The impact of OIS discounting on Libor-benchmarked liabilities

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- Hedging framework for Libor benchmarked liabilities pre-OIS.
- Changing the framework to account for OIS discounting.
- Discount exposure vs Basis exposure.
- OIS generated errors vs other hedging errors.

Libor-benchmarked liabilities pre-OIS

Libor-benchmarked liabilities pre-OIS

Liabilities cash-flows CF are discounted by the Libor-curve. Present value at time t reads:

$$PV_L(t) = \sum_{i=1}^{N} CF(T_i) DF(t, T_i)$$

They can be hedged “exactly” by the replicating portfolio of the Libor curve instruments.

At any time $T > t$, the combined value of liabilities/hedging portfolio must be given by the initial value of the liabilities compounded by the “growth rate” of the Libor curve:

$$PV_L(t) MMA(t, T) = PV_L(T) + \sum_{k=1}^{M} N_k PV_k(T) + C(T)$$

- $\mathrm{N_k}$ is the notional of the kth replicating instrument, and $\mathrm{PV_k}$ its present value;
- $\mathrm{MMA(t, T)}$ is the money-market account between time t and T: $\mathrm{MMA(t,T)} = \prod_{\mathrm{k=1}}^{\mathrm{D}} (1 + \delta_{\mathrm{k,k+1}}\, \mathrm{r(t_k, t_{k+1})})$;
- $\mathrm{C(T)}$ is the cash-account on which payments are collected.

Effectiveness of the hedge

The Hedge-effectiveness can be conveniently defined as the ratio of values between the combined portfolio (liabilities/hedging instruments) and the compounded account:

$$HE(t,T) = \frac{PV_L(T) + \sum_{k=1}^{M} N_k(t,T)\, PV_k(T) + C(T)}{PV_L(t) MMA(t,T)}$$

Clearly for a replicating portfolio HE will be equal to 1 at all times.

For a non-replicating portfolio of hedging instruments of notionals N_k, the definition can be re-written as

$$HE(t,T) = \frac{PV_L(T) + \sum_{k=1}^{M} N_k(t,T)\, PV_k(T) + C(T)}{PV_L(T) + \sum_{k=1}^{M} \overline{N}_k\,(t) PV_k(T) + \overline{C}(T)}$$

HE will be equal to

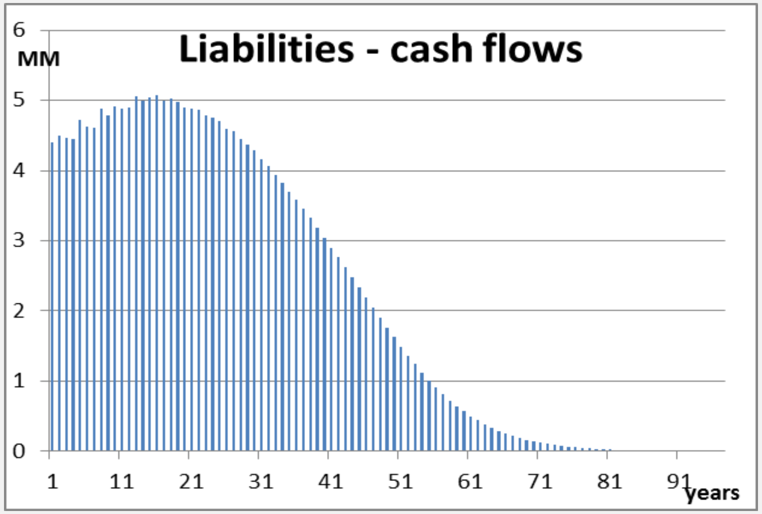
- 1 when the hedging notionals N_k match the replicating notionals $\overline{N}_k$.
- $\neq 1$ for any other set of notionals N_k.

Any deviation from 1 is a measure of hedging error.

Hedging example

Consider a stylized liability profile:

- a set of fixed cash-flows increasing for the next 20 years and decreasing thereafter;
- no derivatives overlay at inception.

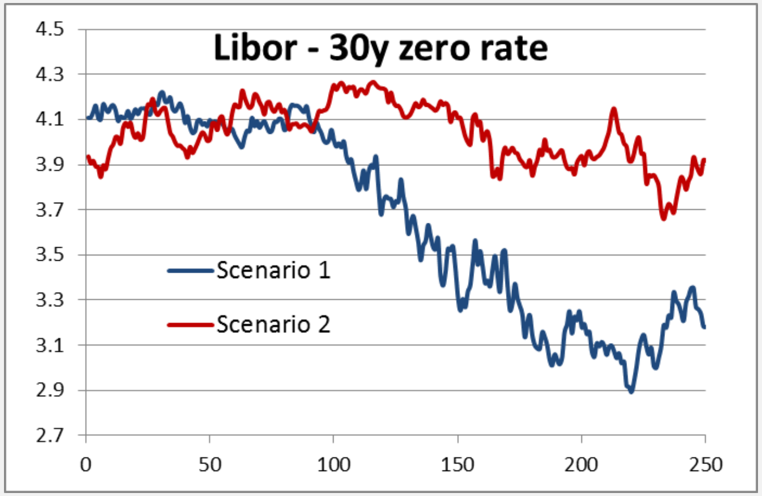


Consider two scenarios for the Libor rates:

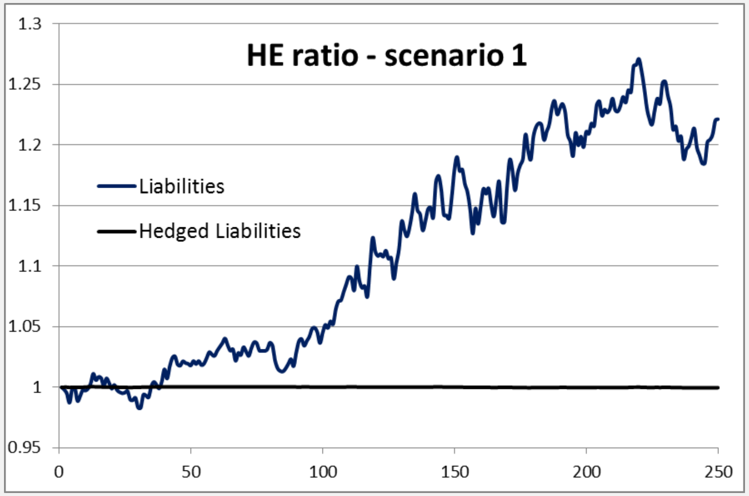
1- Falling rates.

2- Rates moving around their initial level.

