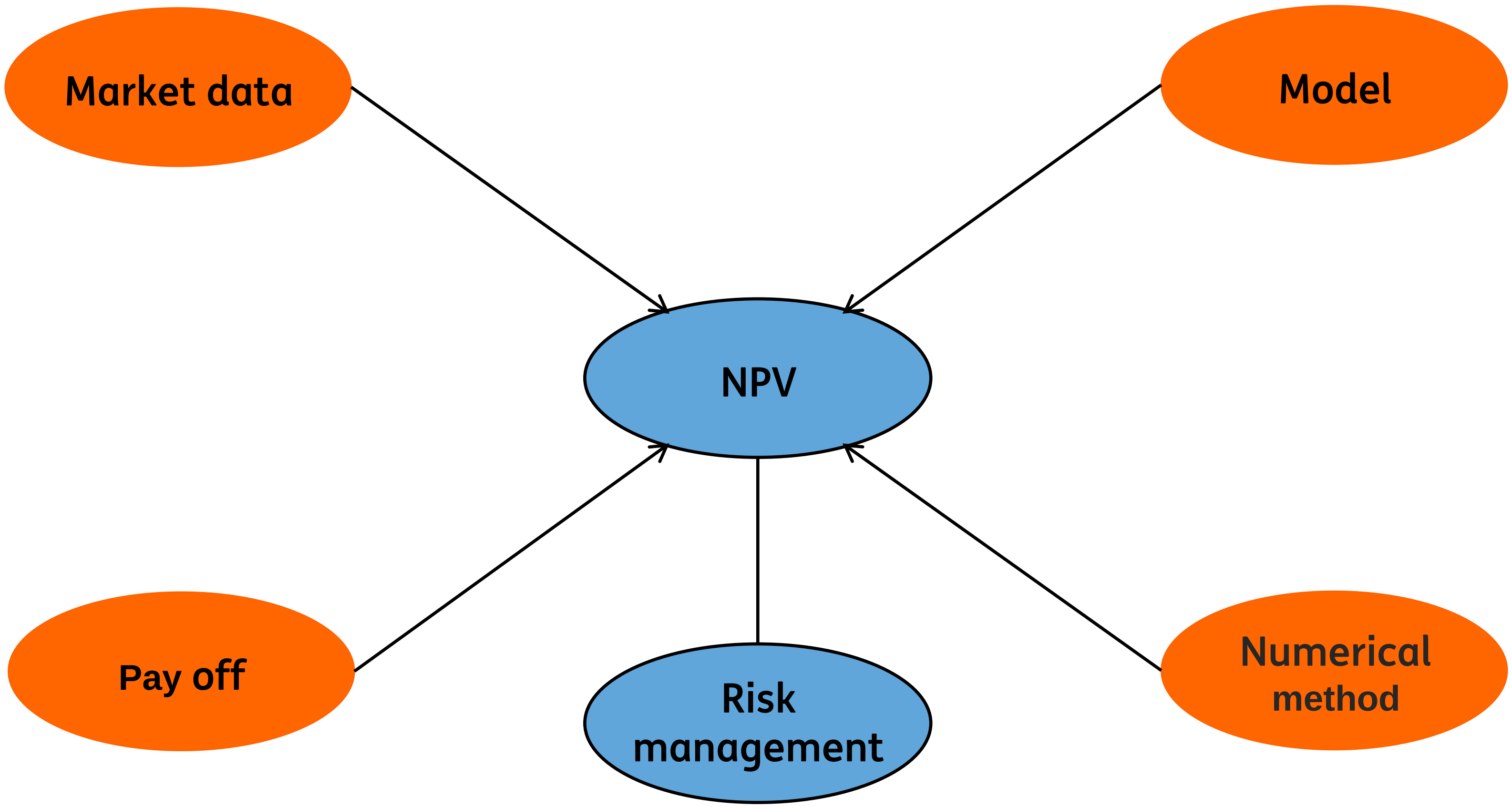
BloombergBusiness | News | Markets | Insights | Video

