Deloitte.

TopQuants: Modelling Liquidity Risk

Mortgage example

Amsterdam, 12 November 2014

Outline of this session

Introduction to liquidity risk

Behavioural modelling

Stress testing

Mortgages

Introduction to liquidity risk

What is liquidity risk?

Different definitions of liquidity risk:

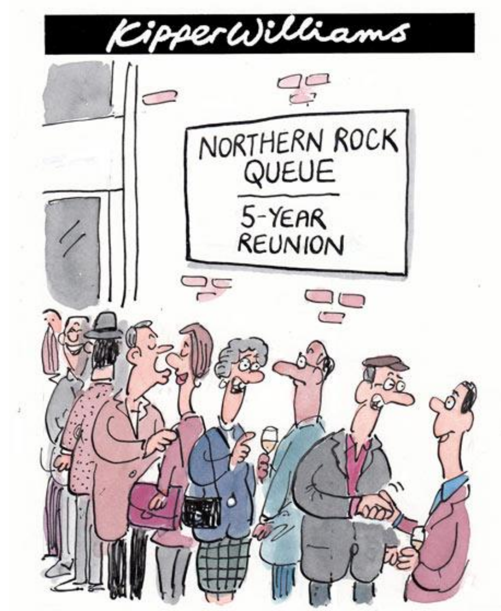
- The risk that a given security or asset cannot be traded quickly enough in the market to prevent a loss (limited scope);
- The risk that a financial firm is not able to meet its debt obligations without incurring unacceptably large losses (limited scope);
- The risk that an organisation is not able to meet its cumulative net cash outflow over a specific period of time.

Important aspects of liquidity:

- Cumulative net cash outflow, so both cash in and outflow;
- Specific period;
- Ability to convert assets into cash.

Possible causes of liquidity risk:

- Bank run;
- Prepayment of mortgage;
- Margin calls.



Introduction to liquidity risk

What is the difference between solvency and liquidity?

There is a distinct difference between solvency and liquidity.

Solvency

The degree to which the current assets of an individual or entity exceed the current liabilities of that individual or entity.

Liquidity

The degree to which an organisation is able to meet its cumulative net cash outflow over a specific period of time.

Examples of mutual influence

- A low level of solvency makes it difficult to attract credit facilities;
- A low level of solvency increases the chances of a bank run;
- Fire sale of assets to meet liquidity need leads to a reduction of asset prices.

Introduction to liquidity risk

Why do we need to focus on this?

Unexpected and material liquidity outflows both under normal and stressed market conditions are serious threats for banks.

In recent years we observed three major trends that effects how liquidity risk is perceived by banks:

- **Rising funding costs**: The credit/liquidity crunch forced banks to reassess their transfer pricing policies in light of increased costs of funding and occasional shortage of liquidity in the market;
- **More stringent regulatory requirements**: Increased and more stringent regulation (e.g. in UK and The Netherlands) forces banks to increase their focus on liquidity risk;
- **Need for profitability improvement**: As costs are increasing there is a need to allocate liquidity costs more efficient.

Until now focus within liquidity modelling was mainly on **contractual** maturities. Caused by these trends the **behavioural** maturity becomes more important.