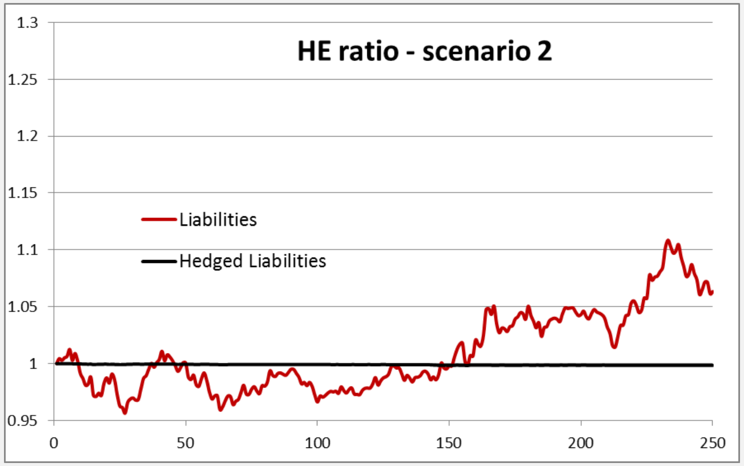
For both scenarios the hedging is performed on a one year period, using a grid of 250 business days.



Hedging example



- Liabilities rise by almost 24%.
- Hedged liabilities show a 0.05% error.



- Liabilities rise by almost 7.5%.
- Hedged liabilities show a 0.17% error.

Note that hedging error (1-HE) is not exactly zero.
It is possible that the error is numerical (e.g., interpolation).

Imperfect hedging

In general liabilities are not replicated exactly for a number of reasons:

1) **Liabilities are updated over time.**
    - Beneficiaries might stop contributing.
    - Beneficiaries might stop receiving benefits.

2) **Buying the whole replicating portfolio might be impractical/inefficient.**
    - Some instruments might be difficult to trade for the fund.
    - Some notionals might be too small.

3) **The liability manager has views.**
    - Leaving X% of the liabilities unhedged.
    - Hedging 100% but leaving exposure to steepening of the curve.

Later the effect of imperfect hedging for a number of cases will be quantified.

Updating the hedge

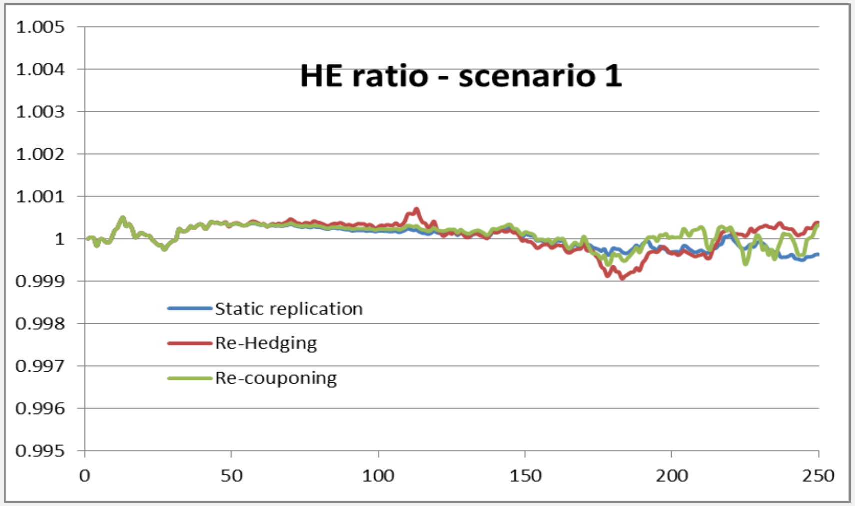
Static replication
The hedge is set at inception as the replicating portfolio and it is left unaltered. If cash flows do not change, static replication is very efficient (transaction costs are only paid at inception).

Re-hedging
The hedging portfolio is updated, at regular or irregular frequency, perhaps closing old positions and entering into new ones, or placing a new overlay on the old portfolio.

Re-couponing
The swaps are reset to par, and their notionals are altered such that the old PV01 is matched. It might be applied using a threshold, so that the portfolio remains as close as possible to the initial portfolio.

** Note that in general banks charge for each unit of PV01 in order to enter into a swap.*
Currently for a Libor swap the charge can be a fraction of a basis point.
Regulatory/Balance sheet charges are also common, though they are not included in this analysis.

Updating the hedge: example



Without transaction costs all updating methods are equally effective. With transaction costs it is a different story:

- Static replication is the most efficient since costs are only paid for PV01 at inception.
- The efficiency of re-hedging by overlaying new swaps on the old hedge depends on the PV01 difference being re-hedged. The bigger the re-hedged PV01 the smaller the efficiency.
- Re-couponing pre-OIS would be free-of-charge on the PV01 (since it is the same), though the old PV would be monetized at around mid.
- Re-couponing post-OIS is charged only for the amount of OIS PV01.

OIS discounting

OIS discounted Libor swaps - sizing

Liabilities are in general benchmarked on an old-style Libor curve (the benchmark is chosen by the client and the manager).

The swaps used to hedge the liabilities are now OIS discounted. Their PV is:

$$PV_{swap} = r \sum_{k=1}^{M} \alpha_{K,K+1}\, DF_{OIS}(T_{K+1}) - \sum_{k=1}^{M} \alpha_{K,K+1}\, L(T_K, T_{K+1})\, DF_{OIS}(T_{K+1})$$

How to size OIS discounted Libor swaps in order to hedge Libor discounted liabilities?

- A possibility is to **use old-style Libor notionals**. This will be shown to be a bad choice.

- Another possibility is to **scale the old-style Libor notionals** by the ratio of the OIS annuity to the Libor annuity. This makes sense because the PV01 of the swap is now an OIS annuity:

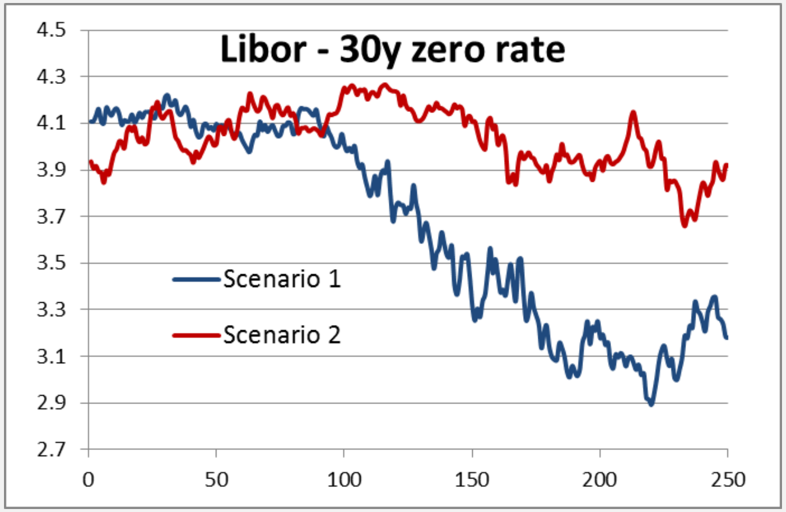
$$PV01_{swap} = \sum_{k=1}^{M} \alpha_{K,K+1}\, DF_{OIS}(T_{K+1})$$

