VARIABLE ANNUITIES

OBSERVATIONS ON VALUATION AND RISK MANAGEMENT

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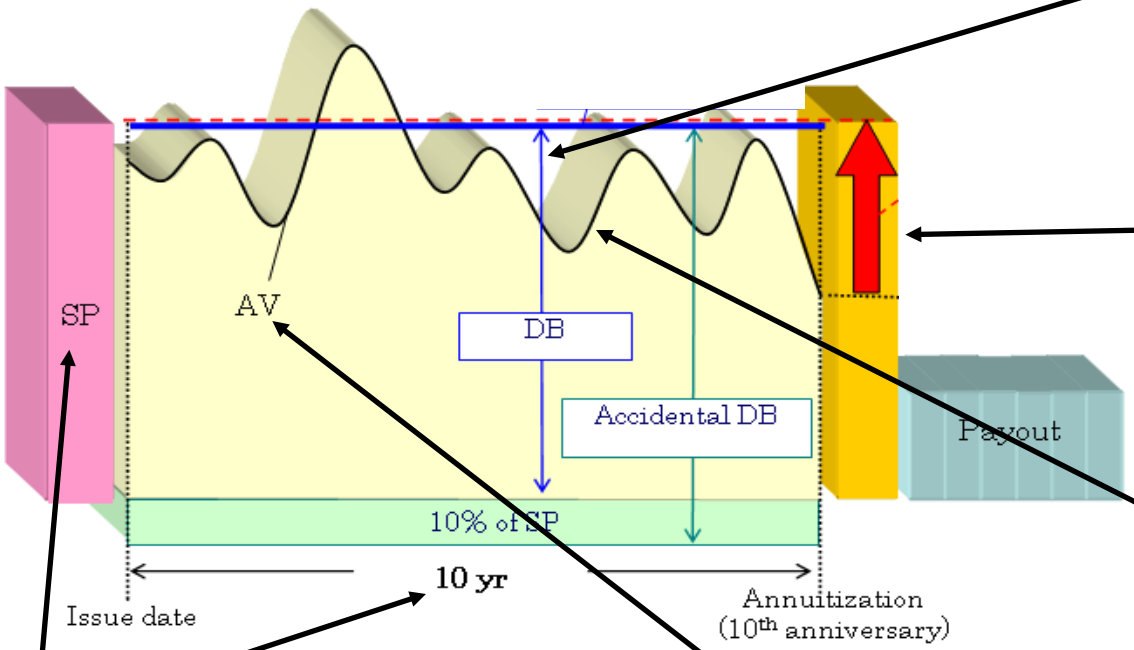
NOVEMBER 7TH 2013

WHAT IS A VARIABLE ANNUITY (VA)



Insurer offers death benefit (MGDB)
→ **Risk for Insurer**

Insurer guarantees to pay at least the single premium back to the customer at the end of the policy's holding period. Payout of account value (AV) if higher than single premium, customer chooses from lump sum or annuity
Survival benefit (MGSB)
→ **Risk for Insurer**

During the contract the customer pays an %-fee for the guarantee and contract related costs (admin, distribution). The customer can lapse the contract at any time and withdraw the account value (American option)

Customer chooses amount to be invested, single premium (SP) & holding period

Insurer invests on behalf of the customers (through investment companies) their assets, single premium → funds → separate account → account value (AV)

THE CASE FOR SAVINGS PRODUCTS WITH GUARANTEE

Why savings products?

- Already now, employees globally are facing a situation where their individual pensions are underfunded because of interrupted employment history or insufficiently funded (Defined Contribution…) pensions
- 1st (Government) and 2nd (Company) pillar pensions will be severely downsized in the aftermath of the Credit and Euro debt crises
- Wage (and hence pension) increases will be replaced by profit sharing arrangements even in traditional socialist North-Western European countries (profit sharing by Volkswagen is a nice example…)

Why guarantees?

- The wealth accumulation phase of a pension should satisfy certain minimum needs of income, however with the current low interest rate there's a need for more upside potential -> *wealth preservation*
- Because of the nature of markets, by not providing guarantees people could (but should not!) risk everything until close to maturity / wealth payout phase (this is mitigated by the use of life cycle funds)

THE CASE … (PART 2)

Legislative landscape: consumer protection

- Several jurisdictions have had cases where disappointing investment returns led to legal charges. Court rulings in favor of plaintiffs in more cases than not.
- Including explicit guarantees in the product can mitigate the reputation and legal risks in investment products. In addition by including an explicit percentage charge the total costs are very transparent.

