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Replication of a Class of Variable Annuities for the Purpose of Economic Capital Calculations

Vanilla options versus path-dependent instruments

TopQuants, 2013

Agenda

Motivation –Variable Annuities are hard to replicate • Difficulties in replicating Variable Annuities	5 min
Replication framework • Principle behind replicating portfolios • Replicating portfolios in practice • Replication building blocks • Financial instrument selection	20 min
Case Study – using plain vanilla instruments	10 min
Case Study – using path-dependent instruments	10 min
Discussion	15 min

Motivation – Variable Annuities are hard to replicate

Market – Variable Annuities provide a series of payments depending on the performance of the underlying investment account for a fixed period or life

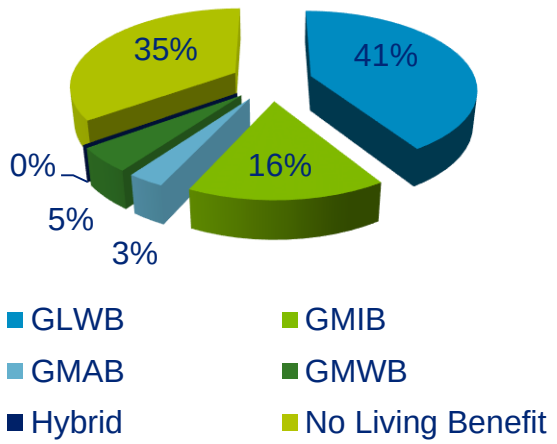
Variable Annuities

- Provide a series of payments to the policyholder during either a fixed period or, in case of a life annuity as long as the policyholder lives.
- The level of the payments depends on the performance of the underlying investment account.

Variable Annuities market

- The United States is the largest variable annuity market (147 billion dollars sold in 2012). In 2002, the Guaranteed Minimum Withdrawal Benefit (GMWB) was introduced as optional rider on VAs. The GMWB usually guarantees an income for life and the ageing of the population helped increasing the demand for this type of guarantee.
- Japan is the second variable annuity market. This market started around 2000 and grew rapidly between 2000 and 2005. Banks were the main distribution channel and foreign banks led the market. Since 2008 many of these foreign markets exited the market or de-risked their products.
- The VA market in the European Economic Area (EEA) measured in technical provision amounted 188 billion euros at the end of 2010.

Variable Annuity sales (Q3 2012) by living benefit



Sources: *LIMRA (2013),*
The Geneva Association (2013), Variable Annuities – An Analysis of Financial Stability
EIOPA, Report on Variable Annuities (2011)

Problem – Insurance companies struggle in replicating variable annuities, but also Solvency II and EIOPA emphasize the difficulty in replicating VAs

Regulator

EIOPA and Solvency II stress the difficulty in determining the Solvency Capital Requirement (SCR) of Variable Annuities *(source: EIOPA, Report on Variable Annuities, d.d. 2011).*

Technical provision:

- "The use of replicating portfolio techniques for calculation of technical provisions may be possible, but **highly unlikely**"

Solvency Capital Requirement:

- "On capital requirements in Solvency II, the task-force considered the **Standard Formula approach to the SCR is generally not sufficient due to the complexity of VA's** so that companies transacting VAs business should usually be required to use an Internal model for the calculation of SCR, according to 119 of Solvency II Directive".

Practice

Replication of VAs for SCR purpose coincides with difficulties in practice

Complex risk profile:

- Special risks: equity volatility, interest rate volatility, policyholder behavior (*source: EIOPA, Report on Variable Annuities, d.d. 2011)*

Unstable and insufficient replication quality:

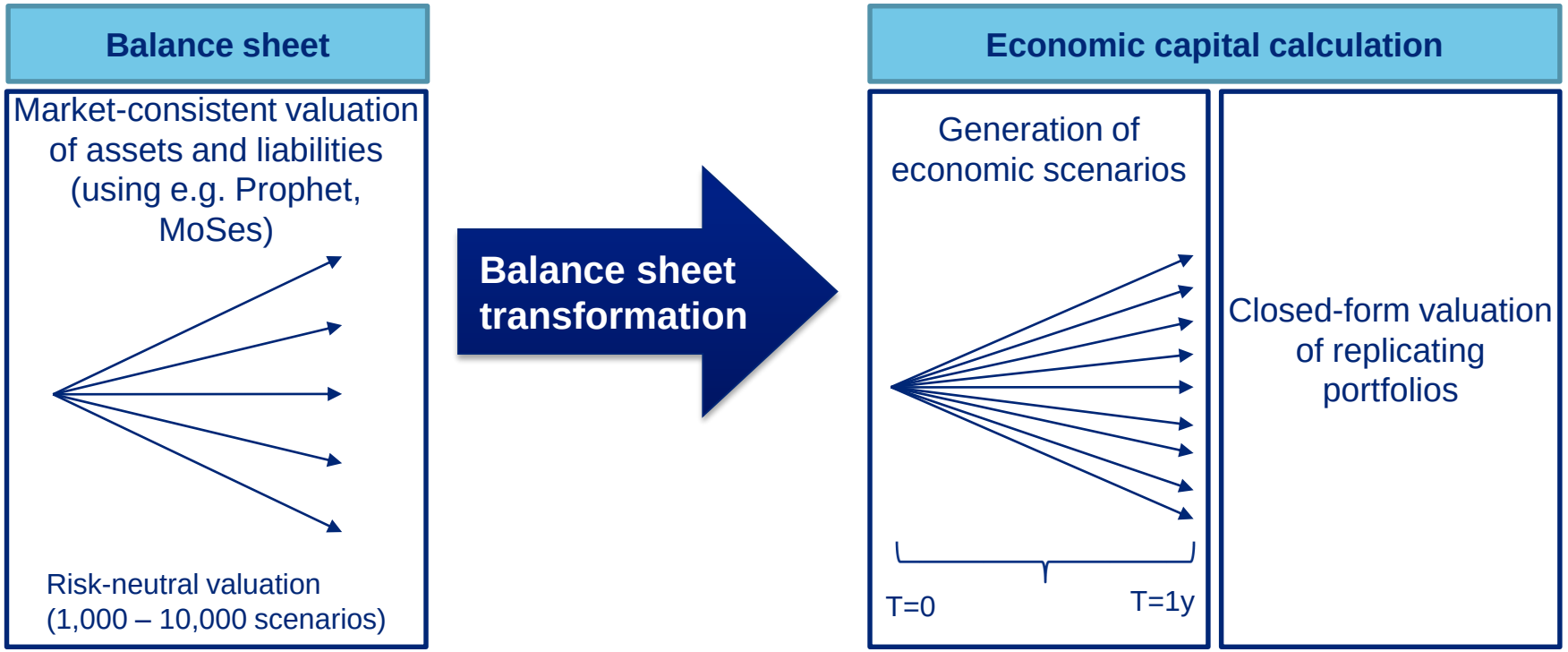
- Correct sensitivities to some risk factors, but large mismatch to others (*Koursaris, 2011*)
- Replication quality unstable over subsequent reporting periods (*Koursaris, 2011*)
- Net present value replication does not capture path-dependency well (*Chavigny and Devineau, 2011*)
- Much manual manipulation needed (*Koursaris, 2011*)

Replication as part of EC

Transformation of assets and liabilities into a replicating portfolio could accelerate the EC calculation due to possible closed-form valuation

EC calculations based on market-consistent value introduce a nested stochastic problem.