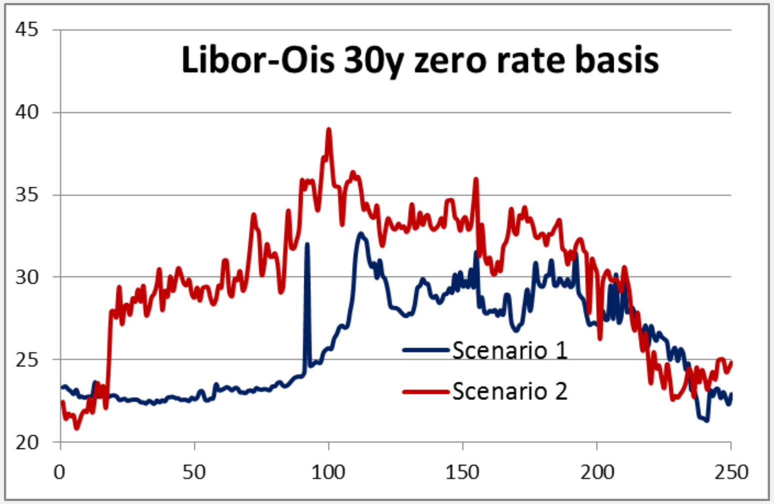
Hedging example - OIS

Liabilities are the same as before. Also the two scenarios for the Libor rates:

1- Falling rates.

2- Rates moving around their initial level.





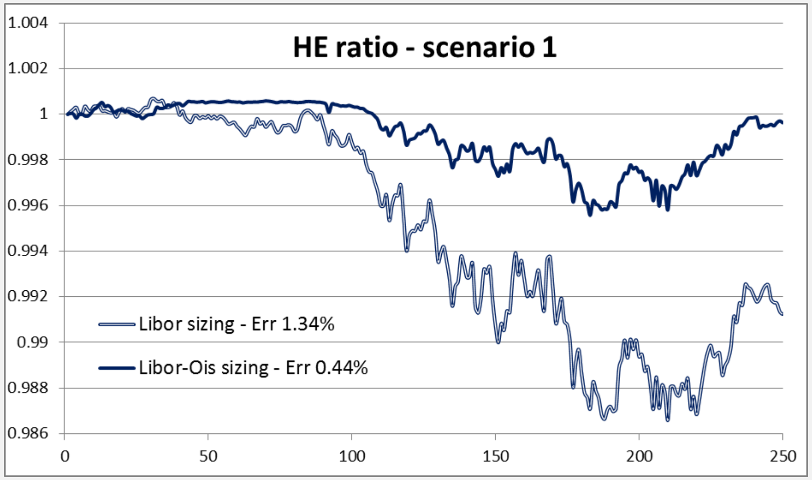
The Libor OIS basis for the two scenarios:

1- From 23.3 bps, widens to 32.7 bps (+9.3 bps) and then almost vanishes (-0.4 residual bps).

2- From 22.5 bps, widens to 39 bps (+16.5 bps), then drops but leaves a 2.4 bps residual.

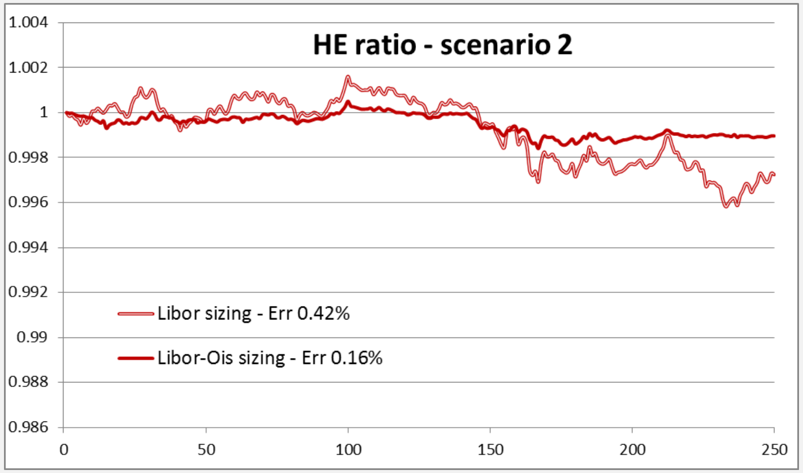
** The residual basis is the difference between the basis at the end and the basis at the beginning of the hedging period.*

Hedging example - OIS



- Old-style Libor sizing gives a 1.34% error.

- Notionals scaling reduces the error to 0.44%.



- Old-style Libor sizing gives a 0.42% error.

- Notionals scaling reduces the error to 0.16%.

Scaling notionals by the OIS to Libor annuity ratio shows to be very effective.

Effectiveness of notionals scaling

It is not surprising that scaling the notionals by the annuity ratio is effective at reducing the hedging error caused by the Libor-OIS basis. The swap PV can be rewritten as the PV of an OIS swap plus a Libor-OIS spread as follows:

$$PV_{swap} = r\sum_{k=1}^{M} \alpha_{K,K+1}\, DF_{OIS}(T_{K+1}) - \overbrace{\sum_{k=1}^{M} \alpha_{K,K+1}\, L_{OIS}(T_K, T_{K+1})\, DF_{OIS}(T_{K+1})}^{\text{-}\, DF_{OIS}(T_0) + DF_{OIS}(T_M)} +$$
$$\sum_{k=1}^{M} \alpha_{K,K+1} \underbrace{\left(L_{OIS}(T_K, T_{K+1}) - L(T_K, T_{K+1})\right)}_{S(DF_{OIS}, DF)} DF_{OIS}(T_{K+1})$$

In the extreme case that the spread $S(DF_{OIS}, DF)$ is constant along the curve, the swap becomes an OIS swap with PV

$$PV_{swap} = (r + S)\sum_{k=1}^{M} \alpha_{K,K+1}\, DF_{OIS}(T_{K+1}) - DF_{OIS}(T_0) + DF_{OIS}(T_M)$$

Effectiveness of scaling notionals

Therefore:

- **if the spread remains constant, the Libor-OIS swap can be a perfect hedge**;

- **a temporary widening of the basis will tend to produce a temporary hedging error,** with magnitude depending on the OIS exposure. The error will vanish as soon as the basis will vanish;

- **any residual basis will have a long lasting effect on the hedging error**, the magnitude of which depends on the OIS exposure.