…for longer dated guarantees

- Short term rates are decreasing
  - Expectations for short term rates to stay lower for longer
  - In a challenging macro environment additional return can come from longer maturity investments.
- Significant component of price of capital protection is the interest rate over the protection horizon.

THE CASE … (PART 3)

In the context of overall investment portfolio of individual

- These products are targeted at wealth accumulation phase and potentially withdrawal phase of retirement

- In addition to this wealth accumulation target a household or individual may have alternative savings targets
  - Mostly these are shorter term
  - Mostly these are of smaller relative size
  - First and second pillar pension schemes both do not have full downside protection (“soft guarantee”)

- Given the importance of retirement savings and little opportunity for diversification or subsidizing between savings goals (either between goals, like a car, house or retirement, or inter-temporal) the need for risk control mechanisms and downside protection (“hard guarantee”) for a wealth accumulation product is obvious

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LIBOR IS THE RATE AT WHICH BANKS DON'T LEND TO EACH OTHER…

MARKET RISK PRICING

FORWARD INDEX LEVELS

MSCI World in JPY vs. 3m JPY LIBOR		
TRS Levels	Size $mm	
Maturity	100	1000
1	-41	-45
2	-56	-60
5	-81	-85

MSCI World in USD vs. 3m USD Libor		
TRS Levels	Size $mm	
Maturity	100	1000
1	-20	-24
2	-17	-21
5	-10	-14

Date: July 26th, 2013

DIFFERENCE WITH BLACK-SCHOLES ASSUMPTIONS

- **TRS (no dividend) but still forward price not determined by swap**
- **OIS? Not really**
  - 1Y LIBOR-OIS spread = -17.5bps
  - 2Y spread = -16.5bps
  - 5Y spread = -44.0bps
- **No free borrowing and lending of stocks! 1Y borrowing cost 23.5bps**
- **Financing cost significant impact on forward levels**
- **But why different in JPY vs USD?**

FORWARD INDEX LEVELS

Wrong

$$F_{index}(t,T) = S(t) * \exp([r-d] * [T-t])$$

Better

$$F_{index}(t,T) = S(t) * \exp([r-d+repo] * [T-t])$$

MOVING FORWARD WITH ARBITRAGE-FREE FX

MARKET RISK PRICING

FORWARD FX PRICES & FX BASIS

USDJPY FX Basis = -21bps

FX Basis explained…

- **Difference over LIBOR rate differentials in FX swap**
- **Deviates from zero since GFC**
- **Funding driven**
- **Has significant impact on forward FX levels**
- **Impacts pricing of guarantees on foreign assets!**

FORWARD FX RATES

Wrong

$$F_{ccy}(t,T) = FX(t) * \exp([r_d - r_f] * [T-t])$$

Better

$$F_{ccy}(t,T) = FX(t) * \exp([r_d - r_f + \text{basis}] * [T-t])$$

"REGRESS-LATER" ESTIMATORS FOR VA (*)

BEHAVIORAL RISK

(*) Joint work with Willem van Ruitenburg,
kudos to Jeanine Kwong

I SUPPOSE *TOPQUANTS* CAN DEAL WITH 2 FORMULAS BEFORE WALKING AWAY…

Lapse model (note that step-function is dense in space of 1D functions):

$$l(t) = b_0(t) + \sum_{j=1}^{J} b_j(t) \cdot \mathrm{I}_{\left[AV^{FWD-T}(t) > B_j \cdot (1-f)^{[T-t]}\right]}$$

Risk-neutral valuation of MGDB / MGSB:

$$G(t, K, x(t), T) = D(t,T) E_t^T \left(IF\left(T, AV^{FWD-T}(s); t < s < T\right) \left[K - AV^{FWD-T}(T)\right]^+ \right)$$