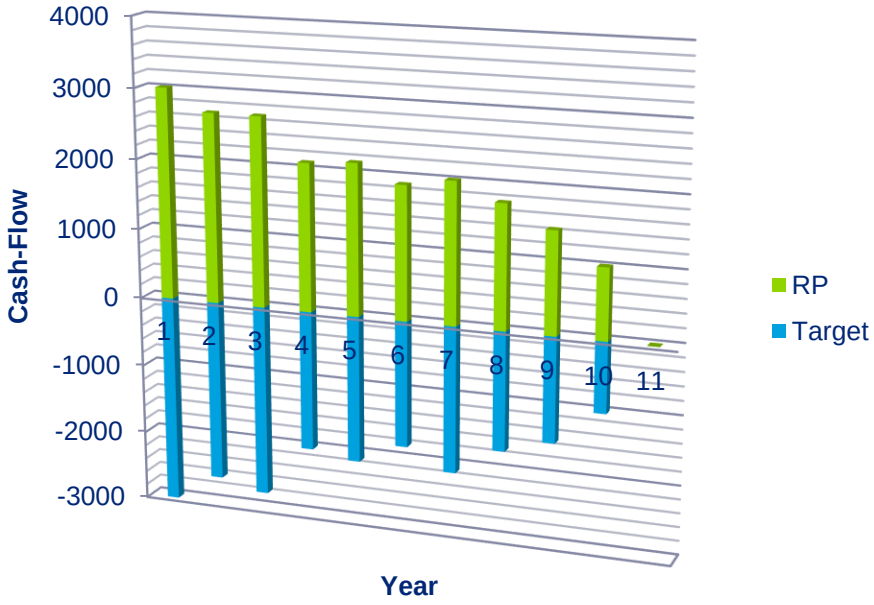
Simplifications are needed to be introduced to obtain a cost effective solution (e.g. through the use of accelerated convergence techniques, such as Least-Squares Monte Carlo, Curve-fitting, Replicating Portfolios, ...)

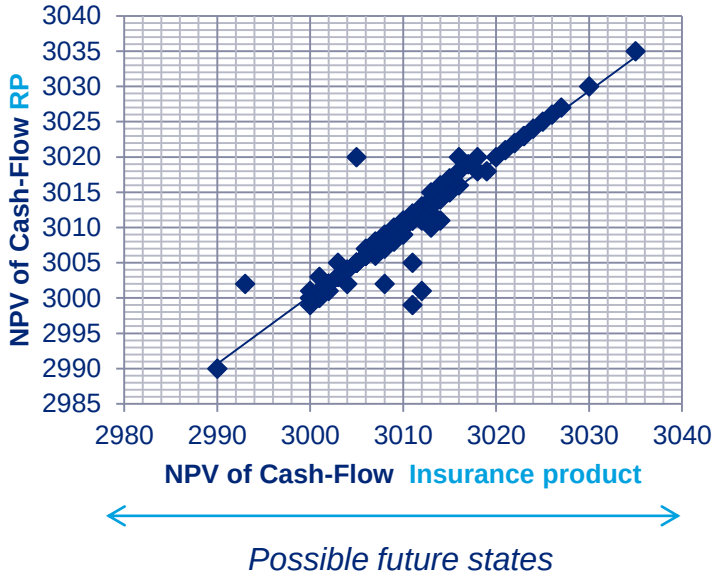


Replicating portfolios intent to resemble the cash-flow pattern of the underlying insurance product in all relevant market circumstances

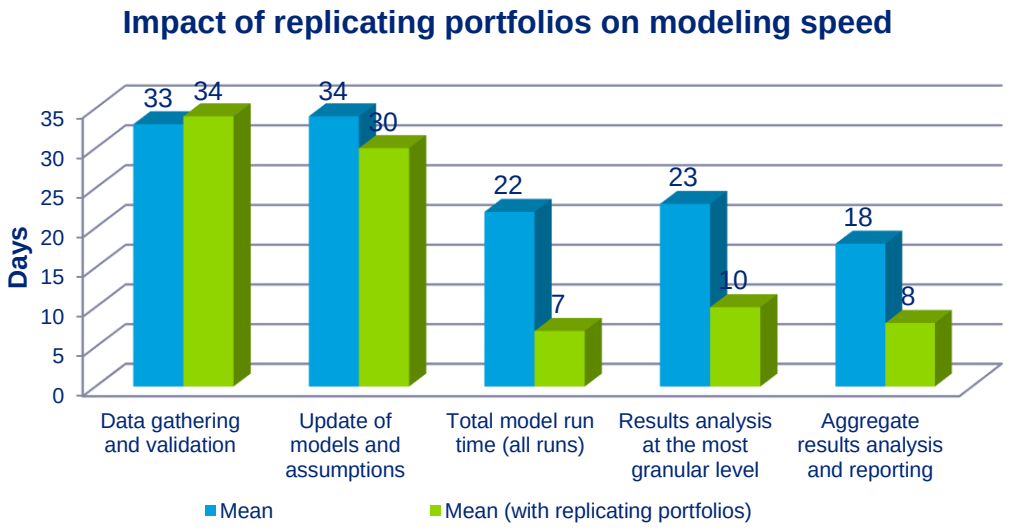
Replicating portfolio = a portfolio of relatively simple assets (**replicating instruments**) that reveals cash-flows that are consistent with the underlying real assets or liabilities under all possible future states.

Purpose This replicating portfolio can then be used as a proxy to value the assets and liabilities under the different economic shock scenarios for EC purposes.

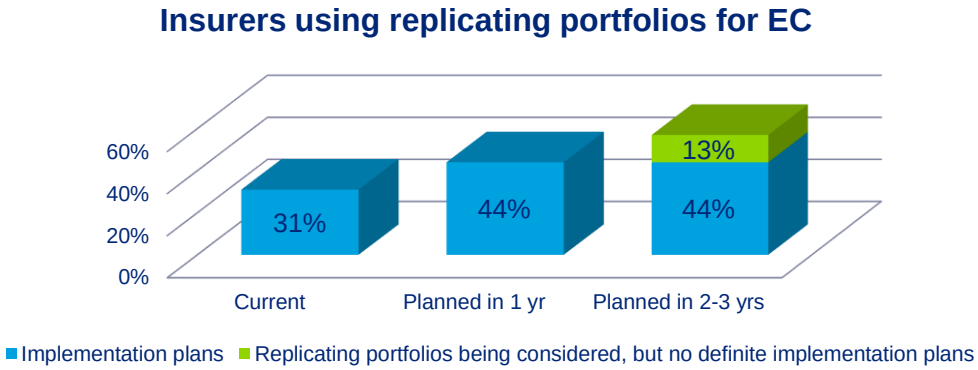




Industry benchmarks CRO forum show that the market acknowledges the potential of replicating portfolios



Replicating portfolio valuation has the potential to significantly **reduce modeling speed** compared to alternative approaches.



