TopQuants Newsletter

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Editorial

Dear Reader,

The TopQuants team is pleased to present the second issue of our newsletter. The inaugural issue, published in March 2013 was very well received among the quant audience and had generated many positive feedbacks. The intention behind the newsletter is to stay in touch with the quant community in between our two events, namely the spring keynote and the autumn/winter workshops and, to provide an alternative forum for people to express themselves. TopQuants will continue to publish the newsletter semi-annually from now onwards and as with all our efforts, it is intended to serve the quantitative community in the Netherlands.

TopQuants have witnessed an increased number of submissions for the current newsletter issue and we hope the enthusiasm continues. We will strive to maintain the quality of the technical contents in the newsletter and ensure that it is relevant and beneficial to the quantitative community. As always, we are open to your suggestions, and are looking forward to receiving your comments and contributions. TopQuants is very open to the nature of submissions in our newsletter which may include technical articles, research results from masters/PhD work, personal blogs, surveys, opinions (e.g. on newly proposed regulations), reviews of books or articles, coverage of interesting events etc. If there is a topic you would like to bring to the attention of your fellow quants in this country, why not write an article about it for this newsletter? We encourage you to contact us to discuss how it may be done.

This issue of the newsletter starts with a brief coverage of the TopQuants spring event in May 2013, held at the SNS Reaal headquarters in Utrecht. The summary includes the interesting presentation by our key note speaker Coen Teulings (professor at the University of Amsterdam) who had expressed his views on the state of the Dutch economy and the possible ways going forward. Also find in the summary, a note on the lively panel discussion hosted by TopQuants committee member Bert-Jan Nauta, that included speakers, Sandra Muijs (Head of Model Development at *SNS REAAL*), Robert Daniels (Senior Client Risk Manager at Cardano), David Schrager (Head of Single Premium Variable Annuity Trading at ING bank) and Coen Teulings himself.

This issue also features five articles with each one being diverse from the others with regard to the technical content and background of the authors. In the first article, Danny Dieleman (Credit Risk manager at ING Bank N.V.) and Onno Steins (Advisor Prudential Regulation at the Dutch Banking Association) jointly discuss the consultation paper '*Revisions to the Basel Securitisation Framework*' that was issued by the Basel Committee on Banking Supervision in December 2012. The second article is by Marco van der Burgt (ING Bank, Market Risk Management, Model Validation) who presents a potential investment strategy that can be adopted by pension funds to prevent underfunding.

The next two articles present the internship work of Joris Chau and Toni Budimir done respectively at the Financial Services Risk department at Ernst & Young and the Group Risk Analytics, Quantitative Review team at Royal Bank of Scotland. The work of Joris Chau focuses on obtaining robust estimates of the operational risk regulatory capital charge (Value-at-Risk or VaR) via the Loss Distribution Approach while the work of Toni Budimir focussed on an alternate way to obtain haircuts for collateral securities via bond simulations as compared to the regulatory proposed haircuts.

The final article provides a brief introduction to the Econometrics Game event that is organized annually by the study association for Actuarial Science, Econometrics & Operational Research (VSAE) at the University of Amsterdam. The event has participants (econometrics masters/PhD students) from prestigious international universities covering all over the world. The article includes in particular, the case study presented by the 2013 Econometrics Game winning team from Universidad Carlos III de Madrid, Spain, which had Guillermo Carlomagno, Andrés García-Suaza, Salvatore Lobello, Michelle Sánchez, Pedro Sant' Anna as its members. Their work was about forecasting the GDP growth of Spain in a data-rich environment.

We hope you will enjoy reading the broad range of topics offered in this newsletter and we look forward to seeing you at the upcoming TopQuants event(s).

Aneesh Venkatraman
(on behalf of TopQuants)

Macro-economic forecasting in times of crisis

— Presentation by Coen Teulings followed by lively panel discussion and comments from the audience at SNS headquarters in Utrecht

For the first time since the founding of TopQuants in 2011, the networking organization has conducted an event outside the country's capital. The 2013 TopQuants event which focussed on "Macro-economic forecasting in times of crisis" took place on 22 May 2013 in Utrecht at the SNS Reaal headquarters and had a participation of more than 120 people.