OK, 1 MORE SINCE YOU ASKED

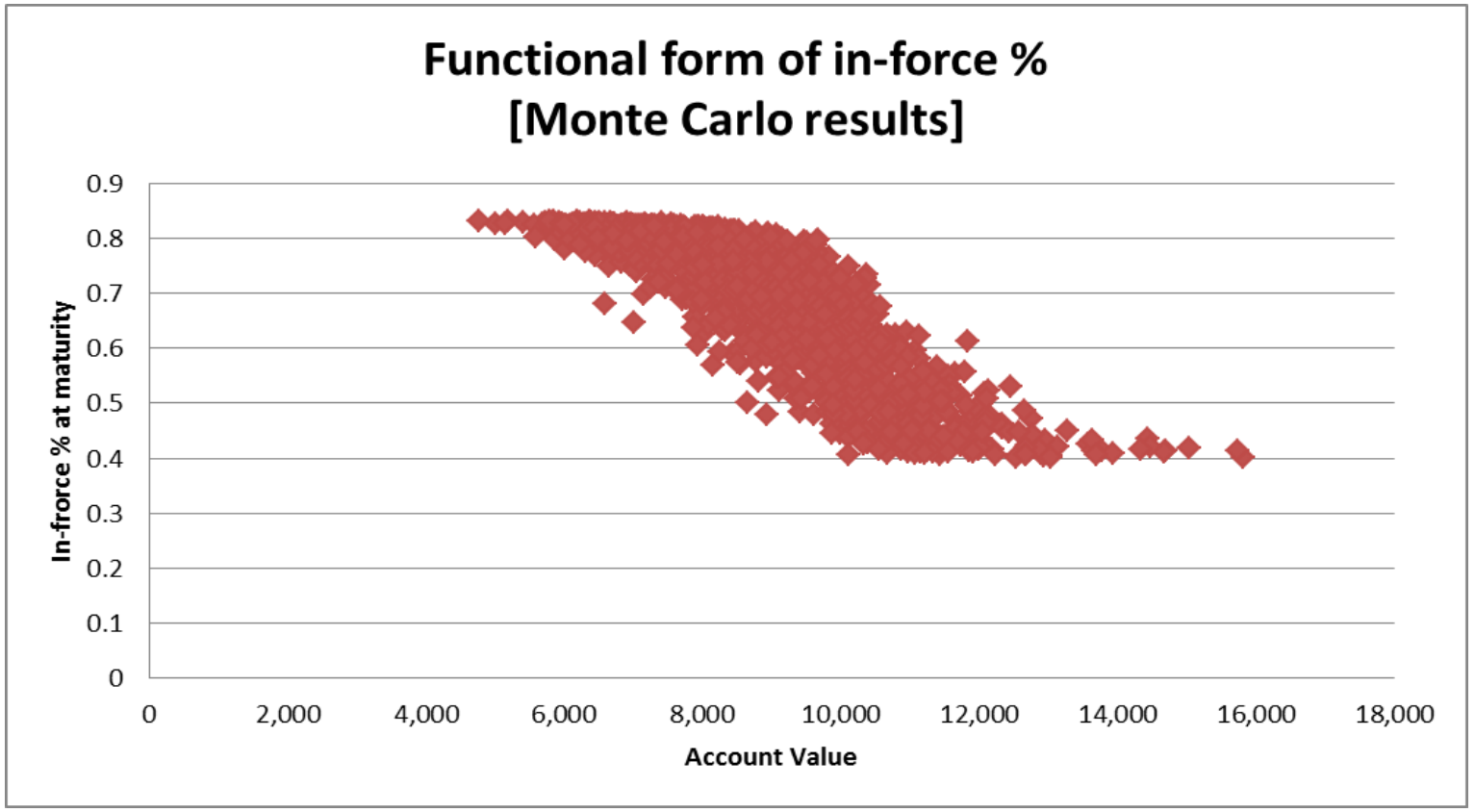
We approximate the in-force function by a piecewise continuous function:

$$IF\left(T, AV^{FWD-T}(s); t < s < T\right) = \delta_0 + \sum_{l,m} \delta_l^m \left[AV^{FWD-T}\left(t_l^{**}\right) - \gamma_m\right]^+$$