This is what is seen relatively clearly in the examples so far. The errors vanish with the basis. For both examples, the residual basis has virtually no influence on the error because of the relatively small OIS exposure.

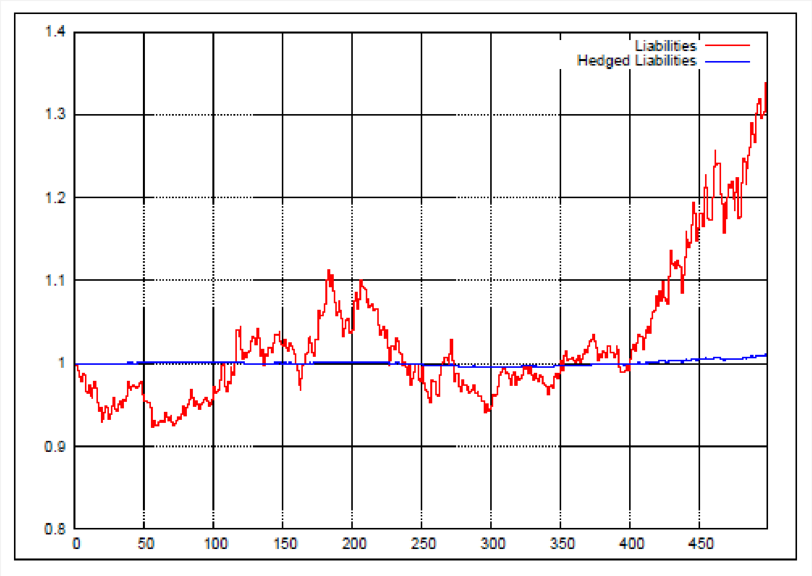
What is known from Bonds

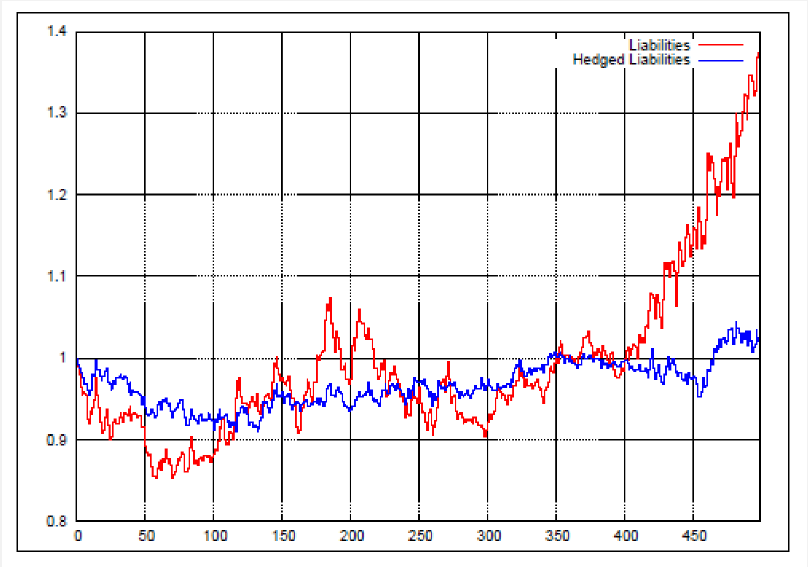
Hedging bond-benchmarked liabilities using swaps is common practice.

A bond-curve-discounted liability cash flow can be decomposed as follows:

$$PV_L = CF\ DF_{BOND} = CF\ DF_{SWAP} DF_{SPREAD}$$

It is easy to see that if the spread does not change, swaps can be a perfect hedge.





Example of bond-benchmarked liabilities hedged with swaps: (left) when swap-bond spread is constant; and (right) when swap-bond spread is not constant.

OIS/discount exposure

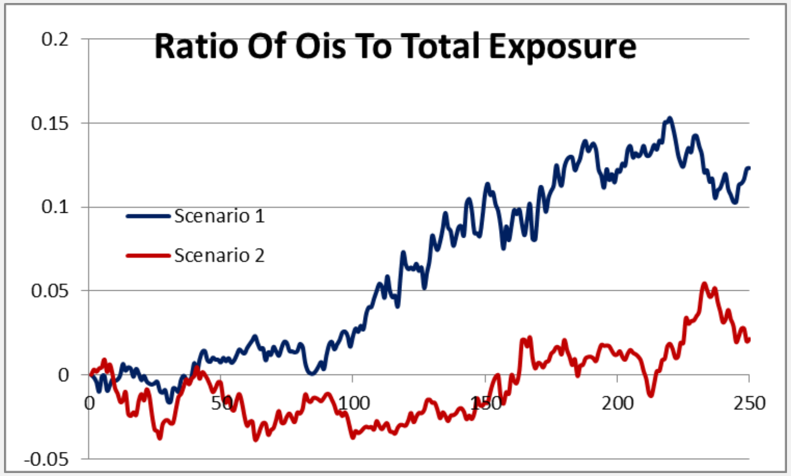
The OIS exposure can be emphasized when the PV is written in term of the ATM rate:

$$PV_{swap} = (r - r_{ATM}) \sum_{k=1}^{M} \alpha_{K,K+1}\, DF_{OIS}(T_{K+1})$$

Neglecting the OIS dependency of the rate, one can see how the OIS exposure is effectively driven (amplified or neutralized) by the money-ness of the swap.

Scenario 1 (falling rates) experiences a growth of the OIS exposure from 0% to 15% of the total exposure.

Scenario 1 experiences bigger hedging errors, even if the basis has a smaller widening than scenario 2.