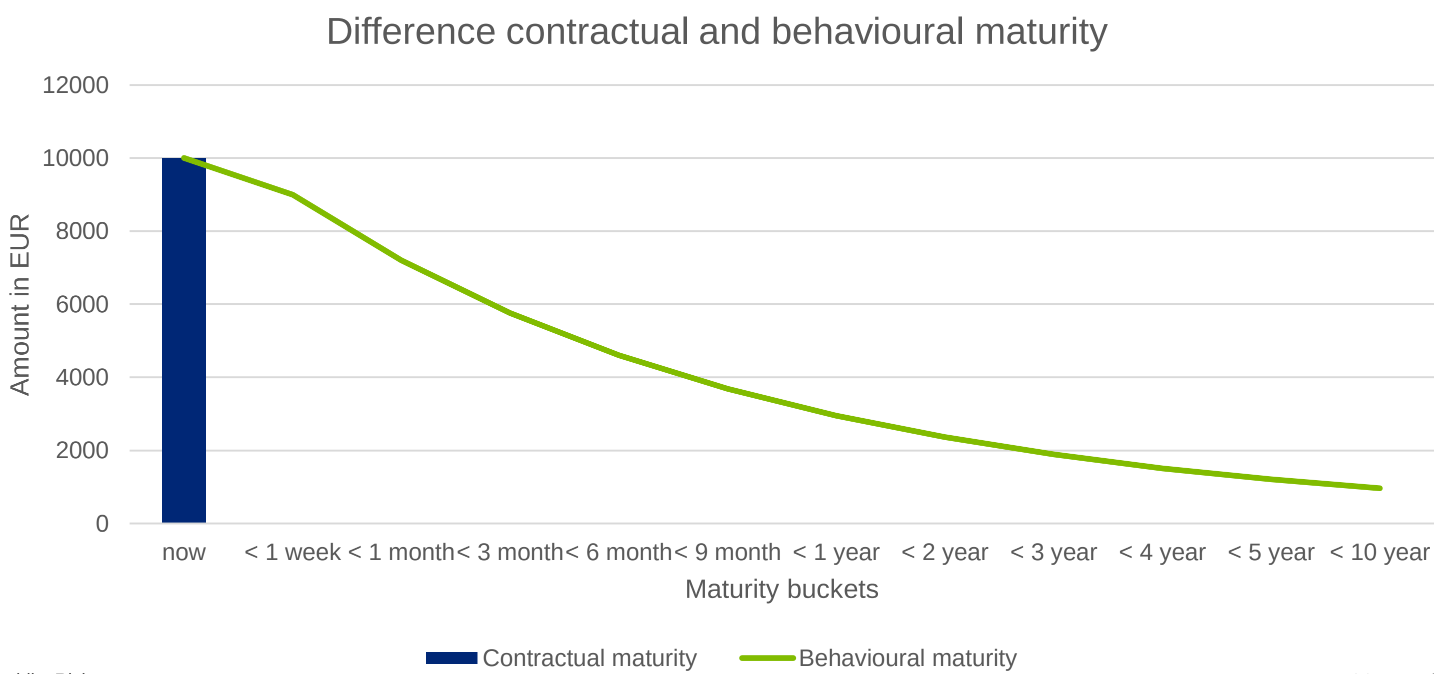
Behavioural Modelling

Objective and purpose

The objective is to develop models that are fit for purpose:

- **Funding:** input for funding plan;
- **Liquidity stress testing:** input for liquidity stress testing framework (survival period);
- **Pricing:** input for direct and/or indirect liquidity costs.

The final outcome of a behavioural model (example for savings):



Behavioural Modelling

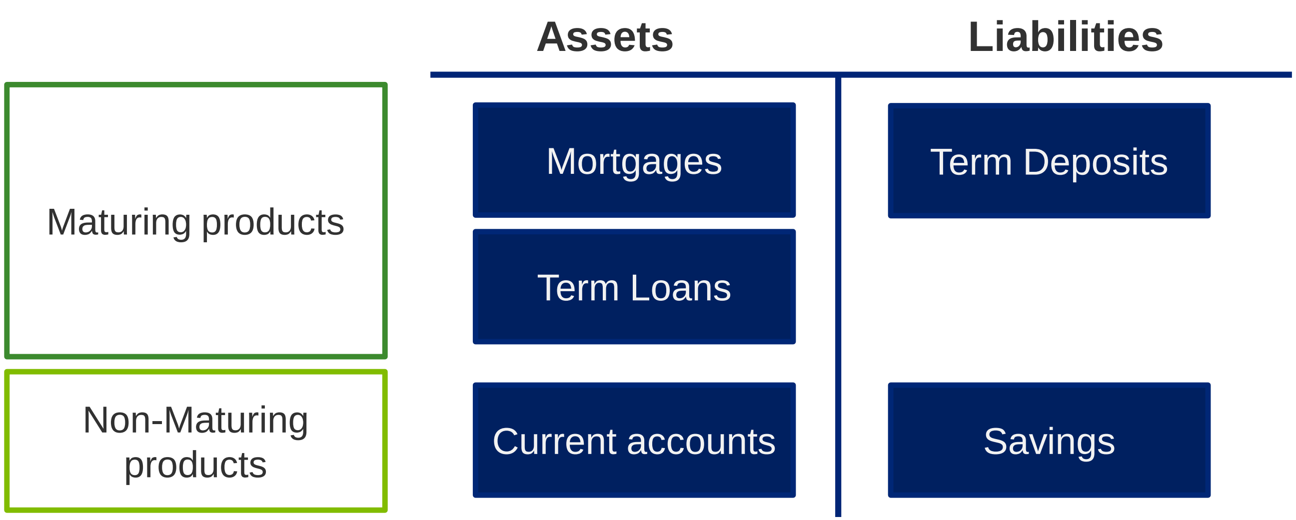
Scope

For a full overview of liquidity risk, **all balance sheet items** need to be taken into account.