Potential is seen by more and more insurers part of the CRO forum

Source: CRO Forum study in 2008 in which 16 large life insurers participated. (Aegon, AIG, Allianz, AVIVA, AXA, Eureko, Fortis, Generali, Groupama, Hannover RE, ING Insurance, Met Life, Munich Re, Prudential, Swiss Re and ZFS).

The replication framework consists out of several building blocks – We will focus on the instrument selection

Replication portfolios are defined by means of minimizing deviations between NPVs of the Cash-flows of the target insurance product with the replication portfolio

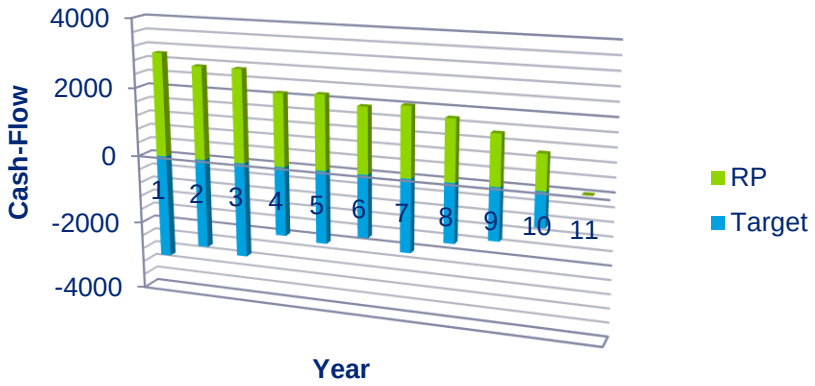
$$\min_{instrument\ weights} \sum_{s=1}^{S} (NPV_{RP}(s) - NPV_{TP}(s))^2$$

Scenario Methodology	**To generate the cash flows** of the insurance product and of the financial instruments used for replication scenarios are required encompassing IR, Eq and FX risk drivers.
Optimization methodology	**To find the replicating portfolio an optimization procedure is used** • Replication method: replication of cash flows, NPV, cash flow buckets • Optimization method: OLS, LAD, WLS, etc. • Optimization constraints
Quality Assessment	**to measure the replication quality,** quality measures are needed. • In-sample • Out-of-sample • Shocked scenarios
Instrument Selection	**To define a replicating portfolio** a universe of financial instruments needs to be defined as input in the optimization: • Vanilla instruments (ZCBs, European Options, etc.) • Path-dependent instruments (Barrier options, Lookbacks, Asian, etc.)

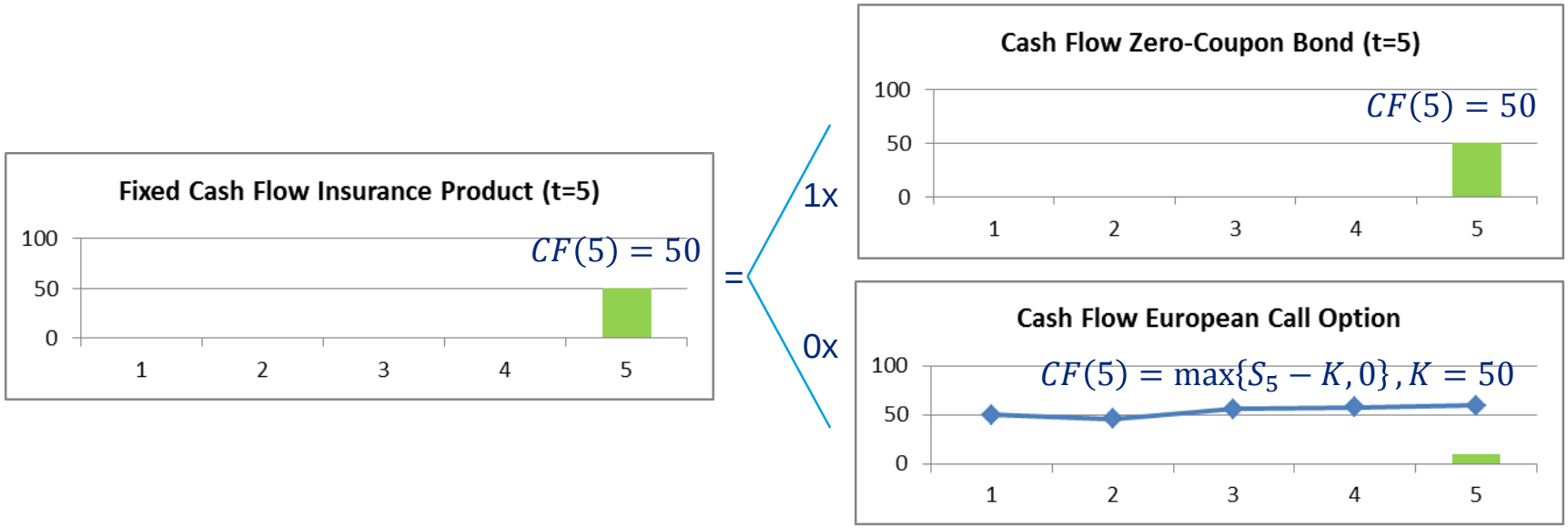
Candidate assets should satisfy a number of design principles

Some design principles

1. Should be able to capture risk features.
2. Materiality of risk features.
3. Replicating instruments should be easy to price (e.g. by means of closed-form formula).