Negative Rates Halt Payments in European Asset-Backed Bonds

April 28, 2015 – 12:22 PM CEST *Updated on* April 28, 2015 – 4:56 PM CEST

- ➢ **How do negative rates affect pricing models and their validation?**

Mind map



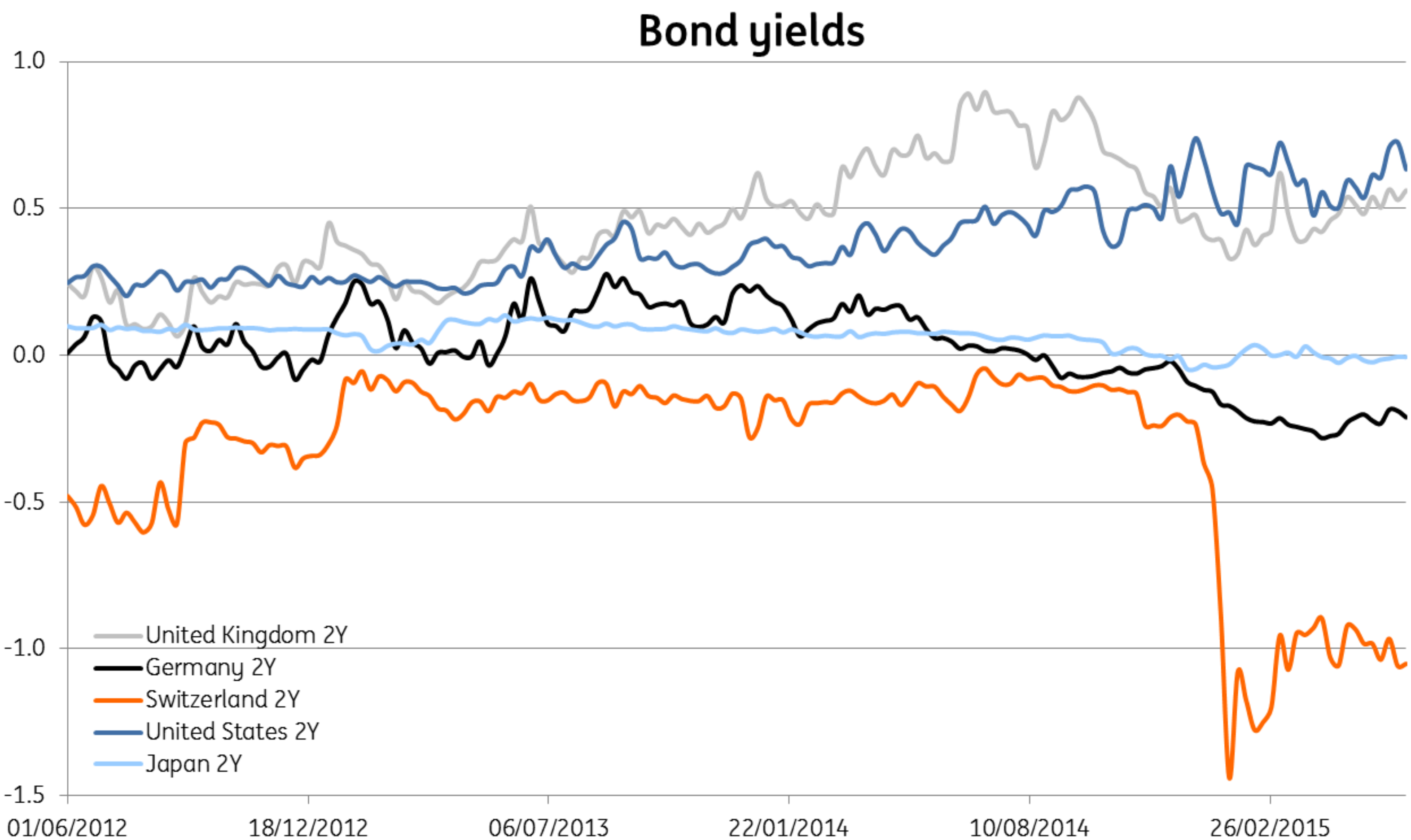
Market data
Model
NPV
Pay off
Risk management
Numerical method

Outline

1. Brief market overview
2. A walk through pricing models
3. Models: are we done with DF>1 and σ_L S → σ_N?
4. And what about validations?
5. Final remarks

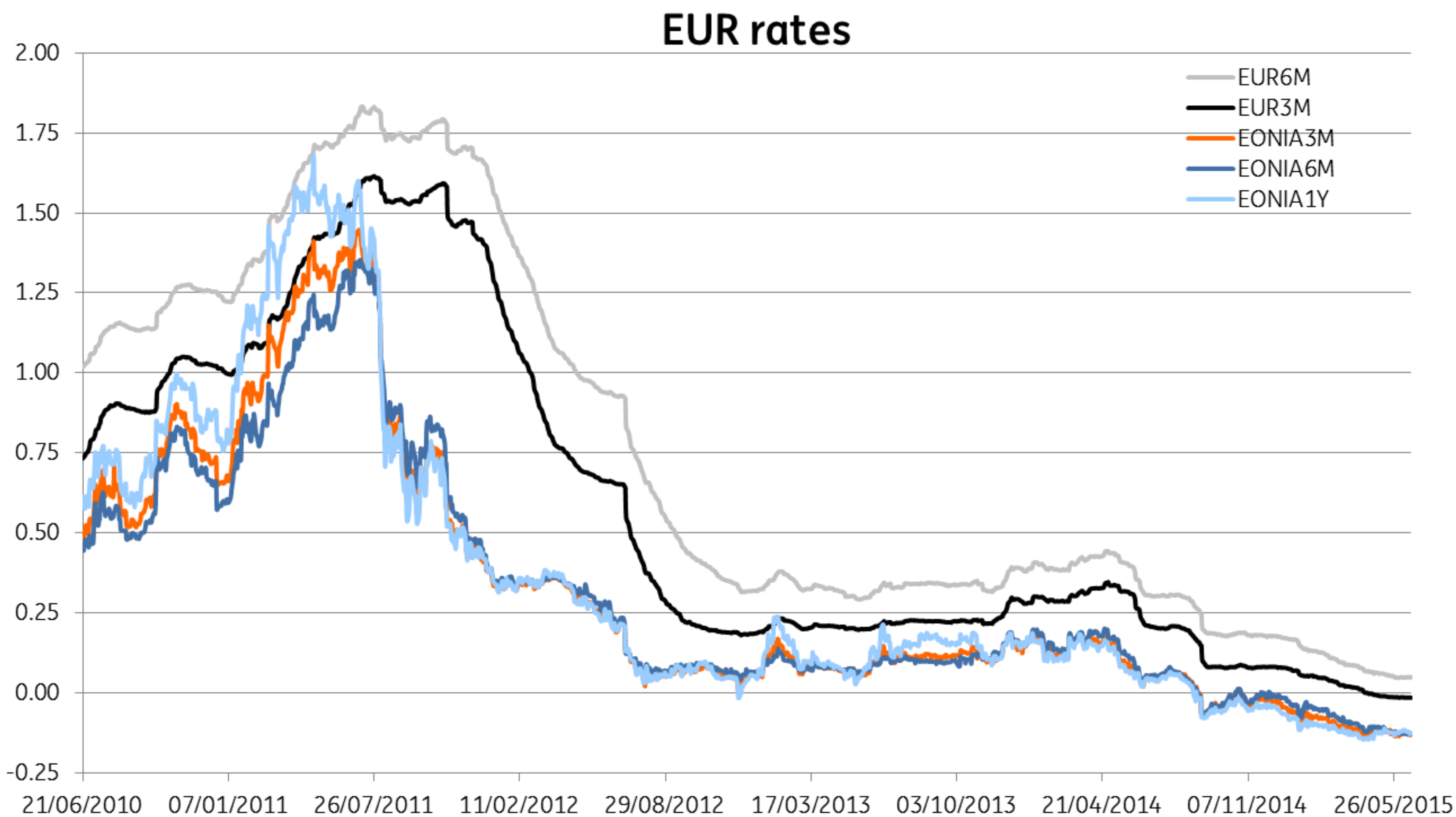
➤**How do negative rates affect pricing models and their validation?**

Brief market overview



Bloomberg data

Brief market overview



Bloomberg data

A walk through pricing models

- **Two general remarks:**
  - All pricing models assuming a lognormal dynamics for the underlying interest rate are not suitable for a negative interest rate environment;
  - For other asset classes than IR, most of the models will simply take DF>1.