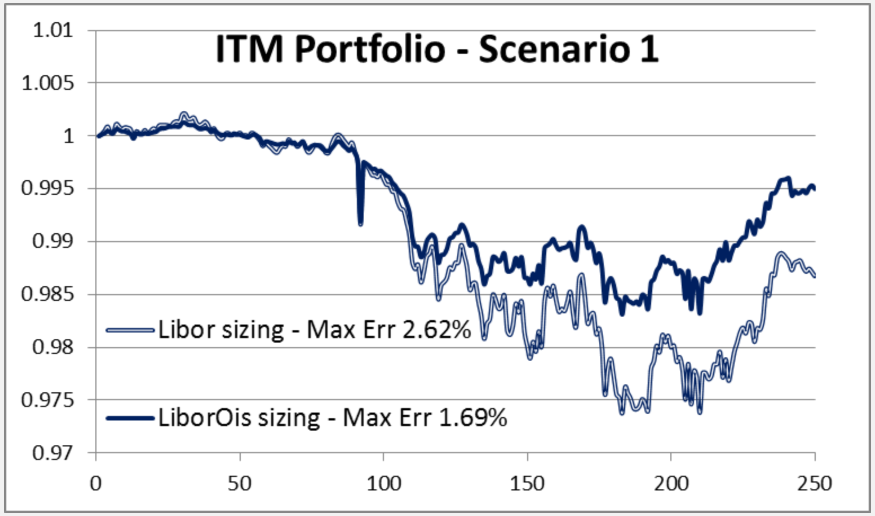


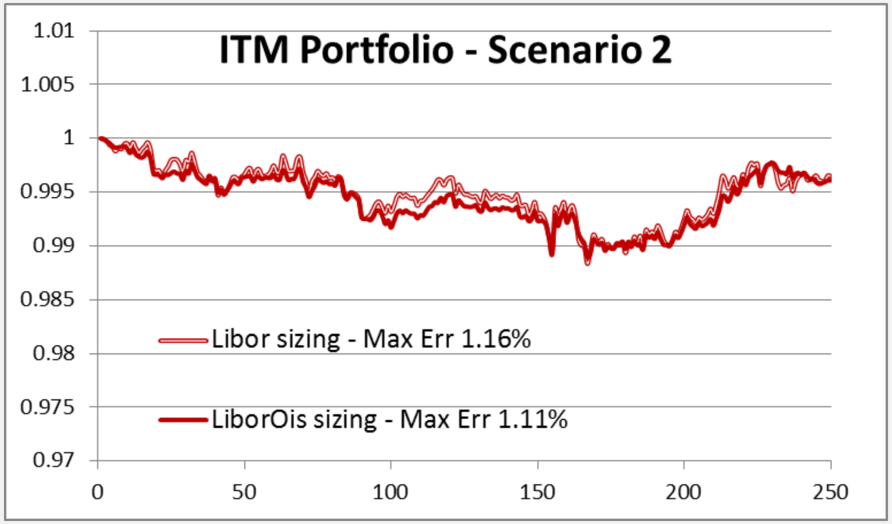
A realistic situation: ITM/OTM portfolios

The examples considered so far did not have an existing OIS exposure: liabilities were hedged using a portfolio of par-swaps. OIS exposure appeared during the hedging.

A more realistic situation is one in which there is an existing overlay, which might be in or out of the money. Next ITM/OTM portfolios with 24% OIS exposure are considered.

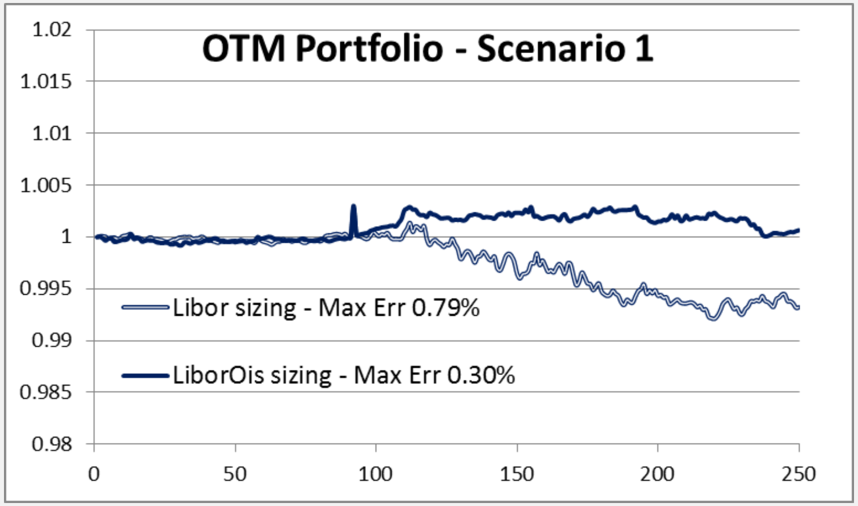
ITM - Libor sizing gives an 2.62% and 1.16% error for scenarios 1 and 2. Notionals scaling reduces the error to 1.69% and 1.11%, respectively.

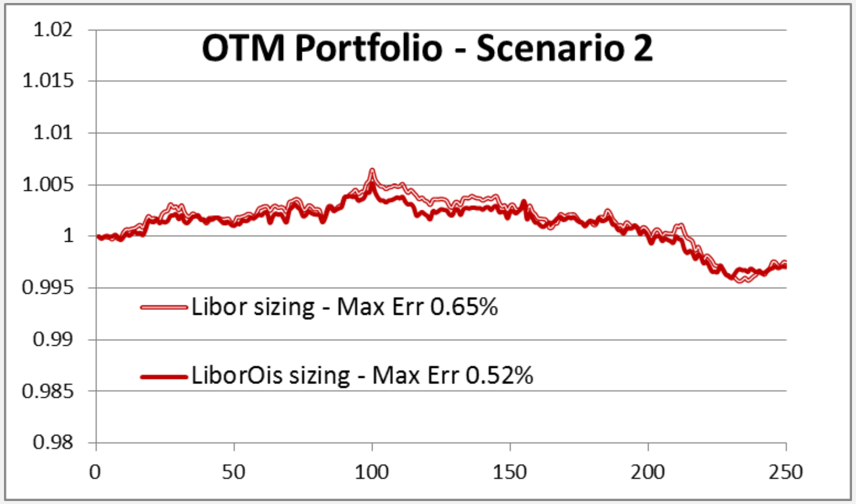




A realistic situation: ITM/OTM portfolios

OTM - For Libor sizing the errors are 0.79% and 0.65% for scenario 1 and 2. Notionals scaling bring the error to 0.3% and 0.52%, respectively.





It is important to notice that:

- The error has a positive effect for the ITM case, and negative for the OTM case;
- The OTM's error magnitude is smaller than the ITM's.
- Residual errors are slightly larger than for the no overlay cases.

Basis risk

OIS exposure vs Libor-OIS spread exposure

Managing the basis should not be understood as hedging the OIS exposure.