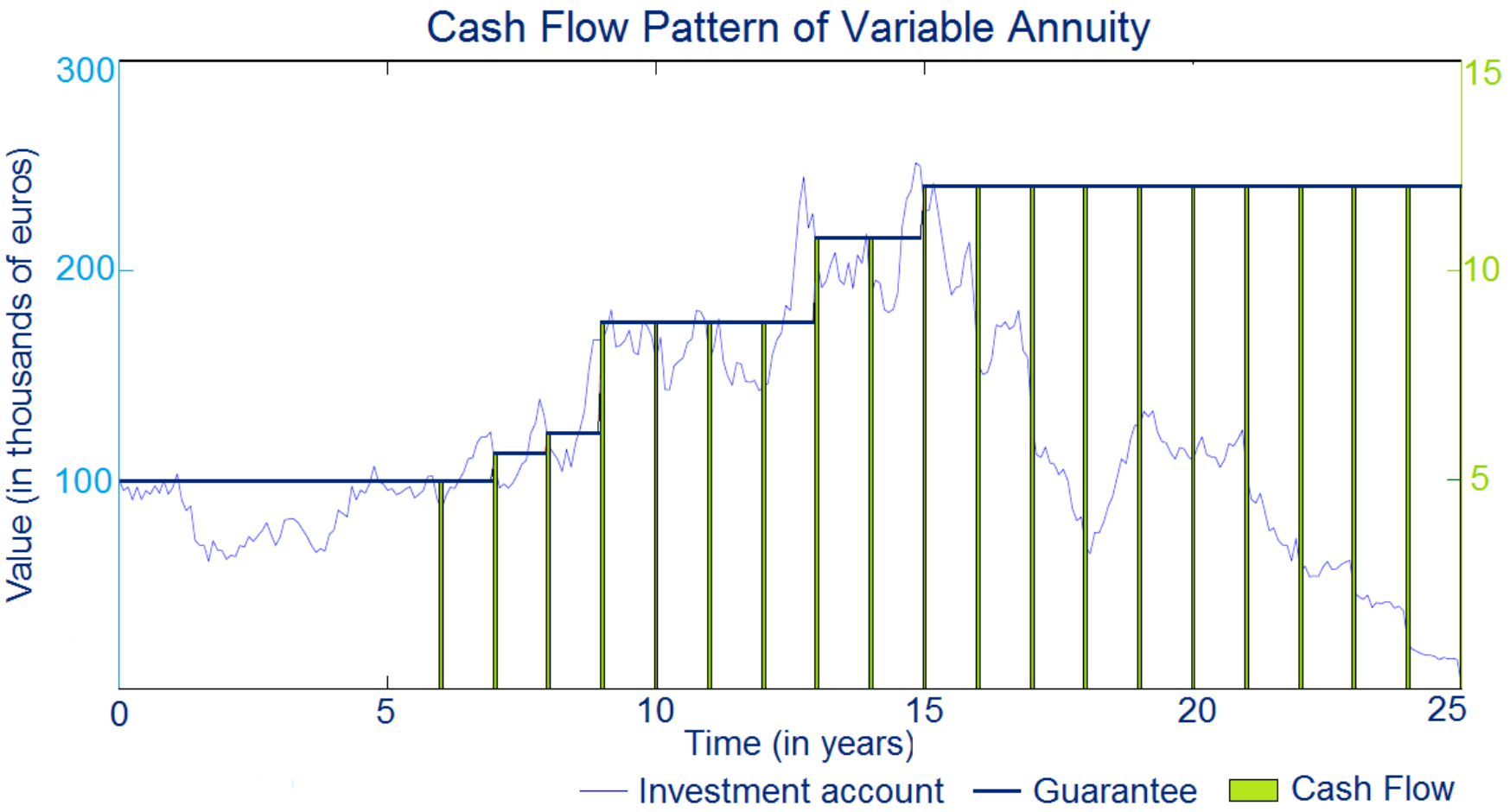


Example



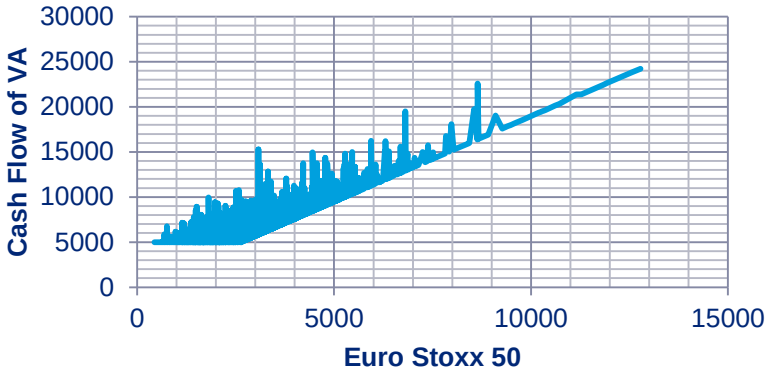
Option 1: using plain vanilla instruments

Understanding the underlying product – Pay-off structure
Pay-off in a specific scenario