A walk through pricing models

❑ Two general remarks

- All pricing models assuming a lognormal dynamics for the underlying interest rate are not suitable for a negative interest rate environment;
- For other asset classes than IR, most of the models will simply take DF>1.

➢ Let's examine some concrete cases:

– Black-76

– Short rate models

– Libor market models

– SABR

A walk through pricing models

Black model

- Until recently, market paradigm for IR options
- Move towards shifted lognormal models/normal models due to low rate environment.

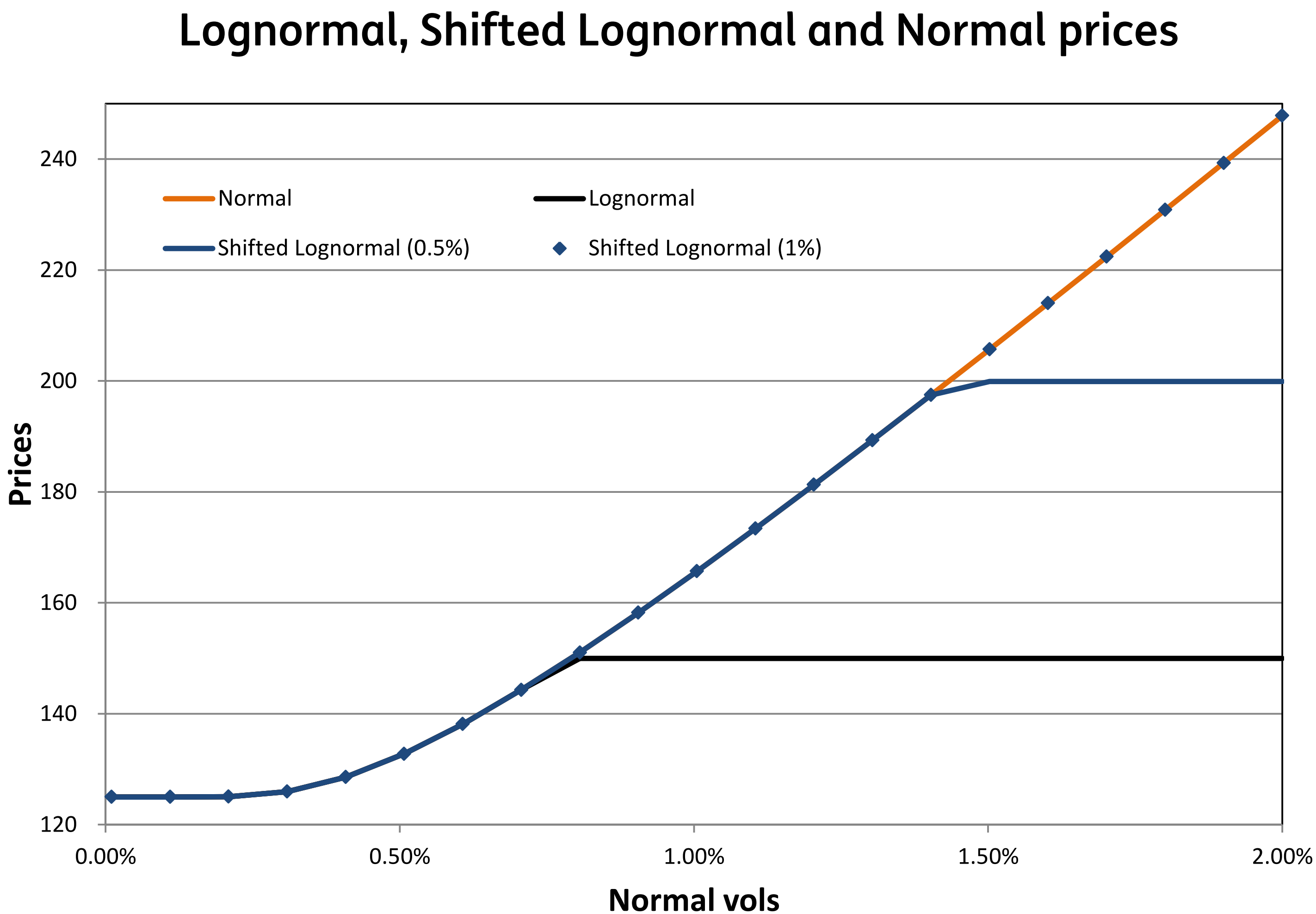
$$dF_t = \sigma_L F_t dW_t$$
$$dF_t = \sigma_S (F_t - S) dW_t$$
$$dF_t = \sigma_N dW_t$$

A walk through pricing models

❑ Black model

- Until recently, market paradigm for IR options
- Move towards shifted lognormal models/normal models due to low rate environment. However,
  - In theory, no lower limit for negative rates in a normal model. Is this realistic?
  - How to fix the shift if a shifted lognormal model is chosen?
  - Beware of tweaking the system!
    - Converting “normal prices” to lognormal volatilities;
    - Creating shifted curves to feed lognormal models;
    - ....