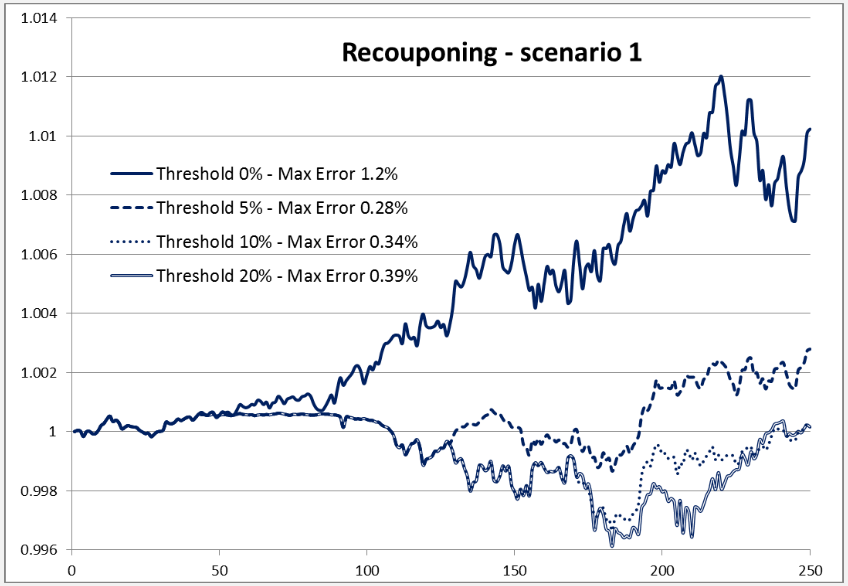
The OIS exposure is the "discount exposure", which is necessary for the swap to function. It is dependent on the money-ness, and changes according to the rate level. Hence, hedging the discount exposure is a dynamic exercise.

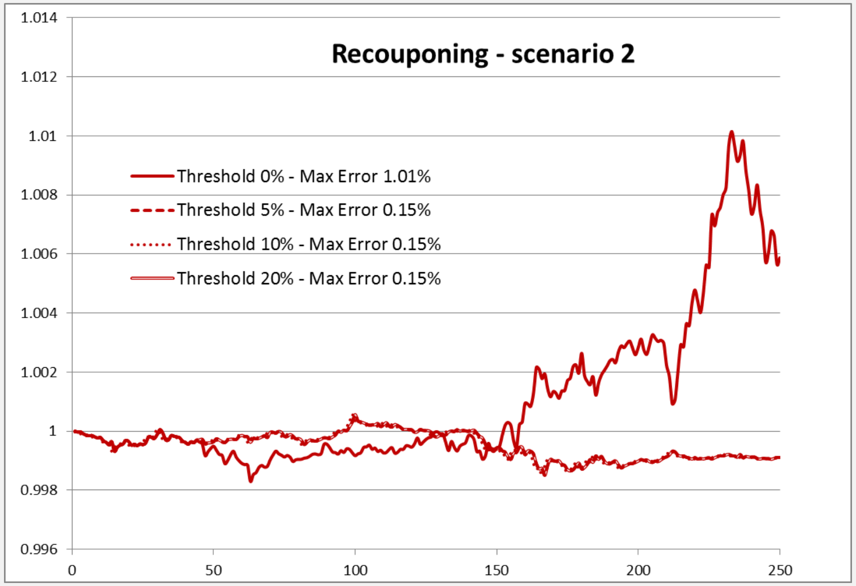
Most telling example is that of hedging with rates moving up and down symmetrically. Neutralizing the OIS exposure would require having a notional amount of OIS swaps when rates go up, and that same amount, but with negative sign, when rates go down.

Such an exercise does not make much sense since the same effect could be achieved by resetting the hedge (e.g., re-couponing) at a fraction of the cost.

Active basis management via re-couponing

Re-couponing the portfolio reduces the discount exposure and indirectly limits the influence of the basis.





- Total re-couponing increases the error as the portfolio changes to the point that it has no longer its original replicating properties (PV01 hedging vs Deltas hedging).

- A threshold is necessary in order to have a trade-off between resetting the discount exposure and avoiding the hedging portfolio to change too much.

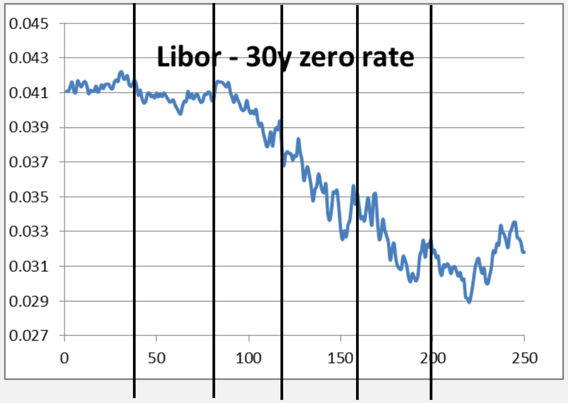
** X% threshold means that a swap is re-couponed only if PV01_OIS/PV01_TOTAL > X*

The costs of re-couponing

In general, pre-OIS re-couponing would not be charged. Nowadays a charge is applied in terms of OIS PV01. At the time of running the example used here, the typical charge was around ½ basis point. The same as for the PV01 of a Libor swap.

Consider the scenario 1 example in the previous page, with 5% threshold and re-couponing happening every 40 days during the 1 year period.

Initially the PV01 is 158K, and the PV = 94100K.
At the 1th and 2nd date no re-couponing is triggered.
At the 3rd date 10.4K of OIS PV01 are re-couponed.
At the 4th date 10.1K of OIS PV01 are re-couponed.
At the 5th date, only 1.6K.



re-couponing dates: 1 2 3 4 5

*The total cost is therefore (158 + 10.4 + 10.1 + 1.6) x 0.5 = **90K**.*
*This is a fraction of the initial cost of setting the hedge, which is 158 x 0.5 = **79K**.*

In terms of performance drag/hedging error:
*Total cost % = 90/94100 = **0.09%** and Initial cost % = 79/94100 = **0.084%***