- For liabilities the main risk is early withdrawal;
- For assets the main risk is that a loan is extended (rolled over) after the maturity date or prepaid before the maturity date.

In general two types of models can be used:

- For **maturing** products: (multinomial) logistic regression;
- For **non-maturing** products: combination of logistic regression (survival) and an ARDL model (avg. balance).



Liquidity stress testing

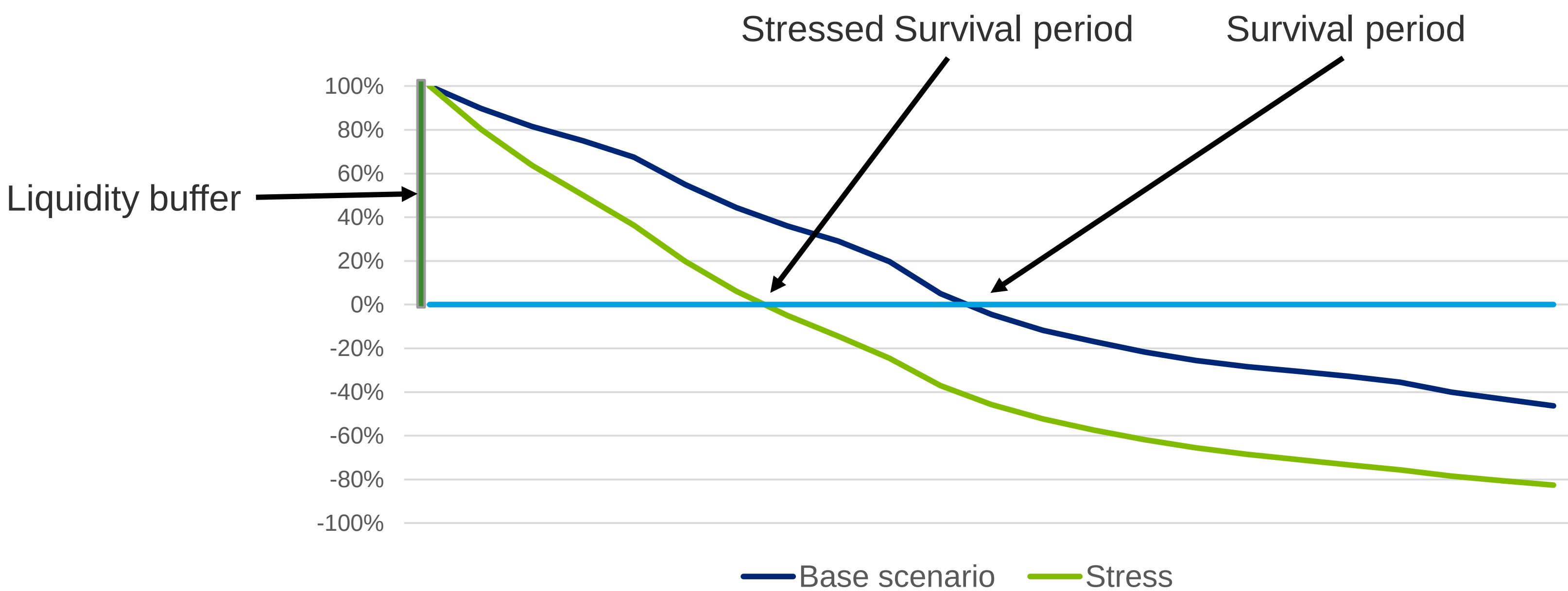
Why liquidity stress testing?