A walk through pricing models



A walk through pricing models

❑ Short rate models

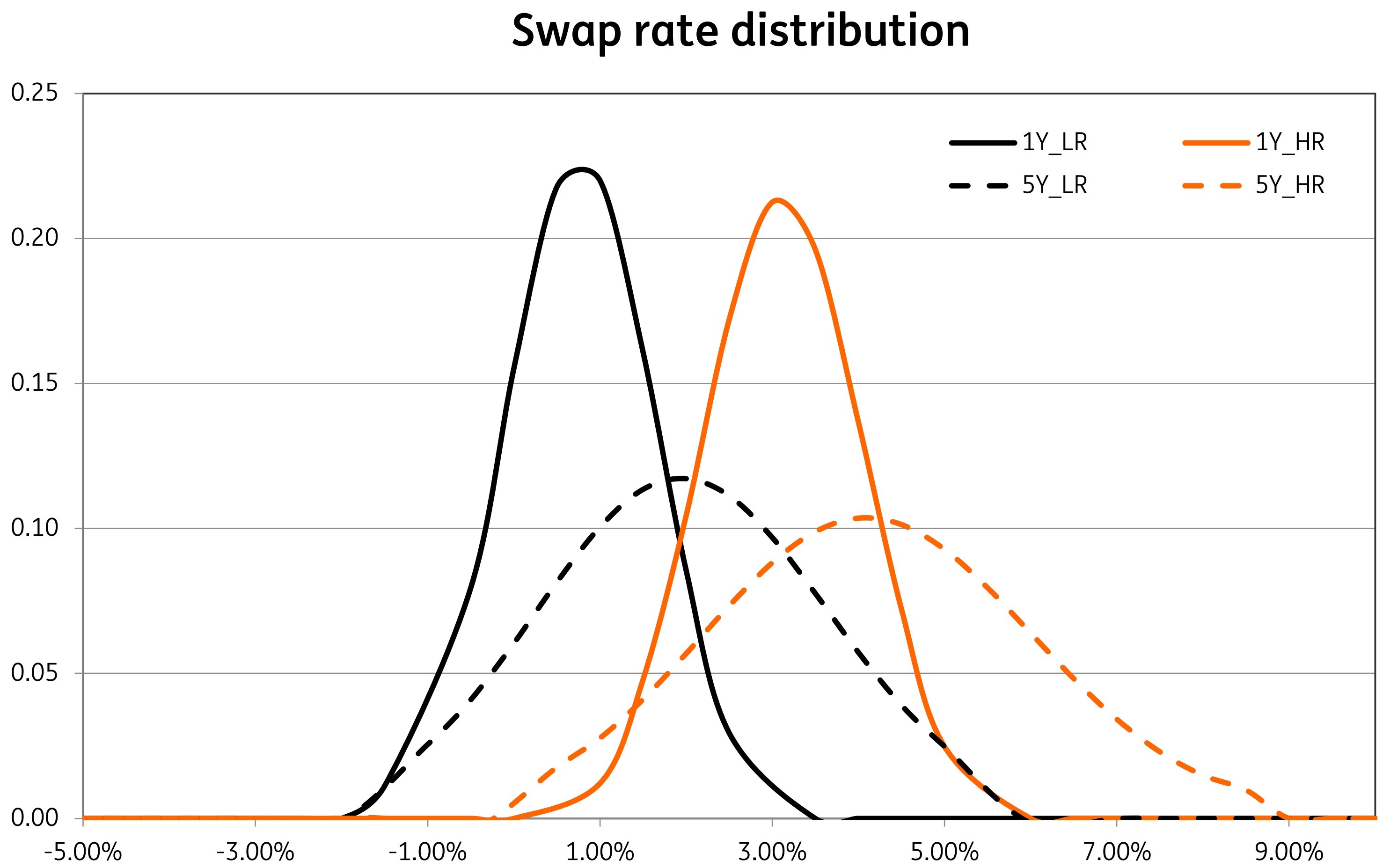
- The most popular allow for negative rates (Vasicek, Hull-White, etc…).

- However,
  - Is the implied level for negative rates compatible with reality (old problem)?
  - Beware if calibration is done to lognormal volatilities!

$$dr_t = [\theta_t - a_t r_t]dt + \sigma_t dW_t$$

A walk through pricing models

A walk through pricing models



A walk through pricing models

Libor market models

- Lognormal and normal formulations possible.
- However,
  - Beware if calibration is done to lognormal volatilities/correlations!

$$dL_n(t) = \sigma_n(t)^{\intercal} dW^{n+1}(t)$$

SABR

- Hagan et al 's formula is the market convention for interpolating swaption volatilities.

- This formula corresponds to an expansion of the SABR model

$$\begin{cases} dS_t = \alpha_t S_t^{\beta} dW_t^1 \\ d\alpha_t = \nu \alpha_t dW_t^2 \\ dW_t^1 dW_t^2 = \rho dt \end{cases} \xrightarrow{\alpha\sqrt{T} \ll 1, \nu\sqrt{T} \ll 1 \text{ and } \frac{|S_0 - K|}{\alpha\sqrt{T}} = \mathcal{O}(1)} \sigma_{L \text{ or } N} = f(\alpha, \beta, \nu, \varrho)$$

A walk through pricing models

❑ SABR

- Hagan et al 's formula is the market convention for interpolating swaption volatilities.

- This formula corresponds to an expansion of the SABR model

$$\begin{cases} dS_t = \alpha_t S_t^{\beta} dW_t^1 \\ d\alpha_t = \nu \alpha_t dW_t^2 \\ dW_t^1 dW_t^2 = \rho dt \end{cases} \xrightarrow{\alpha\sqrt{T} \ll 1, \nu\sqrt{T} \ll 1 \text{ and } \frac{|S_0 - K|}{\alpha\sqrt{T}} = o(1)} \sigma_{L \text{ or } N} = f(\alpha, \beta, \nu, \varrho)$$

- Hagan et. al. expansion fails to work well for high volatility, long maturities and very out-of-the money options.
  - Negative density probability at low strike for long expiry options (particularly relevant in a low rate environment).

