Impact of negative rates on pricing models

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Disclaimer: The views and opinions expressed in this presentation are those of the author and may not necessarily reflect the views and opinions of her employer.
Negative rates!

Very weird: Corporate bond rates go negative

The Strange Case of Negative Interest Rates

Rare ECB move signals strange economic times

The (Strange) New World of Negative Yields

The Strange World Of Negative Interest Rates

The Strange Puzzle of Negative Interest Rates

A new economic mystery: negative interest rates
Negative rates!

“If you haven't found something strange during the day, it hasn't been much of a day.”

John Archibald Wheeler
Negative rates to shake up financial system, say experts

Ralph Atkins and Elaine Moore in London
How do negative rates affect pricing models and their validation?
Outline

1. Brief market overview
2. A walk through pricing models
3. Models: are we done with $DF > 1$ and $\sigma_L \rightarrow \sigma_N$?
4. And what about validations?
5. Final remarks

➢ How do negative rates affect pricing models and their validation?
Brief market overview

Bond yields

United Kingdom 2Y
Germany 2Y
Switzerland 2Y
United States 2Y
Japan 2Y

Bloomberg data
Brief market overview

![EUR rates chart]

Bloomberg data
A walk through pricing models

- Two general remarks:
  - All pricing models assuming a lognormal dynamics for the underlying interest rate are not suitable for a negative interest rate environment;
  - For other asset classes than IR, most of the models will simply take DF>1.
A walk through pricing models

- Two general remarks
  - All pricing models assuming a lognormal dynamics for the underlying interest rate are not suitable for a negative interest rate environment;
  - For other asset classes than IR, most of the models will simply take DF>1.

- Let’s examine some concrete cases:
  - Black-76
  - Short rate models
  - Libor market models
  - SABR
A walk through pricing models

- **Black model**
  
  - Until recently, market paradigm for IR options
  
  - Move towards shifted lognormal models/normal models due to low rate environment.

\[

dF_t = \sigma_L F_t dW_t \\
\]
\[
\]
\[
\]
\[
\]
\[
\]

Source: ING, thinkforward

IR models
A walk through pricing models

- **Black model**
  
  - Until recently, market paradigm for IR options
  
  - Move towards shifted lognormal models/normal models due to low rate environment. However,
    
    - In theory, no lower limit for negative rates in a normal model. Is this realistic?
    - How to fix the shift if a shifted lognormal model is chosen?
    - Beware of tweaking the system!
      
      - Converting “normal prices” to lognormal volatilities;
      - Creating shifted curves to feed lognormal models;
      - ....
A walk through pricing models

Lognormal, Shifted Lognormal and Normal prices

Prices vs. Normal vols

- Normal
- Lognormal
- Shifted Lognormal (0.5%)
- Shifted Lognormal (1%)
A walk through pricing models

- **Short rate models**
  - The most popular allow for negative rates (Vasicek, Hull-White, etc...).
  - However,
    - Is the implied level for negative rates compatible with reality (old problem)?
    - Beware if calibration is done to lognormal volatilities!

\[ dr_t = [\theta_t - a_t r_t]dt + \sigma_t dW_t \]
A walk through pricing models

Swap rate distribution

- 1Y_HR
- 5Y_HR
A walk through pricing models

Swap rate distribution

IR models
A walk through pricing models

- Libor market models
  - Lognormal and normal formulations possible.
  - However,
    - Beware if calibration is done to lognormal volatilities/correlations!

\[ dL_n(t) = \sigma_n(t)^T dW^{n+1}(t) \]
A walk through pricing models

- **SABR**
  - Hagan et al.’s formula is the market convention for interpolating swaption volatilities.
  - This formula corresponds to an expansion of the SABR model:
    \[
    \begin{align*}
    dS_t &= \alpha_t S_t^\beta dW_t^1 \\
    d\alpha_t &= \nu \alpha_t dW_t^2 \\
    dW_t^1 dW_t^2 &= \rho dt
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A walk through pricing models

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  - Hagan et. al. expansion fails to work well for high volatility, long maturities and very out-of-the money options.
    
    - Negative density probability at low strike for long expiry options (particularly relevant in a low rate environment).
A walk through pricing models

- **SABR**
  - Hagan et. al. expansion fails to work well for high volatility, long maturities and very out-of-the money options.
    - Negative density probability at low strike for long expiry options (particularly relevant in a low rate environment).
  - Several approaches proposed in the literature but no market consensus yet:
    - Improving the expansion (for e.g. expansion around normal SABR);
    - Analytic approximations from SABR (for instance solution for uncorrelated case + mapping to the correlated case);
    - Improving Hagan’s implied density;
    - ...
A walk through pricing models

- SABR
Are we done with $DF>1$ and $\sigma_L S \rightarrow \sigma_N$?

- Negative interest rates can also trigger implicit floors thus affecting the pay-off... Main examples:
  - Clauses preventing negative coupons in floating rate bonds
    More than 2.2 billion EUR of notes secured with residential mortgages in Europe are among asset-backed securities priced with spreads over Euribor of five basis points or less. [Source: Bloomberg]
  - Clauses preventing negative interest on mortgages
  - CSAs – does the collateral poster need to pay interest if the reference rate turns negative?
Are we done with $DF>1$ and $\sigma_L S \to \sigma_N$?

- What is the impact of a floor @ 0% in the collateral rate?
And what about validations?

- In risk management, negative rates also changed the environment we were used to:
  - Sensitivities
  - Smile
  - Old relations:
    - American call options on non-dividend paying stocks have the same price as European calls...provided IR are positive!
    - ...
And what about validations?

<table>
<thead>
<tr>
<th>EUR 6Y 10Y ATM FLR</th>
<th>Lognormal</th>
<th>Normal</th>
</tr>
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<tbody>
<tr>
<td>BPV</td>
<td>-71,076</td>
<td>101,651</td>
</tr>
<tr>
<td>Vega</td>
<td>319,571</td>
<td>168,570</td>
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Sensitivities

Lognormal
Normal
BPV: -71,076 to 101,651
Vega: 319,571 to 168,570
And what about validations?

### EUR 6Y 10Y ATM FLR

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**Approximating the normal BPV...**

\[
\sigma_N \sim \sigma_L \sqrt{(K,F)} \quad \text{with } F \rightarrow F+1\text{bp}
\]

\[
\sigma_L' \approx \sigma_N / \sqrt{K \cdot (F+1\text{bp})}
\]

<table>
<thead>
<tr>
<th>F</th>
<th>2.11%</th>
</tr>
</thead>
<tbody>
<tr>
<td>\sigma_L</td>
<td>37.0%</td>
</tr>
<tr>
<td>\sigma_L'</td>
<td>36.8%</td>
</tr>
<tr>
<td>\Delta \sigma_{BS}</td>
<td>-0.2%</td>
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![Graph showing sensitivities](image-url)
And what about validations?

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Sensitivities

- **BPV**: Lognormal = 71,076, Normal = 101,651
- **Vega**: Lognormal = 319,571, Normal = 168,570
And what about validations?

10Y swaption volatility

- Logn
- Normal

Thinkforward
And what about validations?

Smile
In short

Market data
- SABR and arbitrage
- Volatility conversion
- Shift size

Pay off
- Implicit floors

Model
- Level of negative rates
- Shift size and variability
- Calibration formulas (volatilities, correlations...)
- Collateral rates (floors)

Risk management
- VaR
- Smile
- Sensitivities behaviour
- Old relations

Numerical method
Questions?