The event was officially opened by TopQuants committee member Marieke van der Klip. This was followed by a warm welcome speech by Pim Poppe (Head Group Risk Management at SNS), who represented SNS Reaal, the host company and event sponsor. Pim said that SNS were happy to make this event possible. He told the audience that the recently nationalized bank and insurer had seen an inflow of several new quants and has plans to continue hiring over the months to come. Pim emphasized that technical skills were obviously required to work in quantitative finance, but encouraged all quants to also engage in interacting with different businesses and understand operational processes. The goal is not only to develop models and apply there but also explain their models well to all relevant stakeholders. He expressed his strong belief that communication skills often made the difference, as good quants facilitate the translation of model outputs into business decisions.

The friendly welcome was followed by the main presentation of the event, "Macro-economic forecasting in times of crisis" by Coen Teulings (professor at the University of Amsterdam). Until very recently, Coen had been director of the Centraal Planbureau (CPB), the Dutch Bureau for Economic Policy Analysis. Having started his academic career in labour economics, Coen has extensively worked on macro-economic topics over recent years. Not only among fellow economists but also to the general public in the Netherlands, Coen is well-known for his outspoken criticisms of the post-2008 austerity measures. However, Coen's presentation at the TopQuants event covered besides the optimal timing of austerity also a range of other topics, including the pros and cons of different pension schemes in the Netherlands, the state of the Dutch housing market, the role of Bayesian methods in macro-economic forecasting etc.

How can the Dutch economy get out of its current slump? Are the current budget cuts going too far or do we need even more austerity measures? Will the Dutch housing market recover? How reliable are econometric forecasts? How can the models underlying these forecasts be improved? While one might think that each of these questions by itself was complex enough, Coen Teulings managed to address all of them in his talk.

Coen began the presentation by providing a comparison between the Dutch and German economies over last few years and emphasized that, since the aftermath of the 2008-2009 crisis period, the difference in the economic growth rates of Germany and the Netherlands has increased to 6%, with Germany being better off. Coen explained this by differences in private consumption and investment in the real estate industry in the Netherlands. Albeit himself not a quant, Coen clearly enjoyed explaining his analysis of the state of the Dutch economy and his opinion on the policy measures it requires, to the quant audience at this key note event. A terminology very akin to that spoken in banking, he described the Dutch balance sheet as "long pension claims – long real estate – short mortgages". With a wink, he said that in this respect the Netherlands looked like "one big hedge fund". He stated that reductions in mortgage deductibility should go hand in hand with budgetary expansion and structural reforms of the Dutch housing market, arguing that a higher share of commercial rental housing would make the Dutch economy much more flexible. Coen also expressed his opinion that – in the light of financial market volatility – it was time to reassess the pros and cons of pay as you go and funded pension schemes.

This combination of topics from economic research, econometric forecasting and quantitative finance dished up by Coen clearly matched very well the taste of the audience. It triggered questions by several of the over 100 attendees already during the presentation and was followed by an equally lively panel discussion.

Coen's talk was later followed by a panel discussion which was facilitated by TopQuant committee member Bert-Jan Nauta and the participants included Sandra Muijs (Head of Model Development at SNS Bank), Robert Daniels (Senior Client Risk Manager at Cardano), David Schrager (Head of Single Premium Variable Annuity Trading at ING bank) and Coen Teulings himself.

Bert-Jan Nauta confronted the panel members with a number of statements and asked them to comment. The first topic of debate was on the IMF growth forecast of 1.4% for advanced economies in 2008 and the actual outcome. All panel members agreed that econometric forecast models failed abominably in the crisis year 2008. Coen Teulings added that while he supports macro models, standard econometric forecast will not be able to capture crisis and multi-country data needs to be included. Sandra Muijs pointed out that the credit crunch had made decision-makers in finance more aware of model risk, which had made it easier for quants to discuss model risk uncertainty with senior management. Robert Daniels argued that most professionals in finance had lacked the imagination to include a scenario similar to the credit crunch in their simulations prior to 2008, and if they did include it, they had grossly underestimated its probability and impact. He recommended that we "shift the discussion from how likely something is to what we would do if something happened".

The discussion then turned to austerity and if it would help save the day. Coen, said that the austerity measures were going too far, arguing that "politicians nowadays want to be on the safe side – that is part of the problem". Some of the other panel members did not hide their disagreement with the key note speaker. Robert opined that austerity timing is late if it is done after the crisis and emphasized that it has to be pre-emptive. He also questioned critically how the mounting government debt could ever be paid back. Prompted by the case of Japan being cited as a bad example, David Schrager remarked provocatively: "I have lived in Japan for a few years. If that's a country in crisis, then give me 20 years of that!"

Similar to other TopQuants events, the panel discussion was followed by complimentary drinks and snacks during the informal networking part.