A walk through pricing models

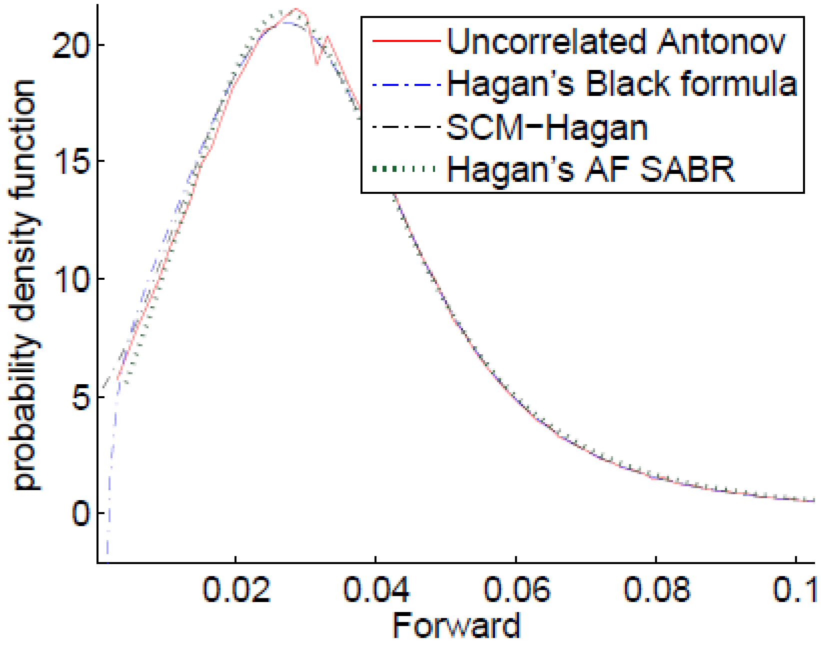
SABR

- Hagan et. al. expansion fails to work well for high volatility, long maturities and very out-of-the money options.
  - Negative density probability at low strike for long expiry options (particularly relevant in a low rate environment).

- Several approaches proposed in the literature but no market consensus yet:
  - Improving the expansion (for e.g. expansion around normal SABR);
  - Analytic approximations from SABR (for instance solution for uncorrelated case + mapping to the correlated case);
  - Improving Hagan's implied density;
  - ...

A walk through pricing models

- SABR



Are we done with DF>1 and σ_L S → σ_N?

- Negative interest rates can also trigger implicit floors thus affecting the pay-off… Main examples:

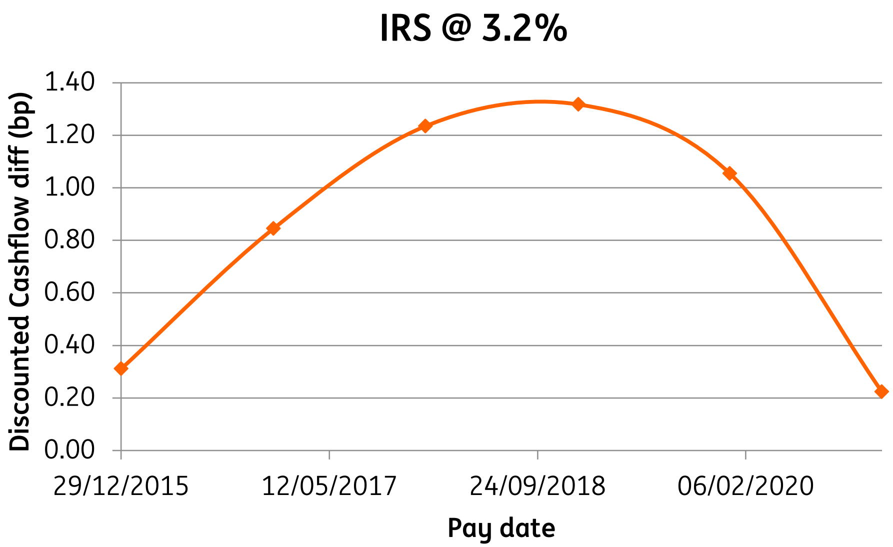
  - Clauses preventing negative coupons in floating rate bonds
    More than 2.2 billion EUR of notes secured with residential mortgages in Europe are among asset-backed securities priced with spreads over Euribor of five basis points or less. [Source: Bloomberg]

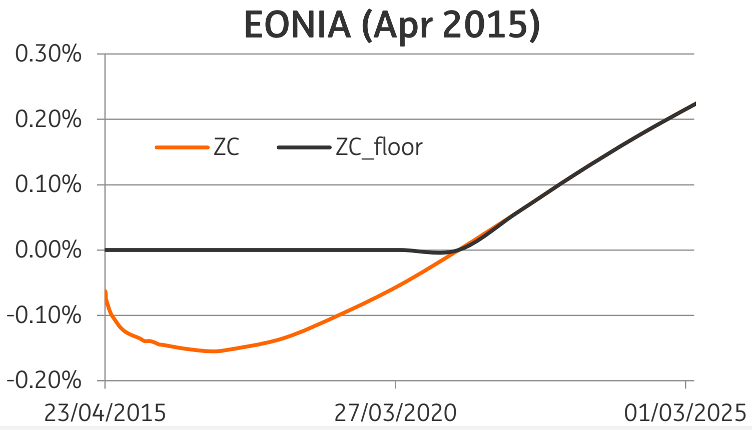
  - Clauses preventing negative interest on mortgages

  - CSAs – does the collateral poster need to pay interest if the reference rate turns negative?

Are we done with DF>1 and σ_L S → σ_N?

- What is the impact of a floor @ 0% in the collateral rate?





And what about validations?