Errors due to hedging assumptions

Error due to OIS vs other errors

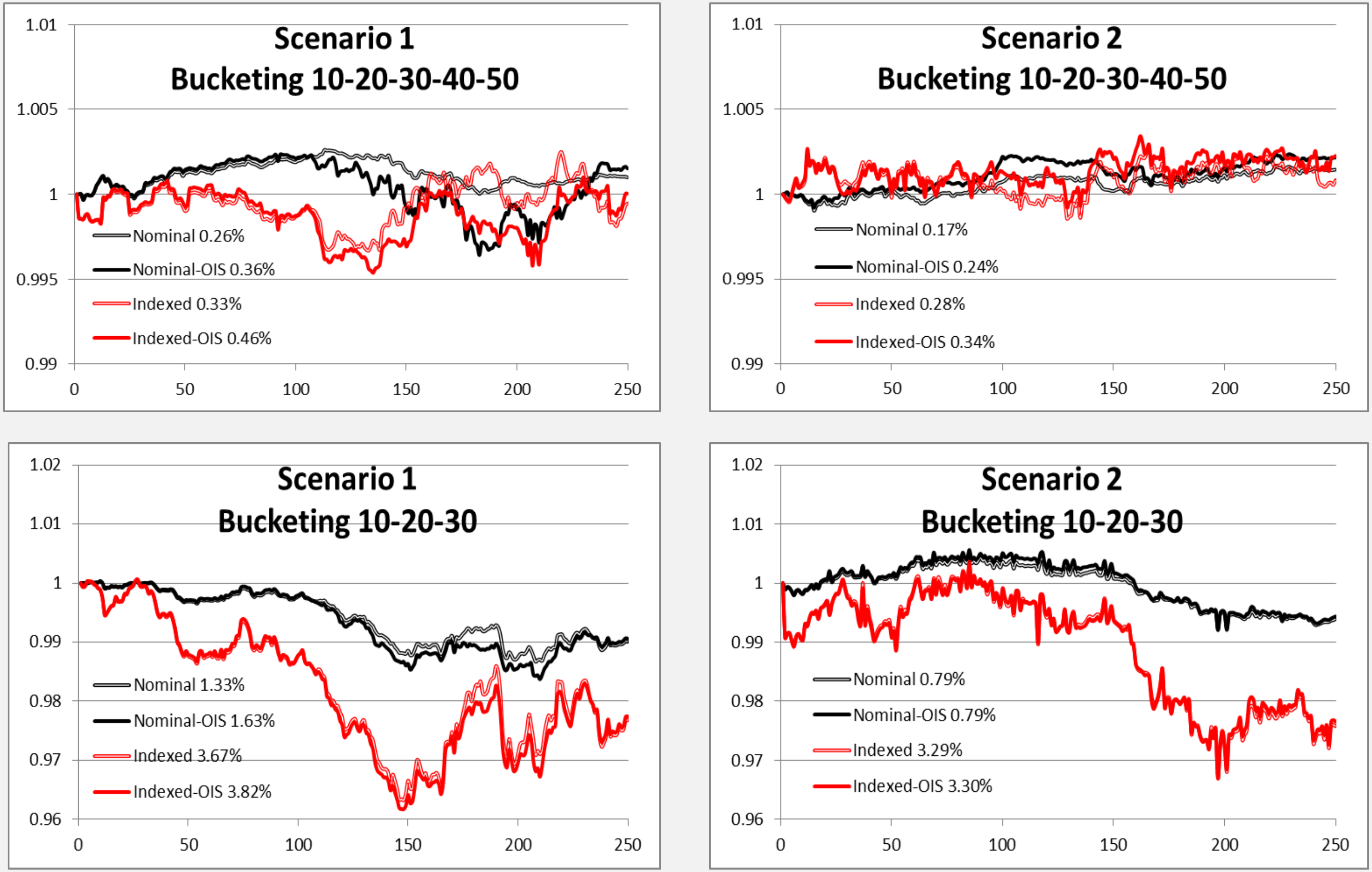
Looking at the error due to OIS in isolation is not a useful exercise. More useful is to compare it with errors generated by routine hedging assumptions.
Hedging is in fact rarely perfect. Bucketing and curve views are often adopted.

- Bucketing divides the deltas of the cash flows in buckets. The deltas of each bucket are then hedged using a swap with maturity falling in the bucket.

- If not done uniformly, bucketing could express a curve view.

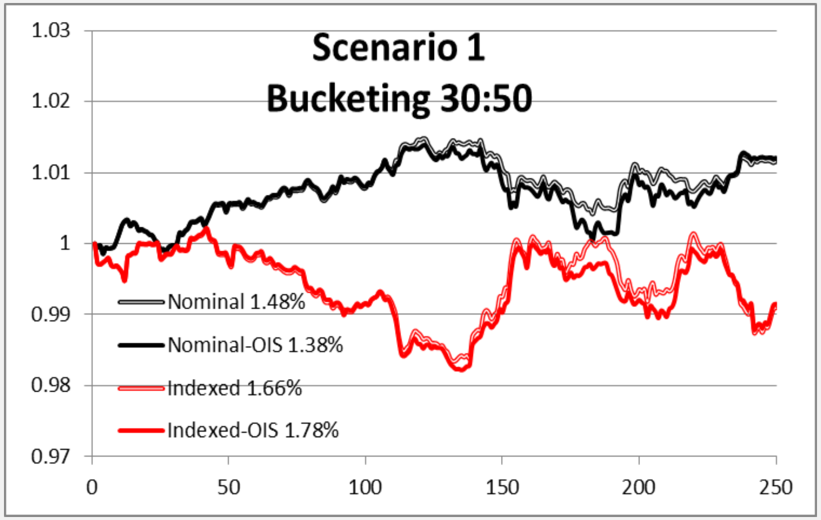
1y	16.72				
2y	247.10				
3y	339.87	5y	1,582.63		
4y	409.13				
5y	569.81				
6y	649.67				
7y	682.96				
8y	848.45	10y	7,174.12	10y	158403.13
9y	978.59				
10y	4,014.45				
15y	9,984.88	20y	32,861.78		
20y	22,876.90				
30y	38,563.64	30y	38,563.64		
40y	36,697.18	40y	36,697.18		
50y	41,523.78	50y	41,523.78		
Total BPV	158403.13	Total BPV	158403.13	Total BPV	158403.13