TopQuants are grateful to SNS Reaal for sponsoring and hosting the event. In particular, we would like to thank Pim Poppe for his kind words of welcome. Special thanks go to the key note speaker, all panel members as well as to the quant audience for making this another successful TopQuants event.

- Tim Mexner and Aneesh Venkatraman

Disclaimer

Policy development: Risk weighting of securitisation positions -an update of the Revisions to the Basel Securitisation Framework

- by Danny Dieleman (Credit Risk manager at ING Bank N.V.) and Onno Steins (Advisor Prudential Regulation at the Dutch Banking Association)

Introduction

This paper discusses the consultation paper '*Revisions to the Basel Securitisation Framework*' that was issued by the Basel Committee on Banking Supervision last December. As a result of the Financial crisis the Basel Committee has the intention to revise the securitisation framework. In certain areas, the proposals result in risk weights that are not commensurate to the underlying risks. Coordinated by the Dutch Banking Association (NVB), the Dutch banks, amongst which ING, jointly provided comments and suggestions to this consultation (A copy of this reaction is available via the website of the Basel Committee via http://www.bis.org/publ/bcbs236/duba.pdf). The consultation paper describes the changes proposed by the Basel Committee. It also provides a simple example to illustrate the workings of the proposed regulation.

What happened since the crisis?

The use of securitisation techniques in the US subprime mortgage market was one of the contributors to the creation and transmission of the global financial crisis. In response to the regulatory shortcomings that were uncovered, the Basel Committee quickly implemented a number of policy changes to address the immediate concerns over securitisations. For example, the regulatory risk weights for re-securitisations were increased, the interests of investors and issuers were better aligned by laying down strict and clear rules around the minimum exposure that issuers need to hold, (the "skin in the game"), and the operational requirements for the credit analysis by banks were enhanced. These changes have become known as Basel 2,5. After the publication of these revisions, the Basel Committee conducted a more fundamental review of the securitisation framework. The proposed changes to the regulatory framework that resulted from this review were published for consultation in December 2012. The paper contains three major areas of change. First, the Basel Committee looks to reduce the reliance on external ratings for establishing the regulatory risk weights. Secondly, updates of the risk weights were proposed. Under the proposed framework, senior tranches would receive significantly higher risk weights, compared to current levels. For mezzanine and equity tranches, the risk weights will decrease. And, thirdly, changes to the hierarchy of the calculation methods to establish the risk weights were proposed.

What are securitisations?

Although the public perception might not distinguish between the various types of securitisations, the asset class is actually very diverse. In securitisation, the entity that originates loans sells those loans on to a special purpose vehicle. In order to finance these assets, the SPV issues bonds. These bonds offer various spreads, depending on the risk associated with the particular bond. Popular assets used for securitisation include residential mortgages, credit card receivables, car loans, commercial real estate mortgages and loans to Small and Medium Enterprises. The diversity of securitised assets, both in terms of nominal values and credit quality is vast. This diversity is further expanded by the structuring techniques used. In Europe, securitisation and covered bonds are important funding tools for banks. The geographical differences in popularity of the instruments used, however, are significant. Germany and Denmark for instance are primarily oriented towards covered bonds, where The Netherlands and the UK tend to use securitisation.

Challenge for regulators:

There is no point in denying the shortcomings of regulation pertaining to securitisations that surfaced during the financial crisis. With the benefit of hindsight one has to agree that, especially for senior tranches of US subprime residential mortgage backed securities, the ratings were too positive, which resulted in ill judged investment decisions and dangerously low levels of capital being allocated to those portfolios. The losses that have materialised in the US subprime market since then exceed the amount of capital that was set aside. On the other hand, the risk weight of 1250% for senior tranches that were downgraded appeared to be too high, as this risk weight implies that all assets in the underlying pool would default and losses would be 100% of the amount outstanding. Such a scenario is next to impossible. All in all, some sort of recalibration of the risk weights had to be done.

The challenge faced by regulators lay in combining the recalibration of risk weights for certain types of securitisation, but having to apply those changes across the board to all types of securitisation. From a modelling perspective, the ideal way to calculate the regulatory capital requirement would be based on data that allows as much granularity as possible. This means that the different types of underlying assets, the different regions and the structural features of the transaction need to be taken into account. However, such an approach will be extremely complex, both in terms of formulating the rules, and in terms of the execution of prudential supervision. Let alone, the calibration of this approach would be very cumbersome, as this requires a vast amount of granular data. The Basel Committee was therefore faced with a trade-off between simplicity of the framework on one hand and the applicability of the calculated risk weights to the underlying exposures on the other hand.