Regulatory perspective

- Identify the risk that a bank is not able to meet it's net cash flow output;
- Liquidity buffer is used to dampen the effect of unexpected cash outflows;
- Stress testing (ILAAP) should take into account stress scenario's that:
  - Last at least 3 months;
  - Unsecured funding is not rolled over during 3 months;
  - Retail outflow doubles to at least 20% during the first month;
  - Facilities are drawn for 50%;
  - Haircuts rise;
  - Market value of liquid assets goes down;
  - FX markets close for 2 weeks;
  - 2 notch downgrade within 3 months.

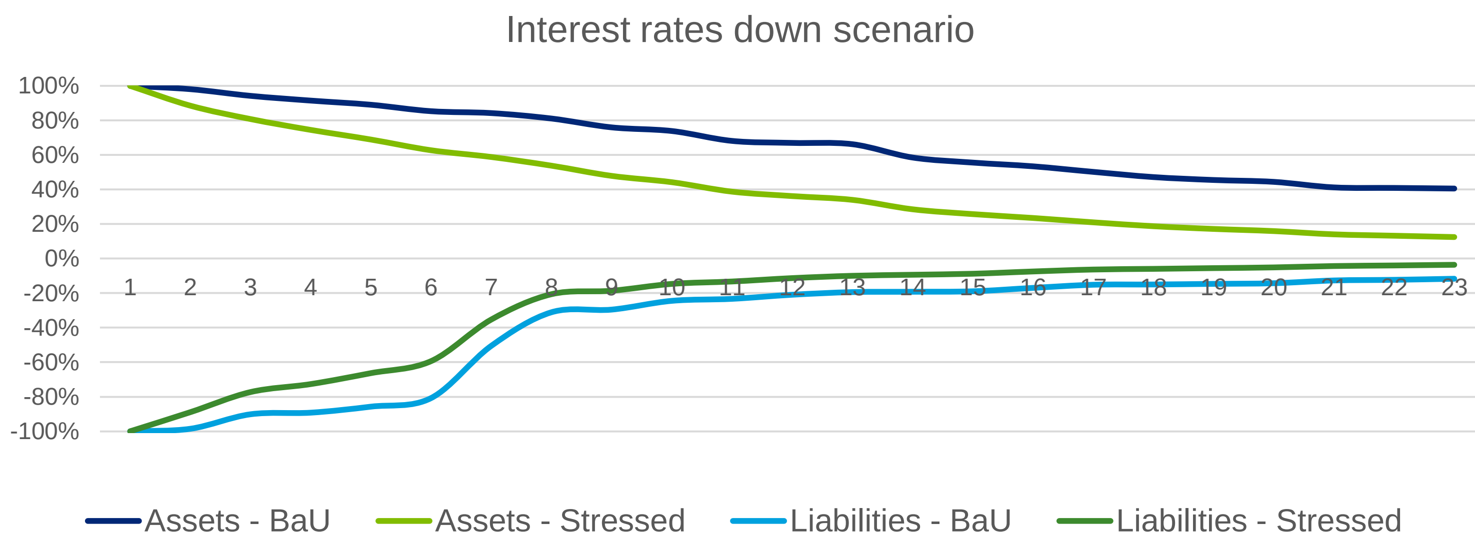
Maturity calendar – net cash-flows

- Behavioural cash flows are based on contractual cash flows and modified for behavioural aspects;
- Based on estimated CPR's and similar metrics, a cash flow calendar can be constructed for a liquidity model;
- Typically separate models are developed for each balance sheet item and interactions are not taken into account.



Aggregating different balance sheet models

- Typically each balance sheet segment has it’s own characteristics and model;
- Challenge is how to correctly take into account diversification effects.



VAR approach for residual aggregation

To aggregate liquidity models, a VAR approach can be used

- Interdependence of these residual series can be estimated using a vector autoregressive model or VAR(p) model:

$$Y_t = \alpha + \sum_{j=1}^{p} Y_{t-1}\Phi_j + U_t, \quad U_t \sim IID(0, \Sigma) \qquad \text{or}$$

$$y_{ti} = \alpha_i + \sum_{j=1}^{p}\sum_{k=1}^{m} y_{t-j,k}\phi_{j,ki} + u_{ti}$$

- Which can be estimated by OLS given normality of the residuals, where Σ can be estimated as follows:

$$\hat{\Sigma} = \frac{1}{n}\sum_{t=1}^{n} \hat{U}_t' \hat{U}_t$$

- The VAR estimates can be used to perform a Monte Carlo simulation that can provide quantile estimates of the maturity calendar.