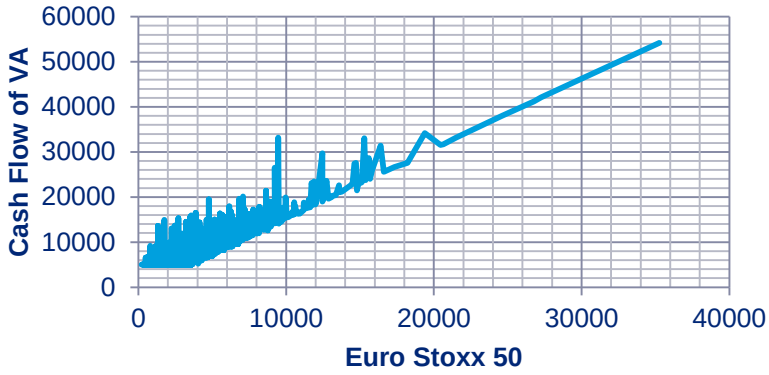


Understanding the underlying product – Pay-off structure

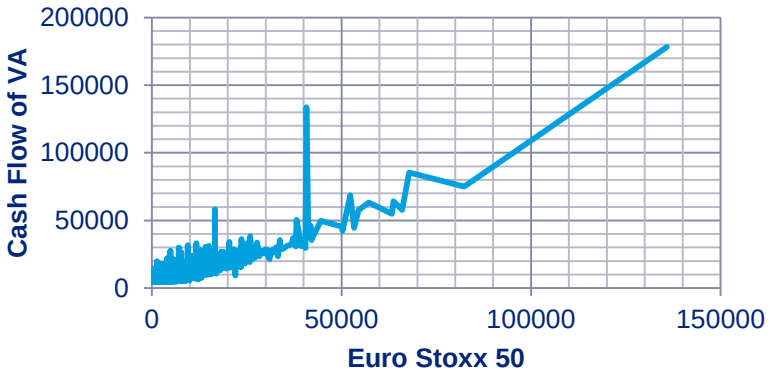
Year 6



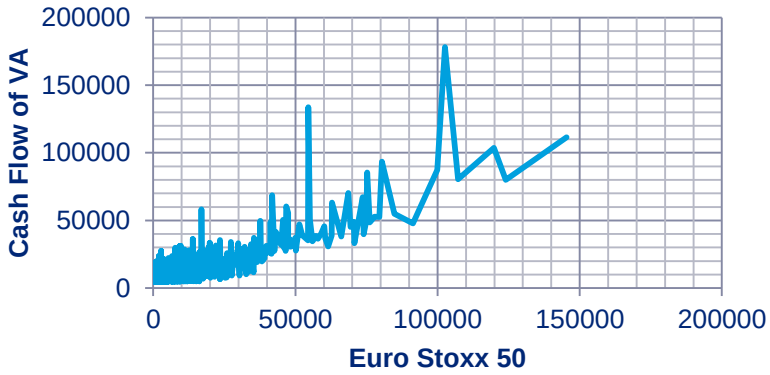
Year 10



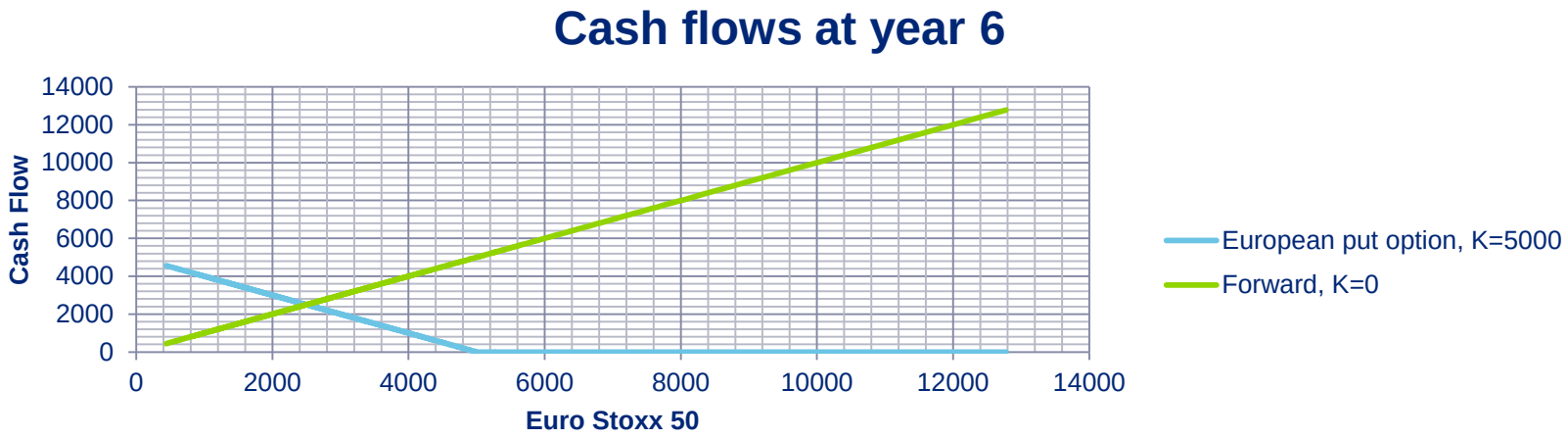
Year 20

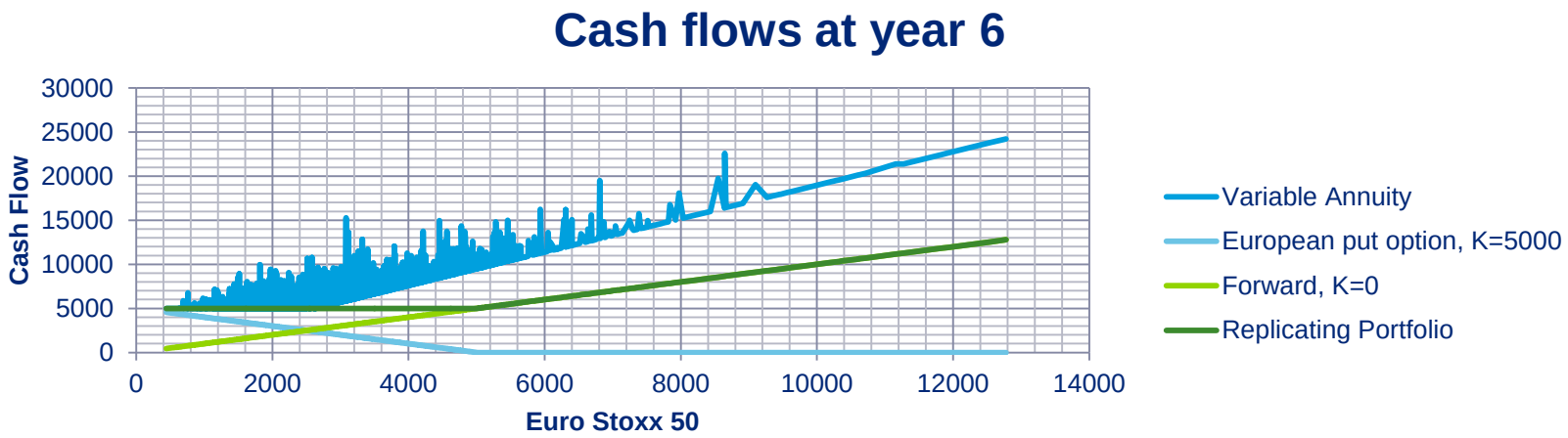


Year 25

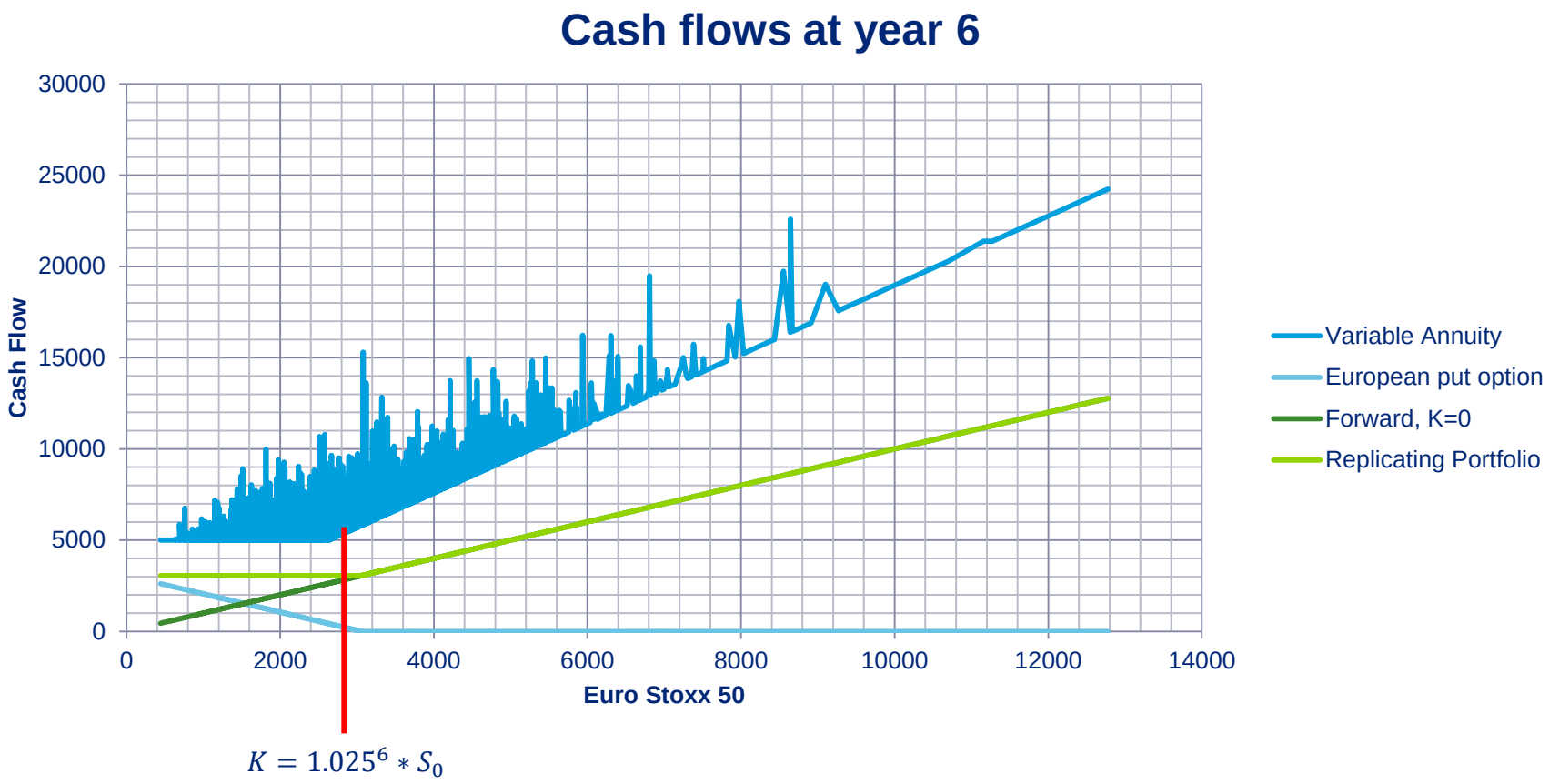


Pre-select the appropriate candidate asset classes, before setting up the replication problem, by analyzing the cash-flows...





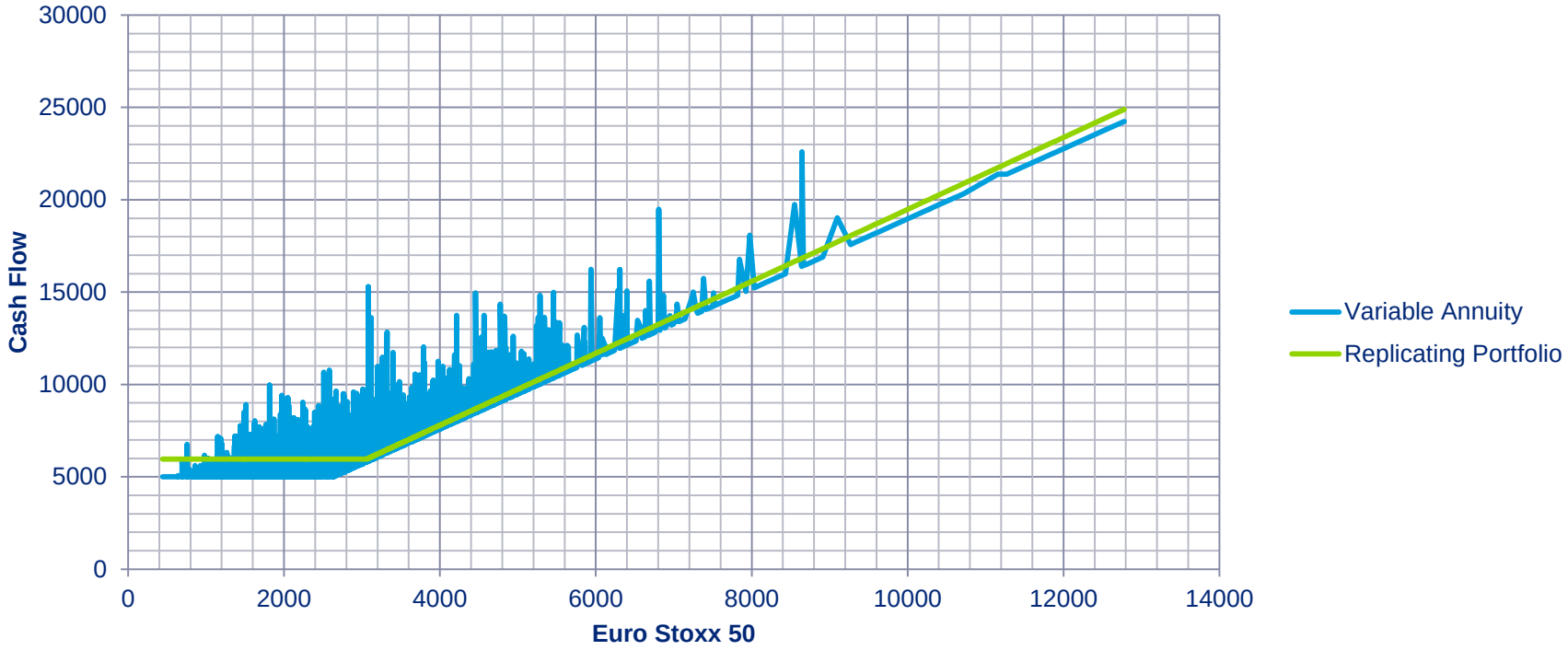
Part of pre-selecting the instrument set is defining the appropriate strike price for the put option ($K = 1.025^6 * S_0$)



Verify the candidate assets by minimizing the sum of squared errors between the cash flows of the product and the replicating portfolio for year 6

$$\frac{1}{S}\sum_{s=1}^{S}(CF_{RP} - CF_{VA})^2 = 1{,}132{,}631$$

Cash flows at year 6



Result - by means of a cash-flow analysis, we selected a likely candidate asset set consisting of plain vanilla instruments.

However, the cash flow is not perfectly replicated due to the difference in risk factors between the variable annuity and the path-independent instruments.

Option 2: using path-dependent options

Financial instrument selection – lookback in combination with forwards best resemble the profile of the GMWB

Payoff Variable Annuity product (simplified GMWB product)

$$CF(t) = \begin{cases} 5\% * IA_{max}(t) & t = 6, \dots, 25 \\ 0 & else \end{cases}$$

where $IA_{max}(t)$ is the maximum investment account value reached at time t using annual observation.

Pre-selected financial instruments

Forward contracts $(K = 0, T = 6, \dots, 25)$ $CF(T) = S_T$

Floating lookback put options $(T = 6, \dots, 25)$ $CF(T) = \max(S_{max}(T) - S_T, 0),$

where $S_{max}(T) = \max\{S_0, \dots, S_T\}$ is the maximum equity price reached at time T using annual observation.