# months in arrears	# borrowers	Notional Amount	Average PD	Average LGD (*)	Average Risk Weight	RWA
0	5.600	1.120.000.000	0,5%	12,50%	10,33%	115.683.490
1	50	10.000.000	20,0%	12,50%	74,53%	7.452.943
2	50	10.000.000	50,0%	12,50%	66,73%	6.672.755
>=3	50	10.000.000	100,0%	12,50%	78,13%	7.812.500
Total	**5.750**	**1.150.000.000**	**1,97%**	**12,50%**	**11,97%**	**137.621.688**

Figure 1: Risk characteristics of the mortgage portfolio

Approaches formulated by the Basel Committee

The Basel Committee developed a number of different approaches for the new securitisation framework. These are:

1. Revised Ratings Based Approach (RRBA)
2. Modified Supervisory Formula Approach (MSFA)
3. Simplified Supervisory Formula Approach (SSFA)
4. Backstop concentration ratio approach (BCRA)

Revised Ratings Based Approach:
The RRBA is an approach that is based on lookup tables. The risk weights depend on the seniority of the tranche, its rating and its expected maturity. The parameters can be looked up and fed into a formula, which produces the capital requirements. This approach is basic and conservative. It does not allow for internal modelling of risks.

Modified Supervisory Formula Approach:
The MSFA is a modification of the current Supervisory Formula Approach. The MSFA is based on banks' internally calculated IRB capital requirements for the underlying pool. The required inputs are:

- Loan-by-loan IRB estimates of the underlying assets.
- Maturity of tranche (M)
- Attachment point of tranche (A).
- Detachment point of tranche (D).

This approach is the most intensive one available. It is generally not available to the investing banks, as those banks will normally not be able to satisfy the IRB requirements for the underlying portfolios they invest in.

Simplified Supervisory Formula Approach:
The SSFA is based on the capital requirements under the standardised approach. Risk weights are assigned based on the subordination level of the tranche.

Backstop Concentration Ratio Approach:
The BCRA is a fall back approach, before banks are required to use the 1250% risk weight. The approach is based on the Standardised Approach capital requirement, which is dependent on the detachment point of the tranche. Junior exposures receive a higher capital requirement than senior exposures. The BCRA is designed to be conservative.

Calibration

In recent years, society and policy makers have changed their stance towards risk modelling. Where in the past complex models were desirable - and even cool - these days simplicity is the preferred way to go. Also, supervisory requirements are now engineered to be more conservative. This change in perspective is reflected in the proposals for the securitisation framework, which were clearly calibrated using assets that are a lot riskier than the assets used in current European securitisation markets. In all cases, the total capital required for senior tranches is higher than the capital requirement prior to securitisation, even though credit enhancements reduce the risk associated with the tranche. In the subsequent example you will see how this works for a low risk mortgage portfolio.

Example

Let's take a look at a fictitious example to illustrate the impact of the proposed changes. For this purpose, a hypothetical portfolio of mortgages has been constructed, with stylised risk characteristics. The pool consists of a homogeneous set of 5.750 mortgages, and every mortgage has a fully drawn notional amount of EUR 200k and there are no concentrations in the portfolio. Further, the pool does not contain NHG mortgages, and all the loans have the same Loss Given default (LGD). The vast majority of the mortgage loans have no arrears, while only a limited number of loans have one or two months arrears. Fifty loans are considered in default, as they are more than 3 months in arrears. Figure 1 provides an overview of the portfolio characteristics.

The bank has regulatory approval to use AIRB models for the loans. The PDs and LGDs in the portfolio are estimated on the basis of historical client behaviour and default experiences of similar loans, while the Risk Weights are obtained by applying the Basel 2 Credit Risk formula for mortgages.

We now assume that this complete portfolio has been securitised, and tranched as shown in Figure 2. The structure consists of an equity (Class C Note) and mezzanine (Class B Note) tranche, and two senior tranches. The class A1 and A2 senior notes are time-tranched. This means that first the A1 notes are redeemed in full before the class A2 notes start to repay. The maturity of the A1 notes is therefore considerably shorter than that of the A2 notes. The risk weights are obtained by applying the current Basel 2 IRB risk weights from the securitisation framework, based on the external rating of the tranches.