Mortgages

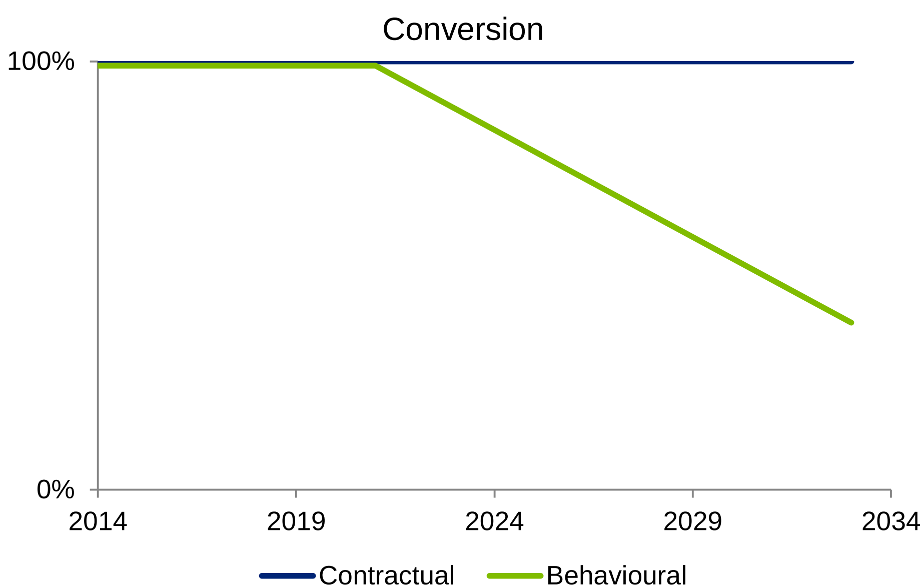
Example

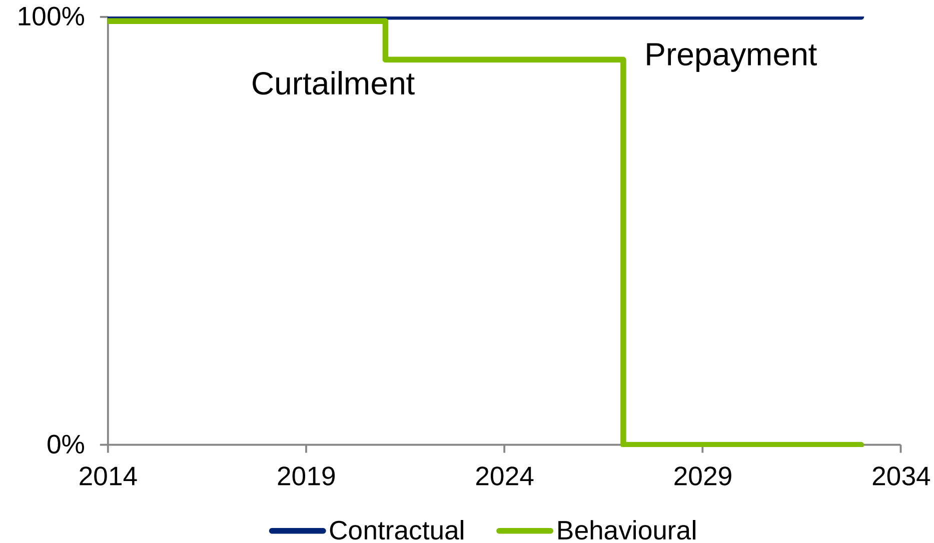
Three options define the behavioral cash flows of a Mortgage

Definition **liquidity risk**: Deviation from contractual cash flows.

Every period we modelled three possibilities that could influence the behavioural cash flows:

- Curtailment (partial prepayment), part of the outstanding is repaid;
- Conversion, the loan(part) is closed and replaced by a new loan(part);
- Full Prepayment, the loan is fully repaid.





To model the three different options a multinomial logit model is used

The multinomial logit model uses multiple logistic regressions to estimate the parameters.