- **Rationale - Combination forward + lookback put, both with maturity T:**

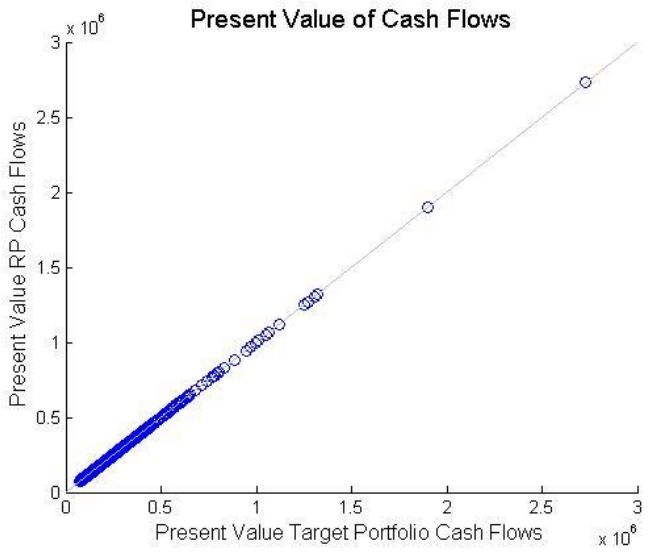
$$CF(T) = \max(S_{max}(T), S_T) = S_{max}(T)$$

Note: distinction between current selection and actual product resembles “withdrawal feature”

In case the payments are not deducted from underlying investment account (IA), using path-dep. instruments gives perfect in-sample replication quality

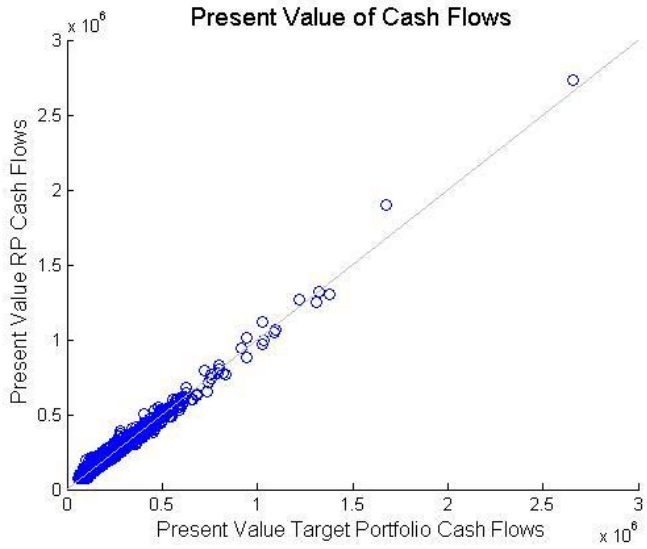
R^2	Measures the total variation in the target portfolio that is accounted for by the variation in the replicating portfolio.
MSE	The average sum of squared differences between the net present value of the target portfolio's cash flows and the replicating portfolio's cash flows.
MAD	The average sum of absolute differences between the net present value of the target portfolio's cash flows and the replicating portfolio's cash flows.

Path-dependent instruments:



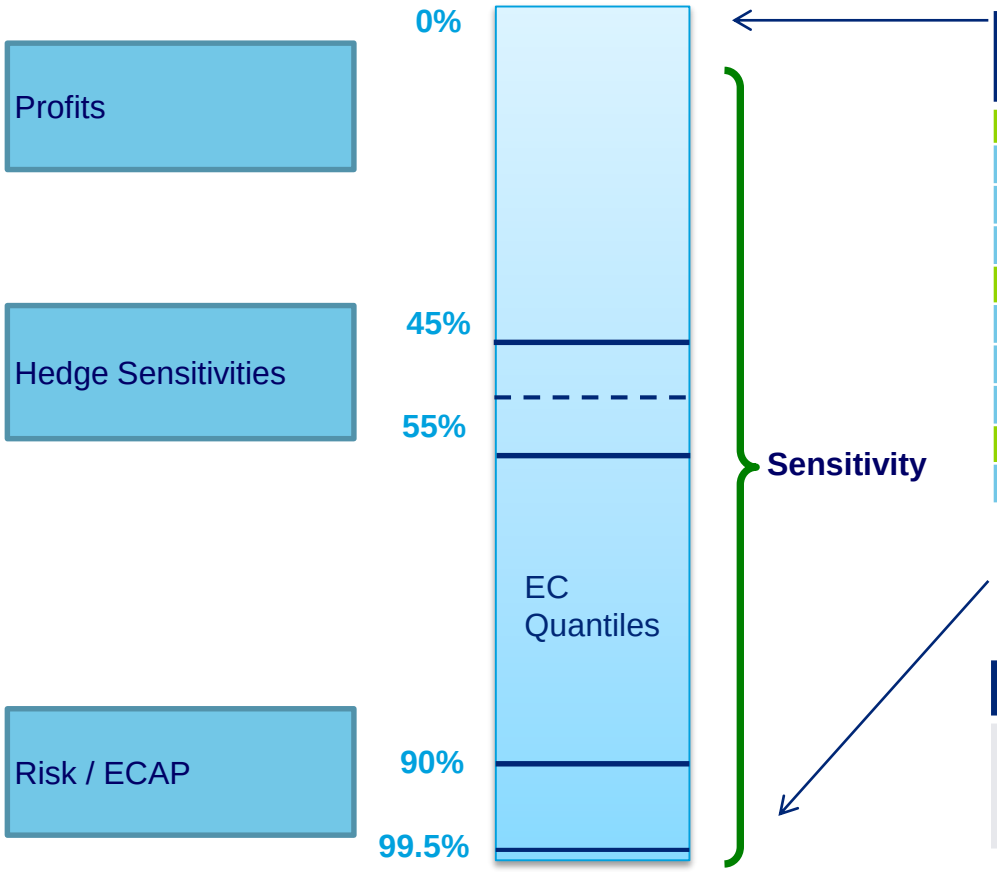
$R^2 = 1$
MSE $= 0$
MAD $= 0$

Plain vanilla instruments:



$R^2 = 0.98$
MSE $= 2.9 * 10^8$
MAD $= 11596$

Path-dependent instruments capture all sensitivities well. However, plain vanilla instruments, in particular do not capture the volatility sensitivity well



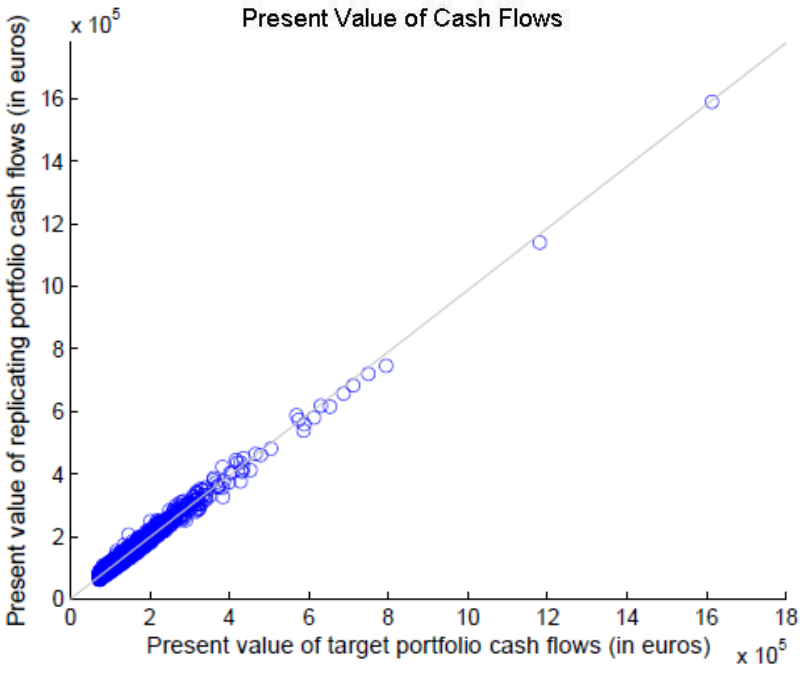
Quality measure	Path-dependent	Plain vanilla
Sensitivity Equity (relative difference)		
Eq 0.1 – quantile	0%	-2.2%
Eq 0.5 – quantile	0%	-1.0%
Eq 0.9 – quantile	0%	1.6%
Sensitivity Interest rate (relative difference)		
IR 0.1 – quantile	0%	2.4%
IR 0.5 – quantile	0%	0.8%
IR 0.9 – quantile	0%	-0.8%
Sensitivity volatility (relative difference)		
double vol	0%	22.1%

Relative difference

The difference between the expected value of the target portfolio and replication portfolio relative to the expected target portfolio value.