Figure 3 provides an overview of the risk weight under the current approach, as well as under the proposed approaches. From this overview it becomes clear that the results for both the RRBA and the BCRA are very conservative. The risk weights for the AAA rated notes - which have the highest notional values – quadruple in the most favourable scenario. The largest increase is tenfold. The class B notes show a 'modest' increase of 2 to three times. For senior tranches, the MSFA and the SSFA produce the lowest - but still substantially increased - results. The average risk weight of the complete portfolio increases from the current 12% to at least 54% under the MSFA. The increased risk weights underline the conservative calibration, since securitisation itself does not impact the total amount of risk, it only redistributes it. In order to mitigate the increase of the risk weights, the Basel Committee offers a risk weight cap for senior tranches. This cap allows a bank to substitute the calculated risk weight by the results of either the AIRB or the SA model for the assets in the underlying pool, depending on the regulatory approval. In this example, the cap has no impact on the results.

Tranche	Rating	Attachment Point	Detachment Point	Current Balance	Maturity	Current AIRB Securitised Risk Weight
Class A1 Notes	AAA	8,00%	100,00%	258.000.000	2	7,00%
Class A2 Notes	AAA	8,00%	100,00%	800.000.000	5	7,00%
Class B Notes	BB	4,00%	8,00%	46.000.000	5	425%
Class C Notes	nr	0,00%	4,00%	46.000.000	5	1250%
Total				**1.150.000.000**		**75%**

Figure 2 Risk Characteristics Securitised Mortgages

Tranche	Rating	Current AIRB Securitised Risk Weight	RRBA Risk Weight	MSFA Risk Weight	SSFA Risk Weight	BCRA Risk Weight
Class A1 Notes	AAA	7,00%	28,09%	20,00%	24,20%	35,00%
Class A2 Notes	AAA	7,00%	49,61%	20,00%	24,20%	70,00%
Class B Notes	BB	425%	1250%	123%	721%	875%
Class C Notes	nr	1250%	1250%	773%	1231%	1250%
Total		**75%**	**141%**	**54%**	**100%**	**142%**

Figure 3 Comparison of Risk Weights per approach

Next to the conservative quantitative impact, the proposed regulations are considerably more complex than the current regulations, due to the introduced sophistication in the framework.

Consequences of the proposals

The increased risk weights after securitisation underline the conservative calibration that was chosen. The changes proposed by the Basel Committee, if left unchanged, will significantly increase the required amount of capital banks have to hold against securitisation exposures. The increased cost of capital will, ceteris paribus, increase the required rate of return. A higher required rate of return means that the originating bank will have to increase the spreads it pays to its investors, increasing the cost of funding for mortgages. For a country as the Netherlands, where the amount of outstanding mortgages is more than double the amount of savings, secured funding is important. Knock on effects in terms of the price and availability of mortgages could occur as a result of the current proposals.

Disclaimer: *The views expressed are those of the authors at the time of writing and do not necessarily reflect the views of their employers.*

The Dutch Banking Association (NVB) is the representative voice of the Dutch banking community with over 90 member firms, large and small, domestic and international, carrying out business in the Dutch market and overseas. The NVB strives towards a strong, healthy and internationally competitive banking industry in the Netherlands, whilst working towards wider single market aims in Europe.

An Investment Model for Pension Funds

- *Marco van der Burgt (ING Bank, Market Risk Management)*

Abstract: Most Pension Schemes in the Dutch market are based on Defined Benefit (DB) agreements. The financial health of these funds is represented by the funding level, i.e. the ratio of assets divided by liabilities. When the funding ratio is below 105%, the fund has to present a recovery plan to the Dutch regulator. Pension funds try to avoid this underfunding by smart investment strategies like splitting their investment portfolio in a matching part, which resembles the liabilities, and a return part, which seeks high returns by investment in risky assets. We propose a method for constructing such a strategy. The method is based on avoiding underfunding within a certain time horizon.

Keywords: Defined Benefit, Pension Fund, Liability-Driven Investment, Return Portfolio.

Introduction

The Dutch Pension system consists of three pillars: a public pension which is applied since 1957, a collective pension administered by a pension fund or by an insurance company and a third pillar, consisting of individual pension products. Concerning the second pillar, Figure 1 shows that most pension funds in the Netherlands are based on the Defined Benefit (DB) scheme. DB schemes guarantee a specified pension benefit payment from retirement until death. Pension funds collect premiums from the participants. Once the premium is invested, the fund will hopefully achieve enough returns to fulfil its liabilities, which consists of the present value of retirement benefits and indexation. As such, these investments have typically a long horizon.

In contrast to DB schemes, Defined Contribution (DC) schemes guarantee a specified premium level during active membership. In this case, the pension benefit depends on investment returns and changing actuarial assumptions.