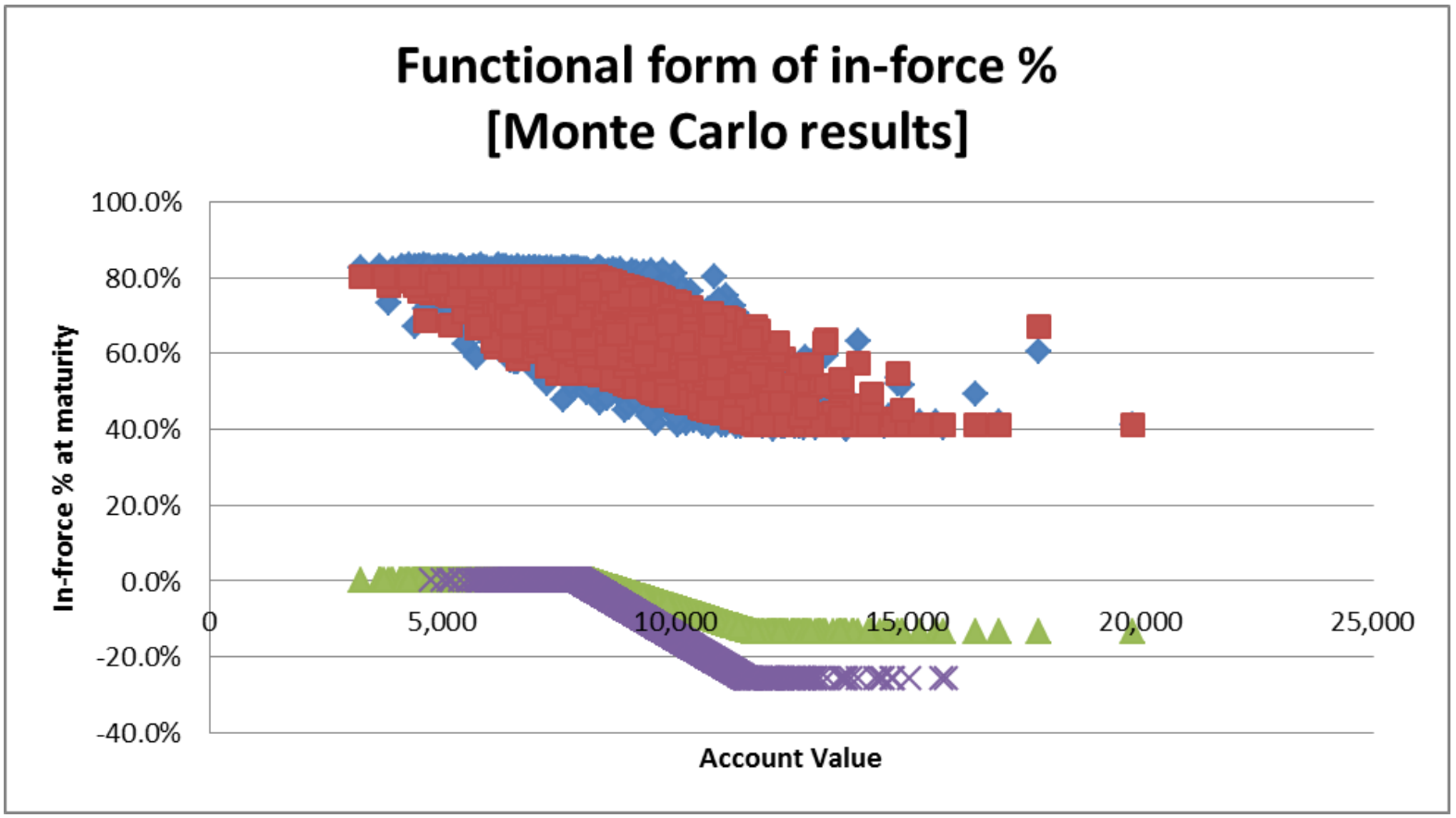
Simplified version that is just dependent on AV @ maturity:

$$\delta_0 + \delta_1 \left[AV^{FWD-T}(T) - \gamma\right]^+$$

DOES THIS WORK?



R^2 = .86


Functional form of in-force %
[Monte Carlo results]
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
-20.0%
-40.0%
In-frorce % at maturity
0
5,000
10,000
15,000
20,000
25,000
Account Value