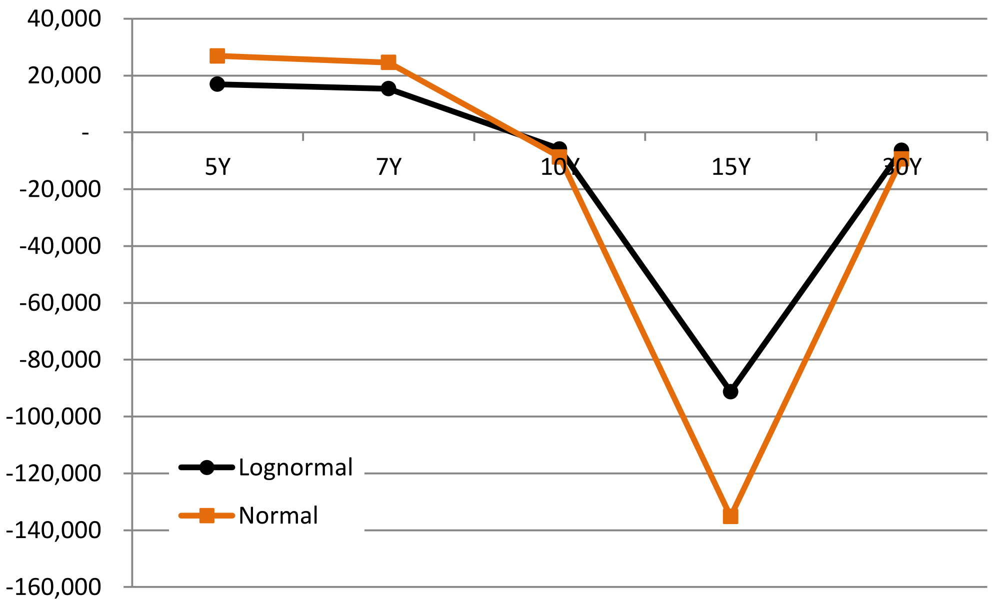
❑ In risk management, negative rates also changed the environment we were used to:

- Sensitivities
- Smile
- Old relations:
  - American call options on non-dividend paying stocks have the same price as European calls…provided IR are positive!
  - …

And what about validations?

EUR 6Y 10Y ATM FLR

	Lognormal	Normal
BPV	- 71,076	- 101,651
Vega	319,571	168,570



And what about validations?