Over the last decade the share of DC Pension schemes is increasing in the Dutch market, but the DB Pension Schemes are still dominating. The financial positions of these DB funds are under pressure due to longevity, the current recession and the low interest rates for liabilities. The fund's financial position is often represented by the funding level, which is defined as the fund's assets divided by its liabilities. The time evolution of the funding level depends critically on the investment strategy of the fund. Most DB pension funds divide their investment portfolio in a matching portfolio, which resembles the characteristics of the liabilities of the fund, and a return portfolio, which seeks for high performance to finance indexation. The matching part consists of fixed income instruments, whereas the return portfolio consists of more risky instruments like equity. This strategy is often referred to as Liability-Driven-Investment (LDI, see Sender 2010). A crucial question is how the portfolio should be split in a return and a matching part. This question is answered by Asset & Liability Management (ALM) studies. Since these simulations require a significant amount of time, finding the optimum splitting of the investment portfolio for a DB pension fund can be a cumbersome task.

In this paper, we propose an investment strategy for a DB pension fund by an optimum splitting of the investment portfolio in a matching and return part. This splitting is related to management statements like "We believe there is a 99% probability that a state of underfunding will not occur in 1 year". The 99% percentage is referred to as the confidence level and the 1 year is referred to as the investment horizon. We derive an equation for the optimum splitting for a specified confidence level and investment horizon.

The next section describes the model, followed by the demonstration of the model with recent data. The last section concludes. An extended version of this paper is published elsewhere (Burgt 2013).

Determining the return portfolio

The nominal liabilities L of a pension fund grow through time with the nominal rate r, but also due to longevity, i.e. the increase in life expectation. This is represented by parameter λ in the evolution of the liabilities L over time:

$$dL = (r+\lambda)Ldt \qquad (1)$$

Based on CBS data (CBS StatLine 2012), we estimated the longevity parameter λ as 0.4%. We assume that the pension fund follows an LDI strategy. As such, the pension fund has divided the portfolio in a return part, which grows with the (expected) return μ of the risky assets, and a matching part, which grows with the nominal interest rate (r). The fraction of the return part is denoted by the return fraction ξ. The matching part is a fraction 1- ξ of the total portfolio. Therefore, the assets of the fund evolve in time according the following stochastic differential equation:

$$dA = [r(1-\xi)+\mu\xi]Adt + v\xi A dW_A(t) \qquad (2)$$

with $W_A(t)$ representing Brownian motion. Here, v is the volatility of the return portfolio. This approach has also been applied by Boulier

(1995). We introduce the Sharpe ratio S as:

$$S = \frac{\mu - r}{v} \qquad (3)$$

Combining equations (1), (2) and (3) with the definition of the funding ratio f = A / L gives:

$$df = \left(Sv\xi - \lambda\right) f dt + \xi v f dW_A(t) \qquad (4)$$

The solution of the stochastic differential in (4) is easily derived from stochastic calculus (see for example Shreve 2004):

$$f = f_0 \exp\left[\left(Sv\xi - \tfrac{1}{2}\xi^2 v^2 - \lambda\right)t + \xi v \varepsilon \sqrt{t}\right]$$
$$\varepsilon \sim N[0,1] \qquad (5)$$

where N[0,1] represents a cumulative standard normal distribution. Equation (5) leads to the following probability of underfunding, i.e. the probability α that the funding ratio after time horizon $t = \tau$ is below a critical level K given as:

$$\alpha = P[f < K] = N\left[\frac{\ln(K/f_0) - \left(Sv\xi - \tfrac{1}{2}\xi^2 v^2 - \lambda\right)\tau}{v\xi\sqrt{\tau}}\right] \qquad (6)$$

In the Dutch pension market, a DB Pension Fund is regarded as underfunded when the funding level is below 105%, in which case K = 1.05. Figure 2 demonstrates (6) by presenting the maximum allowable return fraction ξ as a function of the probability of underfunding for different values of volatility and time horizon: as expected, probability α increases with increasing return fraction, increasing time horizon and increasing volatility in the return portfolio. Equation (6) provides a key to defining an investment strategy in terms of maximum return fraction ξ, given a probability of underfunding α. We can recast equation (6) by writing the return fraction ξ explicitly as in equation (7).

$$\xi = \frac{\left[N^{-1}[\alpha]\sqrt{\tau} + S\tau\right] + \sqrt{\left[N^{-1}[\alpha]\sqrt{\tau} + S\tau\right]^2 - 2\tau\left[\lambda\tau + \ln\left(K/f_0\right)\right]}}{v\tau} \qquad (7)$$

Equation (7) presents the maximum fraction ξ of the return part when the probability of underfunding might not exceed a specified value α. After applying (7), the value of ξ will be capped above 1 and floored below 0 since values outside the interval [0, 1] are not considered to be realistic.