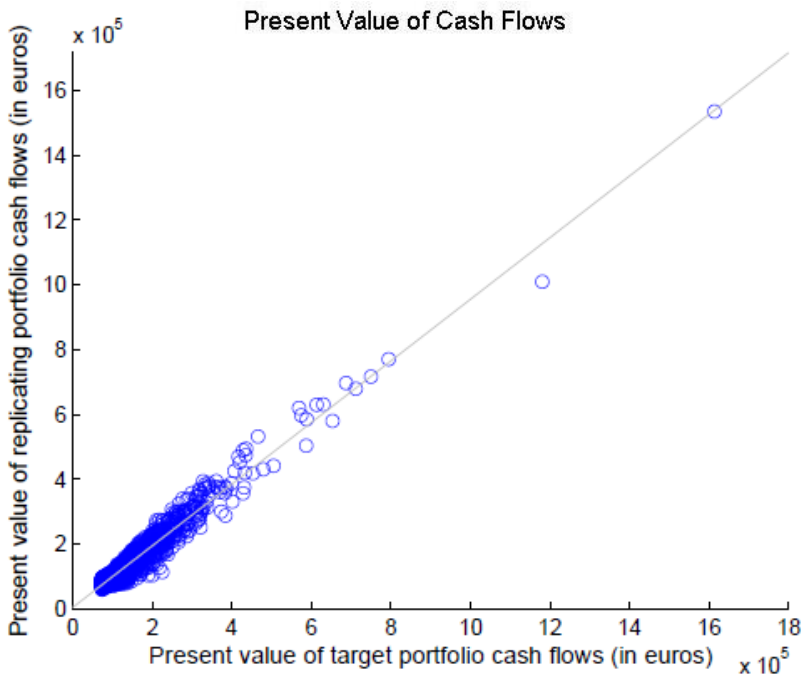
In case payments deducted from IA, we do not find a perfect replication, but path-dependent instruments still give better in-sample replication quality

Path-dependent instruments:



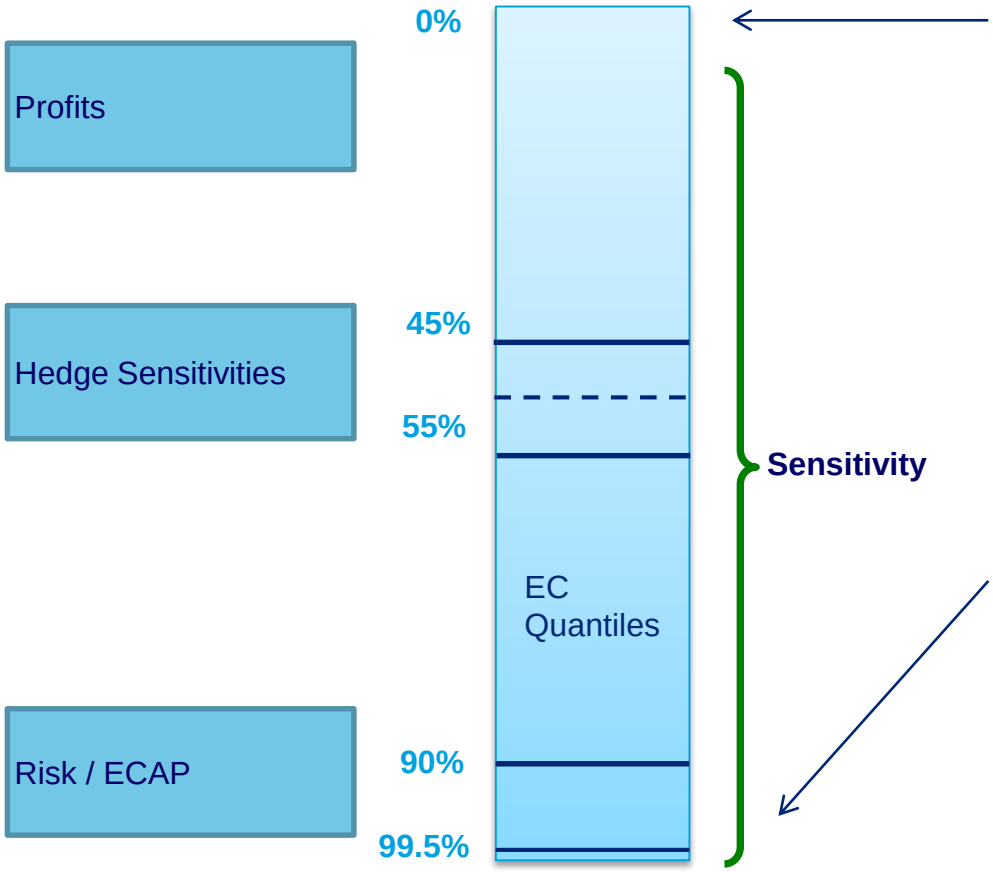
$R^2 = 0.99$
MSE $= 7.0 * 10^7$
MAD $= 6050$

Plain vanilla instruments:



$R^2 = 0.95$
MSE $= 2.8 * 10^8$
MAD $= 11468.1$

In case payments deducted from IA, we do not find a perfect replication, but path-dependent instruments still better capture sensitivities



Quality measure	Path-dependent	Plain vanilla
Sensitivity Equity (relative difference)		
Eq 0.1 – quantile	-0.1%	-3.6%
Eq 0.5 – quantile	0.0%	-1.5%
Eq 0.9 – quantile	0.1%	2.7%
Sensitivity Interest rate (relative difference)		
IR 0.1 – quantile	2.8%	2.2%
IR 0.5 – quantile	1.1%	0.7%
IR 0.9 – quantile	-0.5%	-1.0%
Sensitivity volatility (relative difference)		
double vol	-6.5%	18.1%

Discussion

Some advantages and disadvantages of using path-dependent or plain vanilla instruments for replication of Variable Annuities

Plain vanilla instruments	Path-dependent instruments
In general closed-form formulas available	Closed-form formulas for some path-dependent instruments, but often in simpler models
Hard to capture sensitivities to all risk factors	Better able to replicate variable annuity products
Instrument selection is not intuitive	Intuitive instrument selection

Discussion – What is your opinion?

Statement:

Replication of variable annuities using path-dependent instruments is a better alternative than using plain vanilla instruments.

Thank you for joining this session!

Who are we?

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