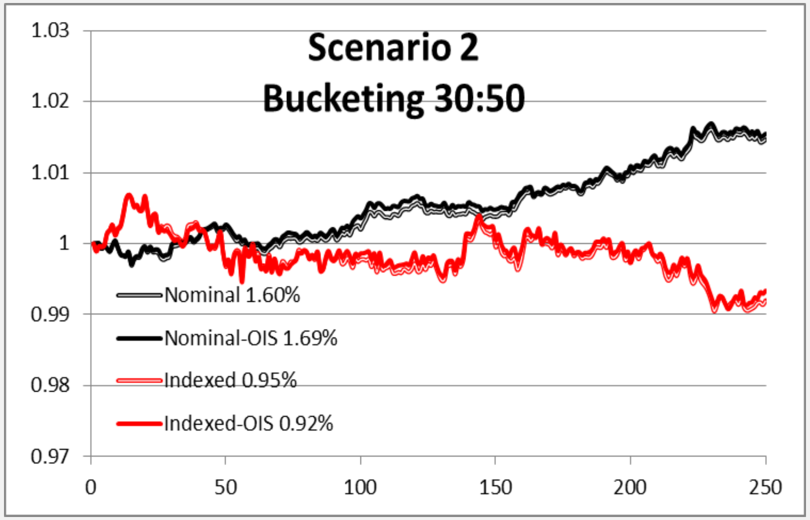
Bucketing uniformly

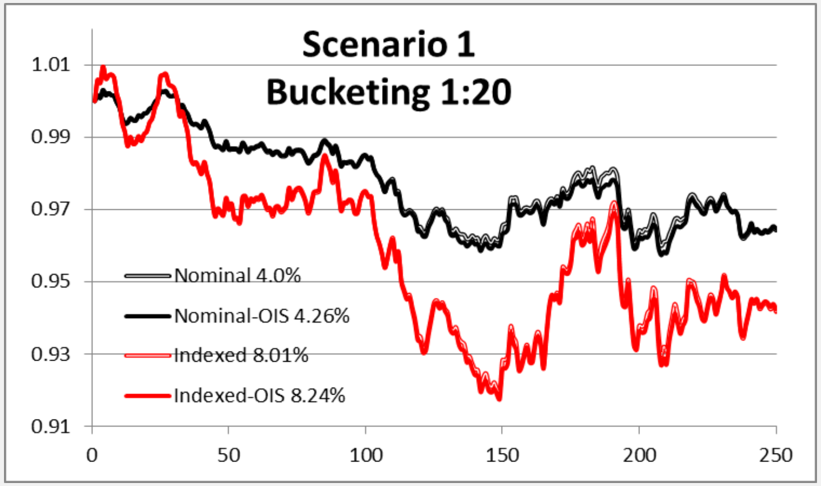


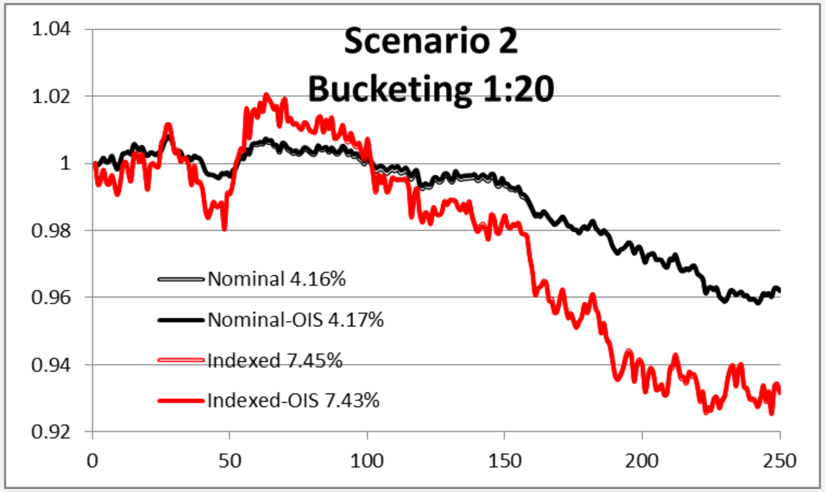
* Note that apart from the addition of inflation swaps to the hedging portfolio, the framework for hedging inflation is very similar. It is not introduced here for brevity.

Bucketing short- or long-end









Summary of errors

	Libor (%)	Libor-OIS (%)	Difference (%)
Scenario 1, Bucketing. 10-20-30-40-50, Nominal	0.26	0.36	0.1
Scenario 1, Bucketing. 10-20-30-40-50, Indexed	0.33	0.46	0.13
Scenario 2, Bucketing. 10-20-30-40-50, Nominal	0.17	0.24	0.07
Scenario 2, Bucketing. 10-20-30-40-50, Indexed	0.28	0.34	0.06
Scenario 1, Bucketing. 10-20-30, Nominal	1.33	1.63	0.3
Scenario 1, Bucketing. 10-20-30, Indexed	3.67	3.82	0.15
Scenario 2, Bucketing. 10-20-30, Nominal	0.79	0.79	0
Scenario 2, Bucketing. 10-20-30, Indexed	3.29	3.3	0.01
Scenario 1, Bucketing. 30:50, Nominal	1.48	1.38	-0.1
Scenario 1, Bucketing. 30:50, Indexed	1.66	1.78	0.12
Scenario 2, Bucketing. 30:50, Nominal	1.6	1.69	0.09
Scenario 2, Bucketing. 30:50, Indexed	0.95	0.92	-0.03
Scenario 1, Bucketing. 10:20, Nominal	4	4.26	0.26
Scenario 1, Bucketing. 10:20, Indexed	8.01	8.24	0.23
Scenario 2, Bucketing. 10:20, Nominal	4.16	4.17	0.01
Scenario 2, Bucketing. 10:20, Indexed	7.45	7.43	-0.02

Overall, looking at different assumptions for nominal/inflation hedging, it seems that the error added by OIS discounting is relatively small if proper sizing is performed.

Especially for assumptions that give large errors, the OIS error is small in comparison.

Alternative to manage the basis

Theoretically using basis swaps (OIS vs 3M Libor & 3M Libor vs 6M Libor) would be the best way to manage the basis.

In practice, hedging with OIS vs 3M Libor swaps might not be viable:

- The market is liquid for short tenors, and less liquid for long tenors;
- The costs in basis points are high compared to Libor swaps;

For the case considered in the previous slide, **the cost could go from 0.084% to 0.43%.** Note that, in the previous table, the **max error due to OIS was < 0.3%**.

Conclusions

- Properly sizing swap notionals is very effective at neutralizing/reducing the impact of the basis;

- The impact is minimal (only volatility) if the basis leaves no residual after widening, and can be long-lasting if a residual basis is left;

- Compared to other hedging assumptions, the impact of the Libor-OIS basis seems to be relatively small.

- Even for ITM/OTM portfolios proper sizing is very effective. Whereas ITM portfolio benefits from the basis, OTM portfolios do not. However, the error in the OTM case is smaller in magnitude than the ITM case;

- Money-ness of the portfolio amplifies the impact of the basis. Therefore re-couponing is a useful and cost-effective tool in mitigating the possible impact of the basis (and it is already available in the LDI manager toolbox)