In order to apply (7), the management of the fund defines a confidence level $1\text{-}\alpha$ and an investment horizon τ. When the management wants a $1\text{-}\alpha$ confidence that the pension fund will not be underfunded in τ years, the probability of underfunding is α. Then (7) is used to determine ξ, given the initial funding level f_0 of the pension fund. Figure 3 demonstrates (7) graphically: the return part of the portfolio decreases at high confidence levels, which is in line with intuition. We will denote the described strategy as Confidence-Level-Based (CLB).

Before proceeding, we address some specific assumptions in deriving (7). First of all, interest rate risk is assumed to be completely hedged by the matching portfolio. Furthermore, the model is based on nominal growth of the liabilities. Above a certain funding level the management might decide to index the pension rights, i.e. an increase in liabilities. We also like to remark that some pension funds apply derivatives like swaps to hedge interest rate risk. Counterparty risk on swap transactions is not included in this model. Finally, the volatility in the return portfolio is assumed to be constant.

Backtesting the model

Machiavelli once said: "Never waste the opportunities offered by a good crisis". In line with this thought, the recent credit crunch and following financial crisis in 2008 provides the possibility to test the described model and validate its resulting investment strategy during the crisis period. We therefore assume a hypothetical pension fund with a funding level of 120% in 1999. The fund has split its asset portfolio in a 30% matching part and a 70% return part, i.e. the return fraction ξ equals 30%. We further assume that the return part resembles a benchmark, the Dow Jones Industrial Index. In our case, the pension fund might choose between three strategies. In the first strategy, the return fraction $\xi = 30\%$ is kept constant. The second strategy is a buy-and-hold strategy: it starts with $\xi = 30\%$, but no rebalancing occurs after each observation period. The third strategy is a CLB strategy with a confidence level of 97.5% and one year horizon. In all three strategies we assume that the management might periodically adjust the investment portfolio with a frequency of one month. The situation of under funding occurs when the funding level is below 105% in line with the Dutch regulation.

Figure 4 demonstrates how the funding level will change during the crisis when the different strategies are followed. The black solid line represents the first strategy of a constant mix. This strategy has the highest funding levels in periods of an economic stable situation. However, during the onset of the credit crisis in 2008, this investment strategy leads to underfunding with a minimum funding level of 96%. When the

FIGURE 1: Overview of percentage of DB pension funds in the Dutch market over the last decennium. Source: (see DNB(2011)).