ALTERNATIVE INTERPRETATIONS OF ESTIMATOR

BEHAVIORAL RISK

A COACH LOVES MULTI-PURPOSE PLAYERS…

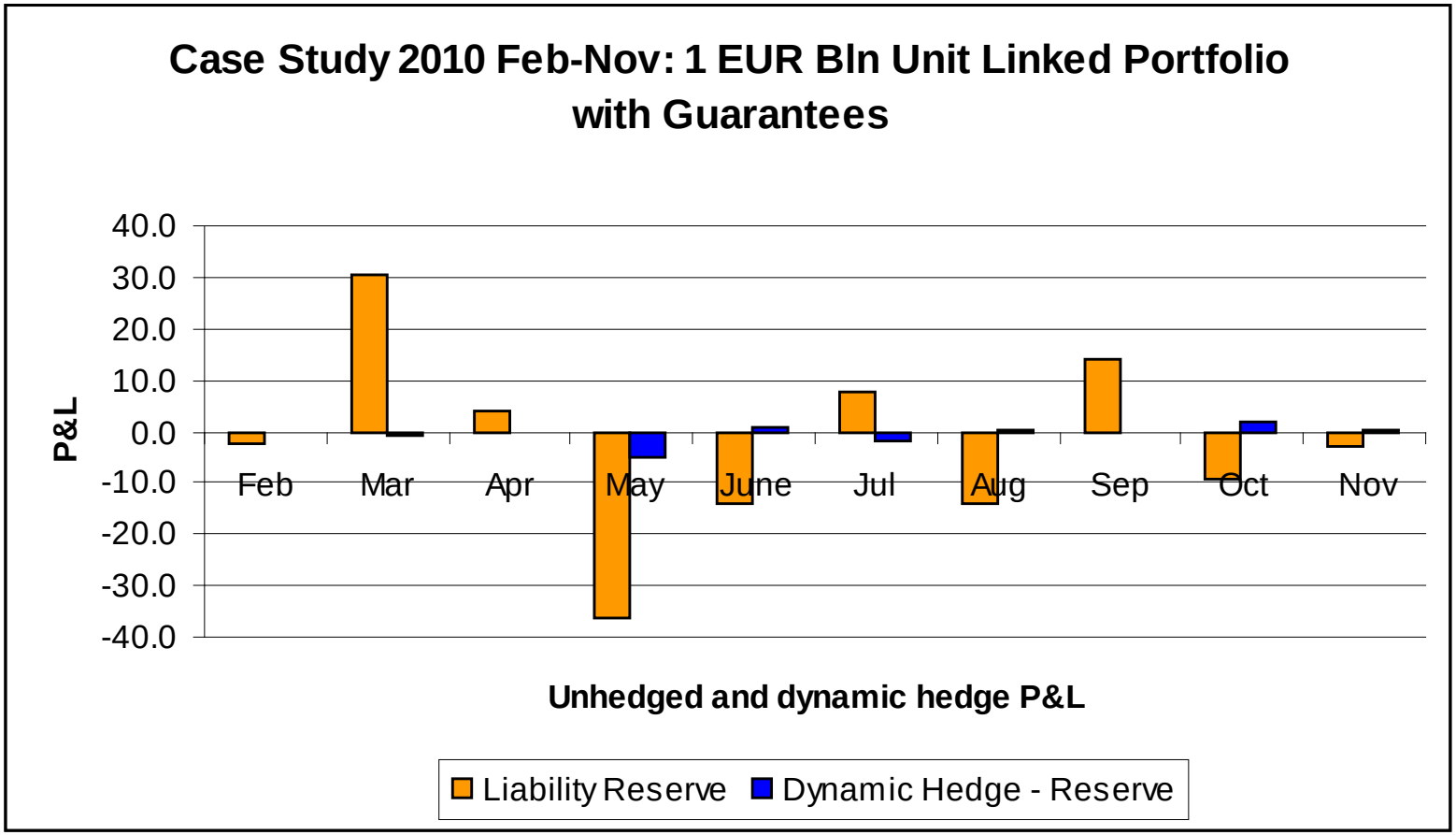
Our estimator is also:

1. **A building block for speeding up Monte Carlo pricing and Greek calculations by serving as a control variate for the actual model.**
2. **Direct intuition into the impact of lapse on the value and remove Black Box nature of the "exact" valuation approach. Upper bound on notional of simple put hedge.**
3. **Structuring of financial derivatives that closely approximate the financial risk implied by the "exact" model.**