The response variable in our case is:

$$y = \begin{cases} 1: \text{full prepayment} \\ 2: \text{curtailment} \\ 3: \text{ conversion} \\ 4: \text{no changes} \end{cases}$$

Based on this model the probabilities are defines as follows (j=1..3):

$$\pi_j = P(y = j) = \frac{e^{\alpha_j + \beta_j X}}{1 + \sum_{j=1}^{3} e^{\alpha_j + \beta_j X}}$$

And for the last category:

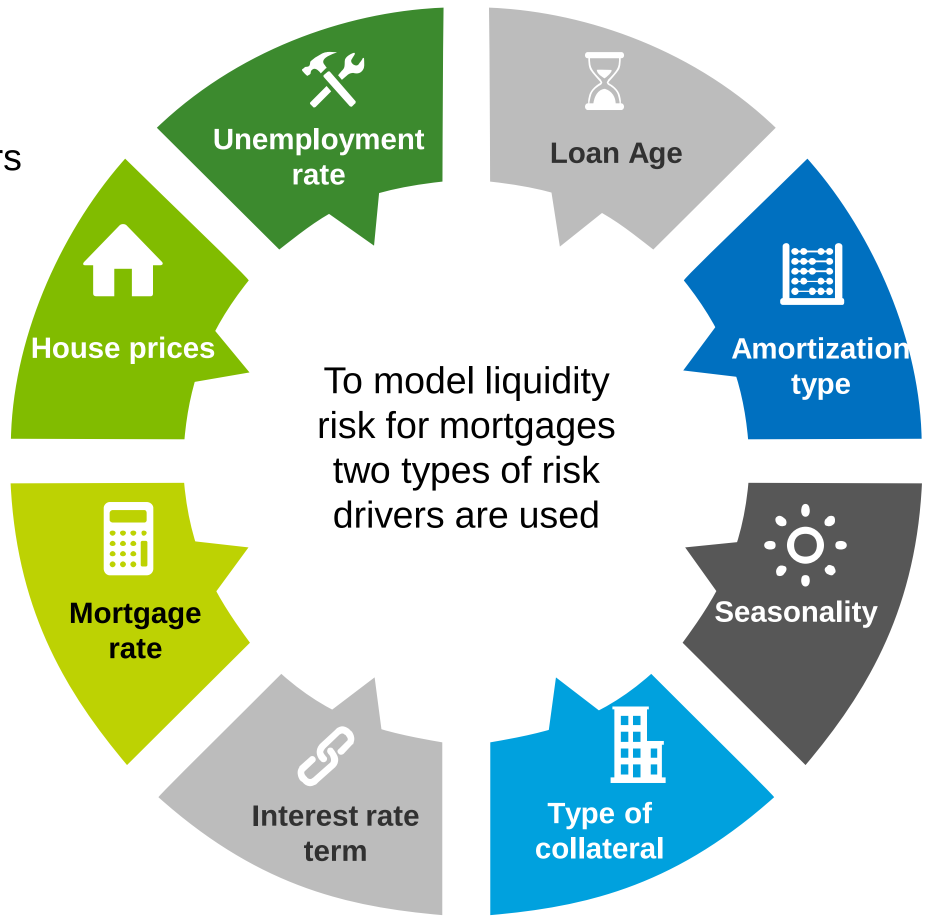
$$\pi_k = P(y = 4) = \frac{1}{1 + \sum_{j=1}^{3} e^{\alpha_j + \beta_j X}}$$

This produces a set of probabilities that sums to one:

$$\sum_{i=1}^{4} \pi_i = 1$$

Possible risk drivers for Mortgages Loans

Macro-economic risk drivers



Contract specific risk drivers

Model fit - Testing

Residual testing

Residual testing is performed on aggregated level:

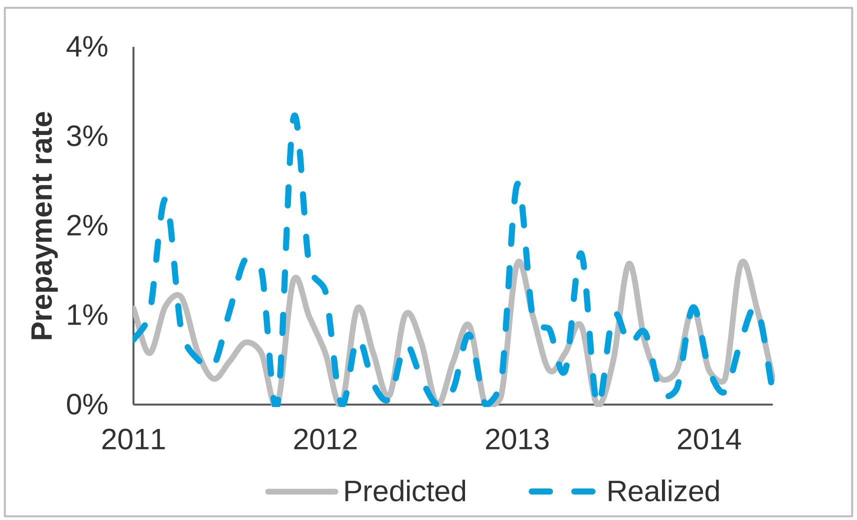
$$ln\left(\frac{\pi_{i,t}}{\pi_{k,t}}\right) = X_{i,t}\,\beta_{i,t} + \epsilon_{i,t}$$

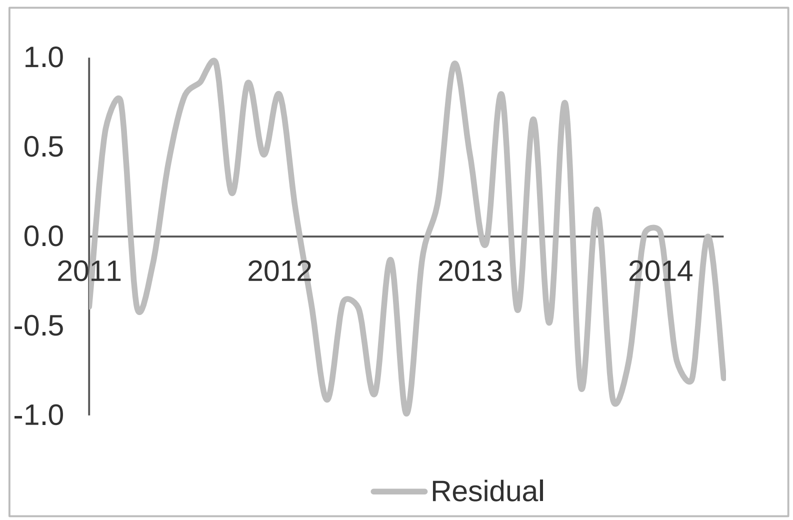
- Assuming the last response ‘no changes’ (k) has a probability close to one, for $i \neq k$:

$$\pi_k \approx 1 - \pi_i$$

- denoting $\tilde{\pi}_i$ as the realized fraction of an event, it follows that

$$\epsilon_i^* = ln\left(\frac{\pi_{i,t}}{1 - \pi_{i,t}}\right) - ln\left(\frac{\tilde{\pi}_{i,t}}{1 - \tilde{\pi}_{i,t}}\right) \sim NID(0, \sigma_i)$$





From model output to behavioural calendars

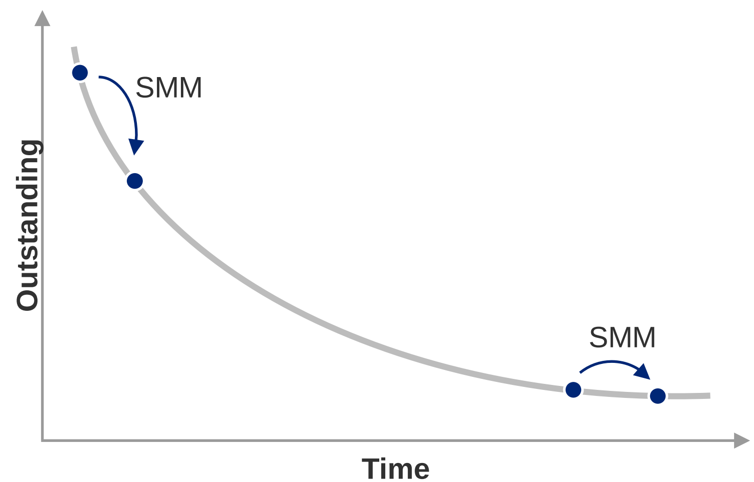
The four probabilities are used to perform a behavioural cash flow projection

- Use a **Monte Carlo Simulation** to simulate paths based on the output probabilities:
- Use the probabilities to define a **Single Monthly Mortality** rates to project the cash flows.