EUR 6Y 10Y ATM FLR

	Lognormal	Normal
BPV	- 71,076	- 101,651
Vega	319,571	168,570

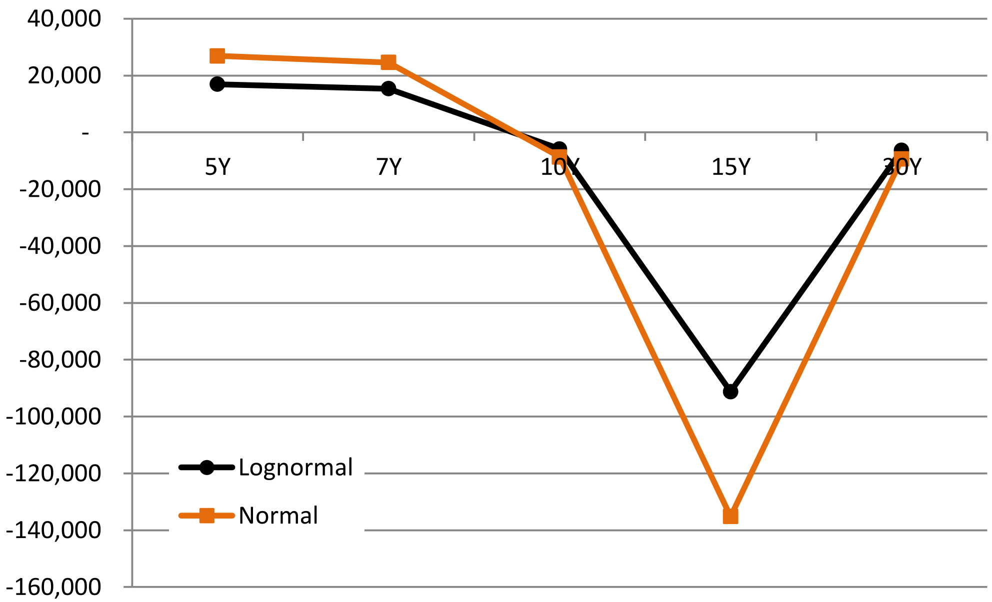
Approximating the normal BPV…

$\sigma_N \sim \sigma_L \sqrt{(K.F)}$ with F→ F+ 1bp

$\sigma_L' \sim \sigma_N / \sqrt{[K.(F+1bp)]}$

F	2.11%
σ_L	37.0%
σ_L'	36.8%
$\Delta\sigma BS$	-0.2%

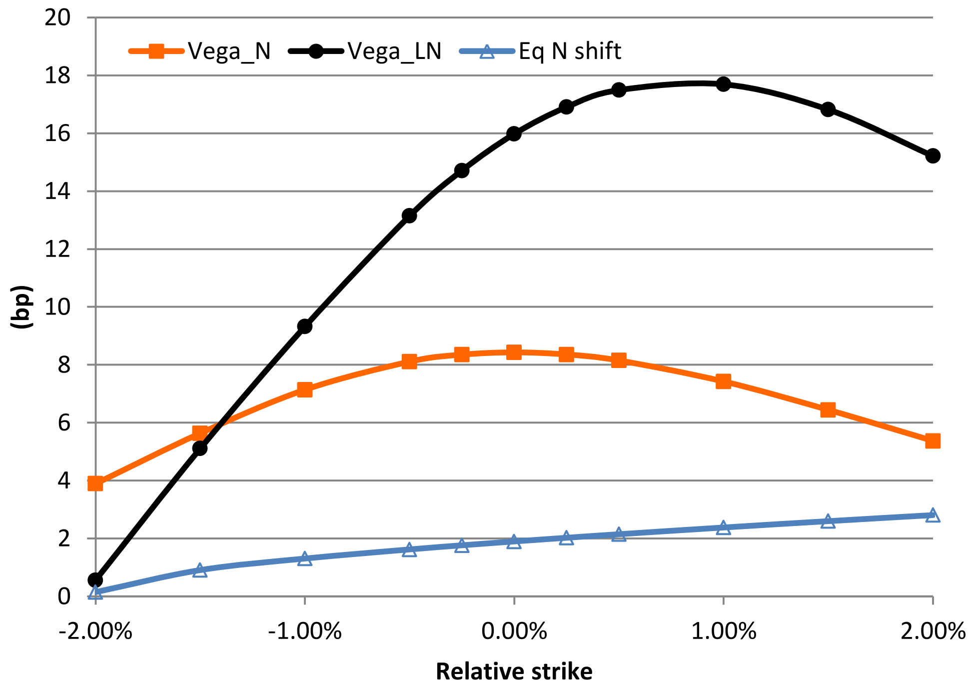
× VegaBS **-55,806**



And what about validations?