CASE STUDY STRUCTURED DERIVATIVES

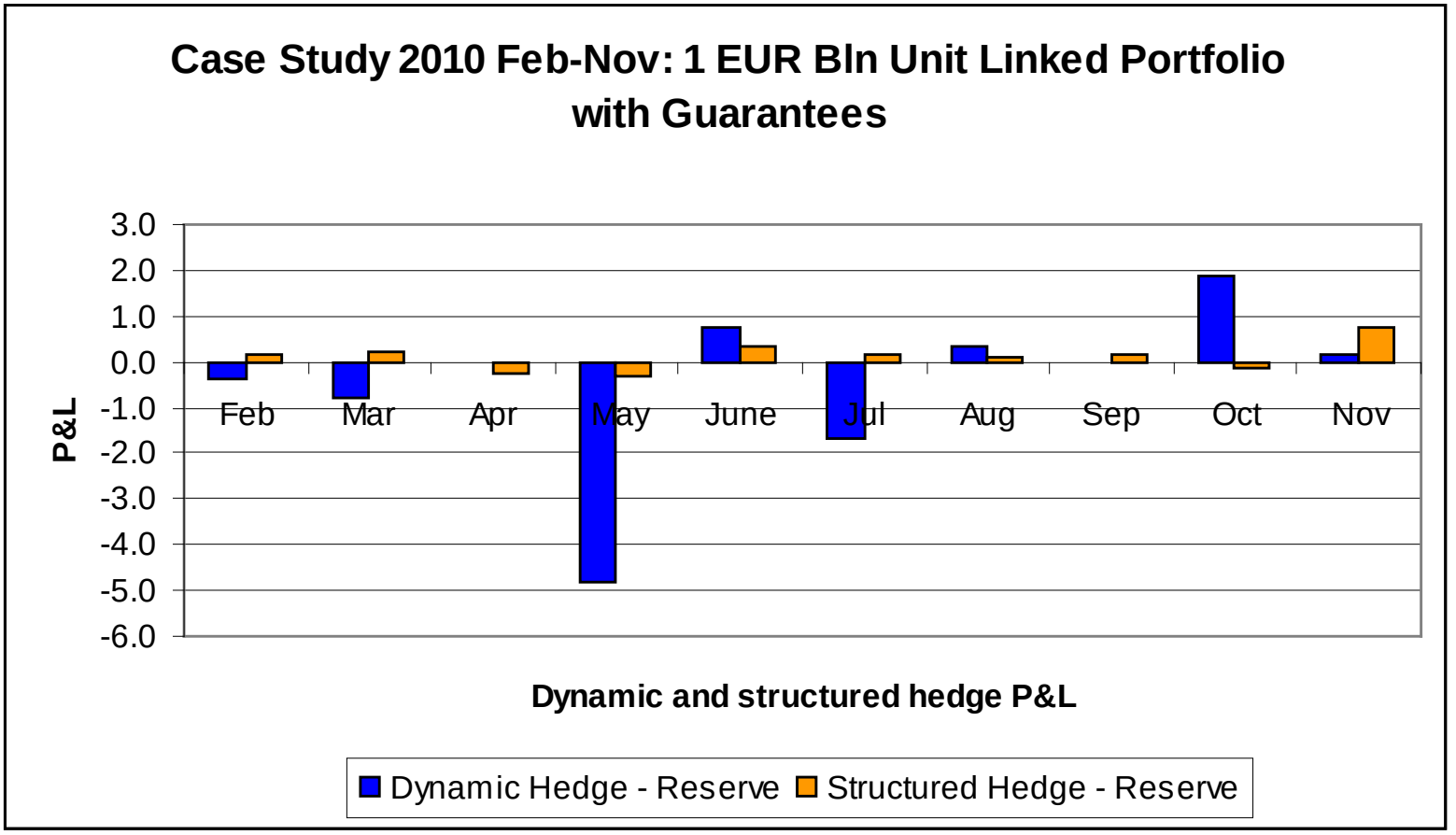
HEDGING

A DYNAMIC HEDGE PROGRAM CAN REDUCE RISK, BUT…



The orange bars represent change in the Fair Value of Guarantee Liabilities while the blue bars represent the P&L of the dynamically hedged portfolio (Assets minus Liabilities)

IT WILL LEAVE SIGNIFICANT P&L VOLATILITY BECAUSE OF UN-HEDGED MARKET RISKS



* Note that compared to previous slide the scale of the graph has changed

The orange bars represent the P&L of the *statically* hedged portfolio (Assets minus Liabilities) while the blue bars represent the P&L of the *dynamically* hedged portfolio