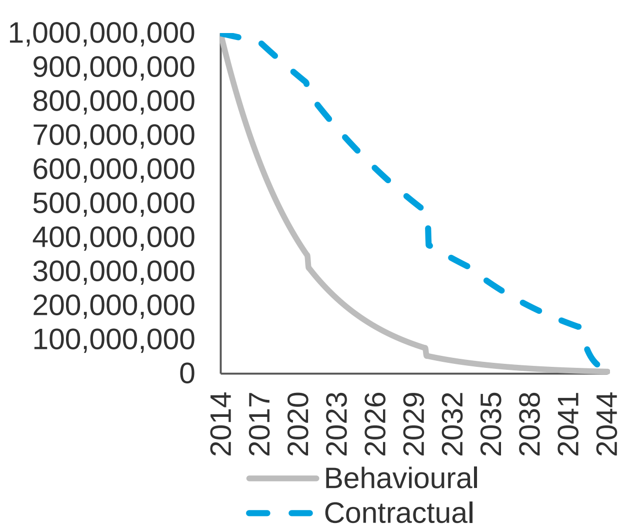
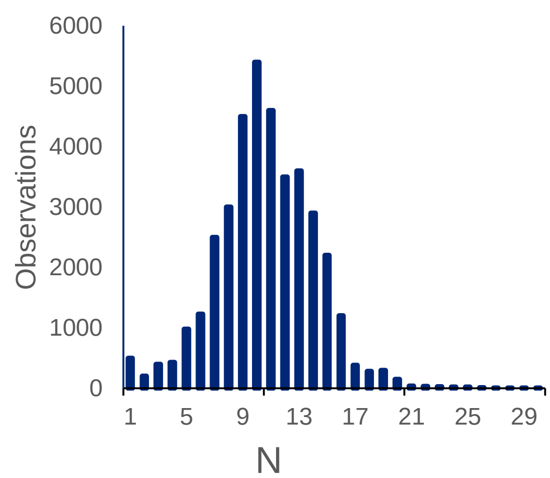
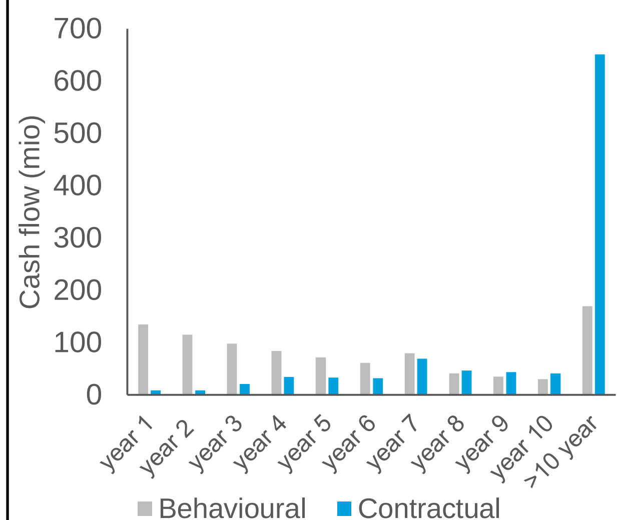
$$SMM = \sum_{i=1}^{k} \pi_i \cdot v_i$$

π_i is the probability that event j will occur and v_i contains the mortality rate per event.

Contractual cash flows have to be subtracted.



Results

<table>
<tr><th>Outstanding over time</th><th>Distribution Weighted Average Life</th><th>Cash flow calendar</th></tr>
<tr><td>Behavioural and contractual outstanding.</td><td>Weighted average life:
$$WAL = \frac{\sum_{t=1}^{N} CF_t * t}{\sum_{t=1}^{N} CF_t}$$</td><td>Cash flows contractual and behavioural</td></tr>
<tr><td>
</td><td>
</td><td>
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Modelling challenges

Challenges in **model estimation**:

- Limited data availability: Liquidity models are first generation models - not all desired data available;
- Regulatory changes influence client behaviour;
- Historical changes:
  - Mortgage types;
  - Portfolio changes.

Challenges in **cash flow projection**:

- Limited historical data with stress;
- Limited availability of benchmarks.

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