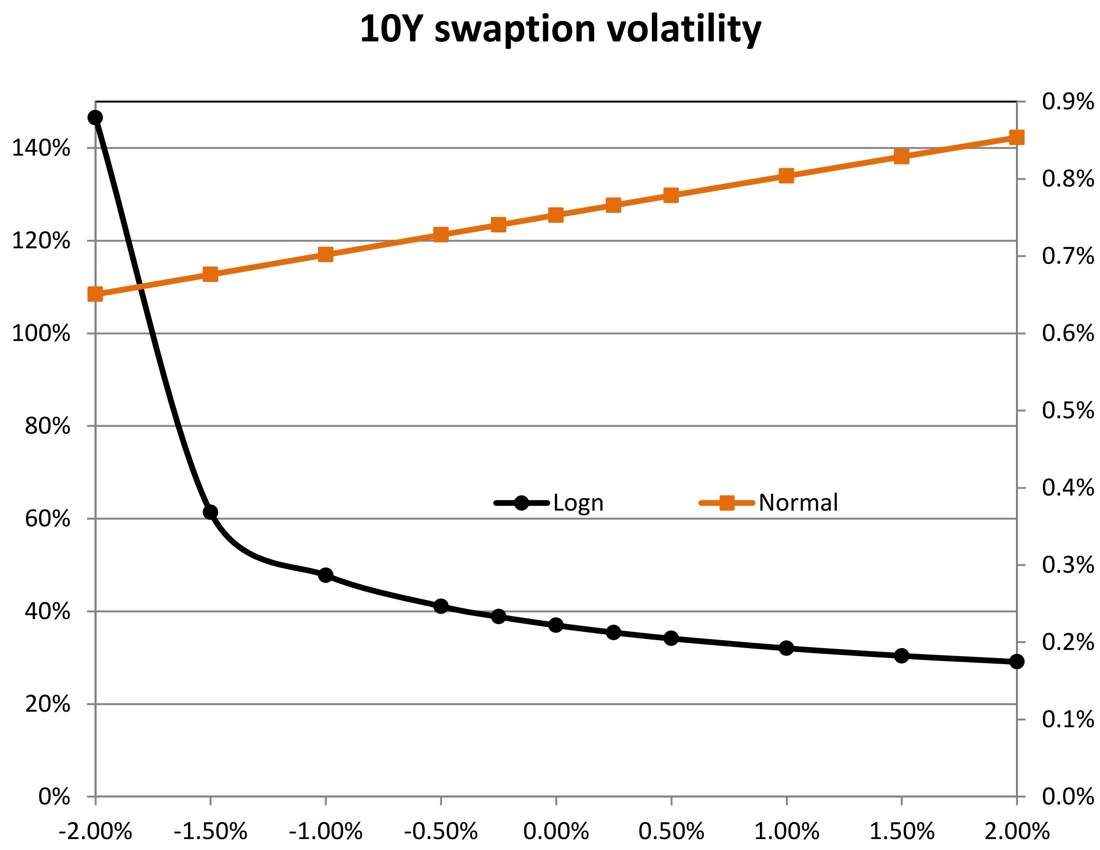
EUR 6Y 10Y ATM FLR

	Lognormal	Normal
BPV	- 71,076	- 101,651
Vega	319,571	168,570



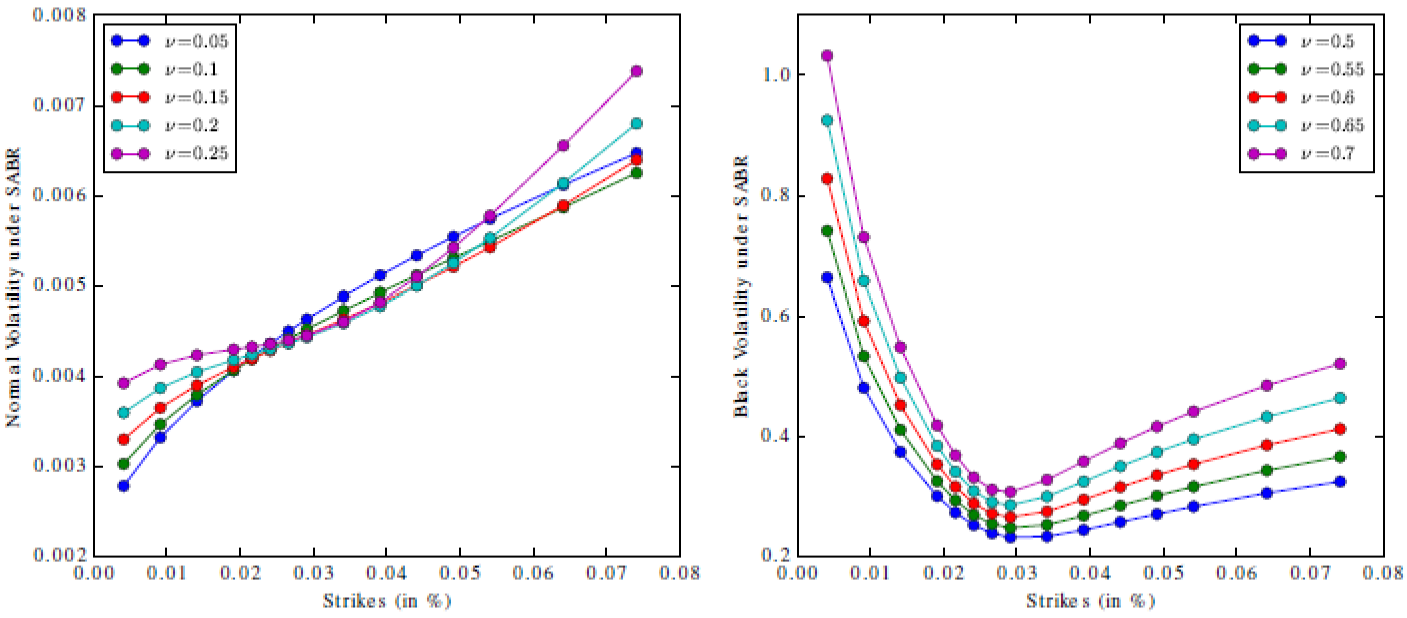
And what about validations?

Smile

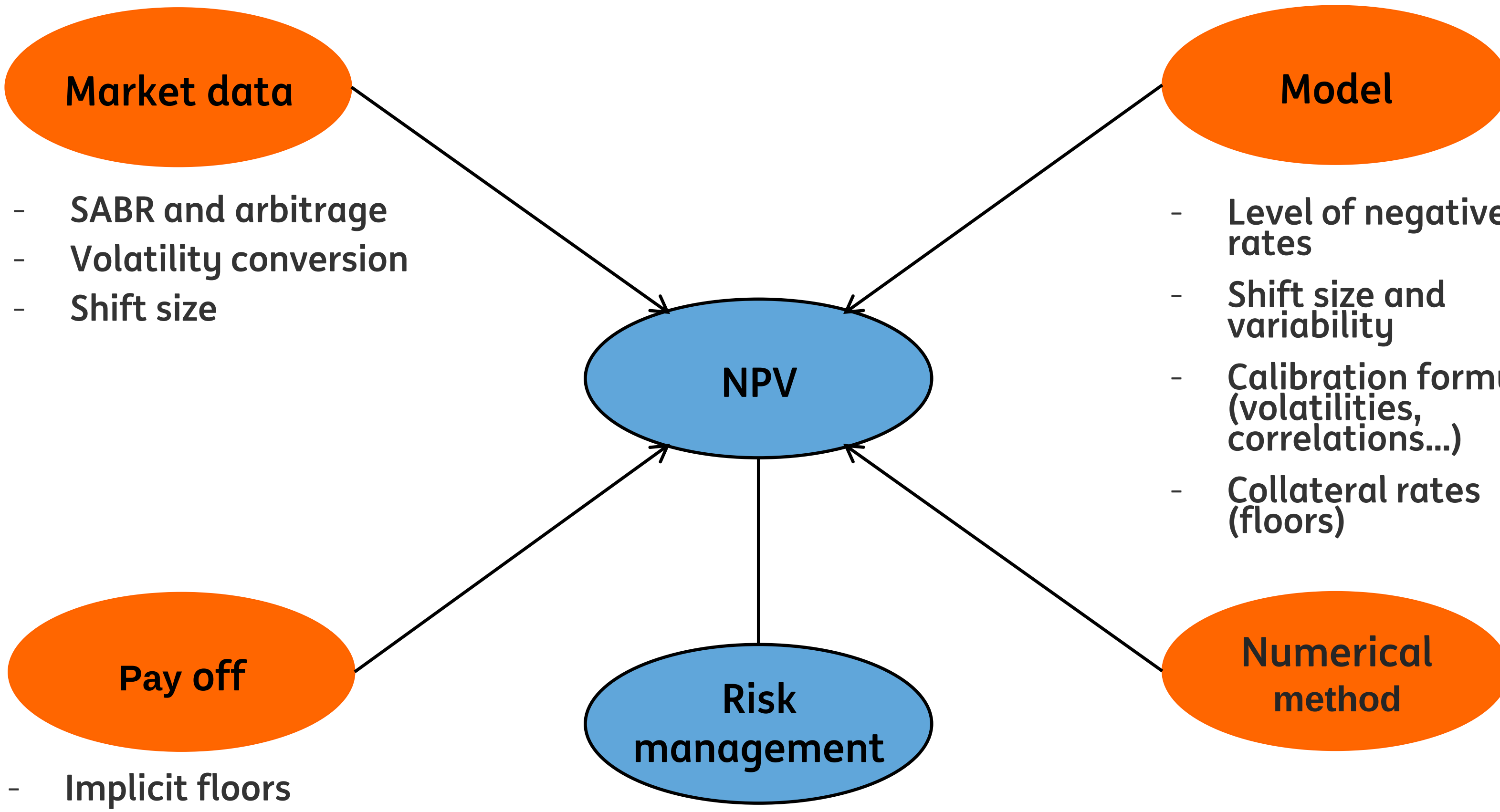


And what about validations?

Smile



In short



Market data
- SABR and arbitrage
- Volatility conversion
- Shift size

Model
- Level of negative rates
- Shift size and variability
- Calibration formulas (volatilities, correlations…)
- Collateral rates (floors)

Pay off
- Implicit floors

Risk management
- VaR
- Smile
- Sensitivities behaviour
- Old relations

Questions?