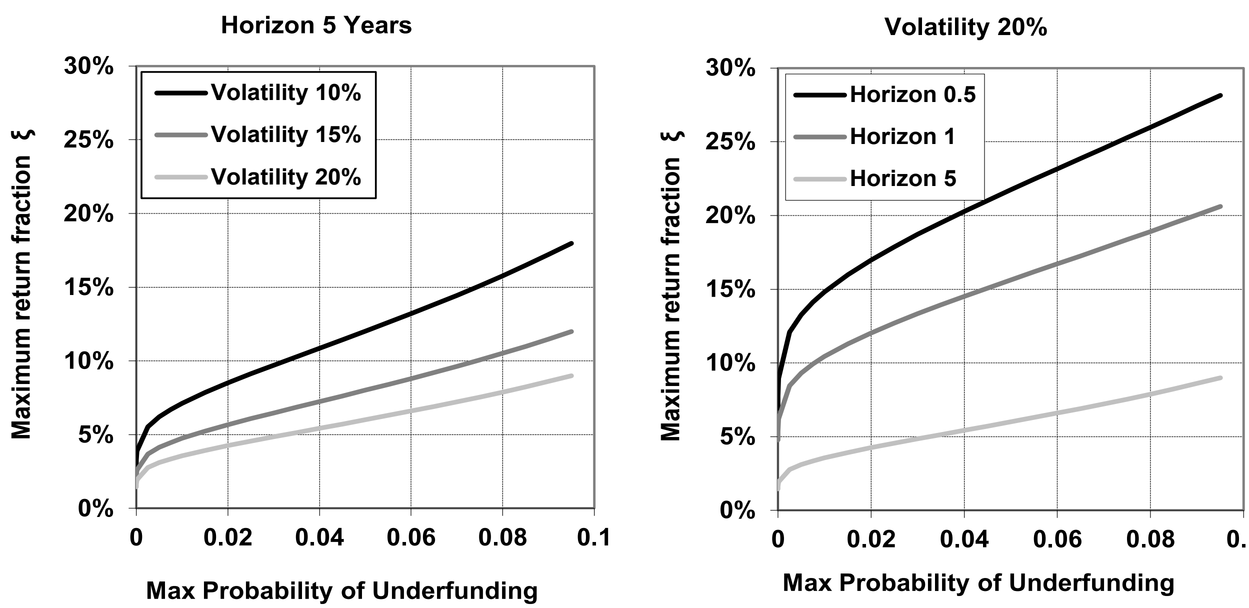

FIGURE 2: Maximum return fraction ξ as a function of the maximum probability of underfunding, shown for different volatilities and horizons. The value of K is 105%. The initial funding level is 110% and the Market Price of Risk (S) amounts to 0.3.

buy-and-hold strategy is followed, underfunding also occurs during the crisis period, as shown by the dark-grey line in the figure. The buy-and-hold strategy leads to underfunding, but less severe than in case of a constant mix. The third strategy, based on CLB, appears to be the safest strategy. This investment strategy might not lead to the highest funding levels, but no underfunding occurs during the crisis in 2008.

Summary and conclusion

In this paper we introduce an investment strategy for DB pension schemes, assuming that the pension fund splits its investment portfolio into a matching part and a return part. Our conclusion is that the CLB investment strategy represents the safest strategy. Other strategies might lead to higher funding levels,

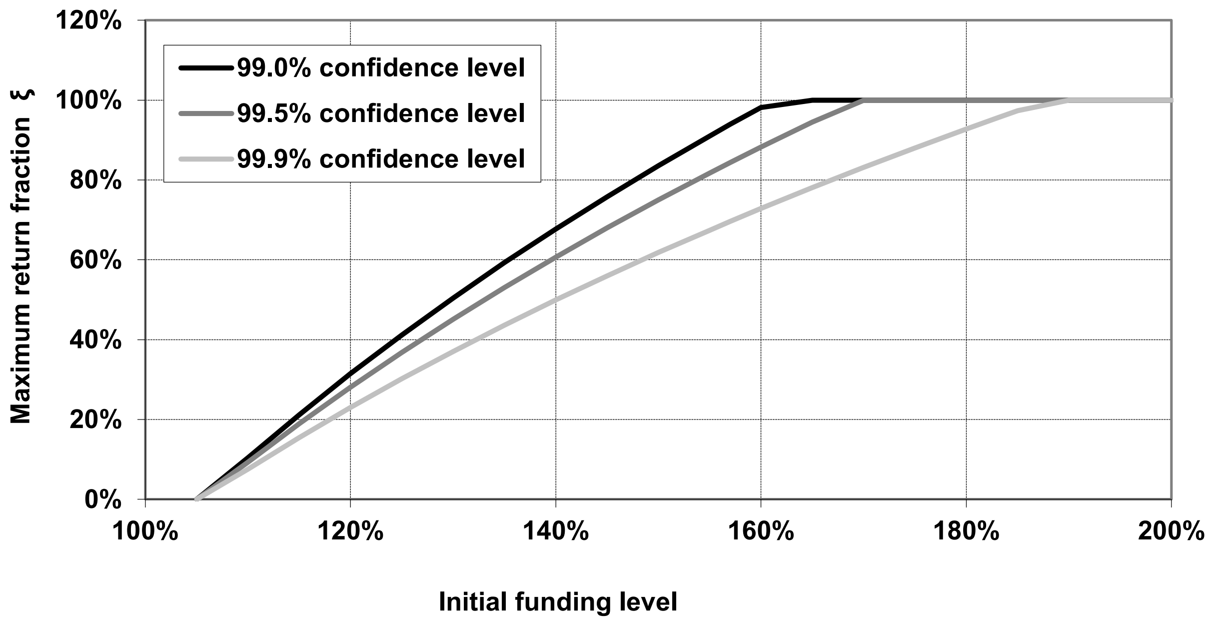

FIGURE 3: Maximum return fraction ξ of the portfolio as a function of the initial funding level, at different confidence levels. The value of K is set to 105%, which means that the Pension Fund might only invest in the return portfolio when its funding level is higher than K. The volatility in the return portfolio is assumed to be 20% and the allocations are shown for a time horizon of 1 year. The fraction ξ is maximized to 100%.

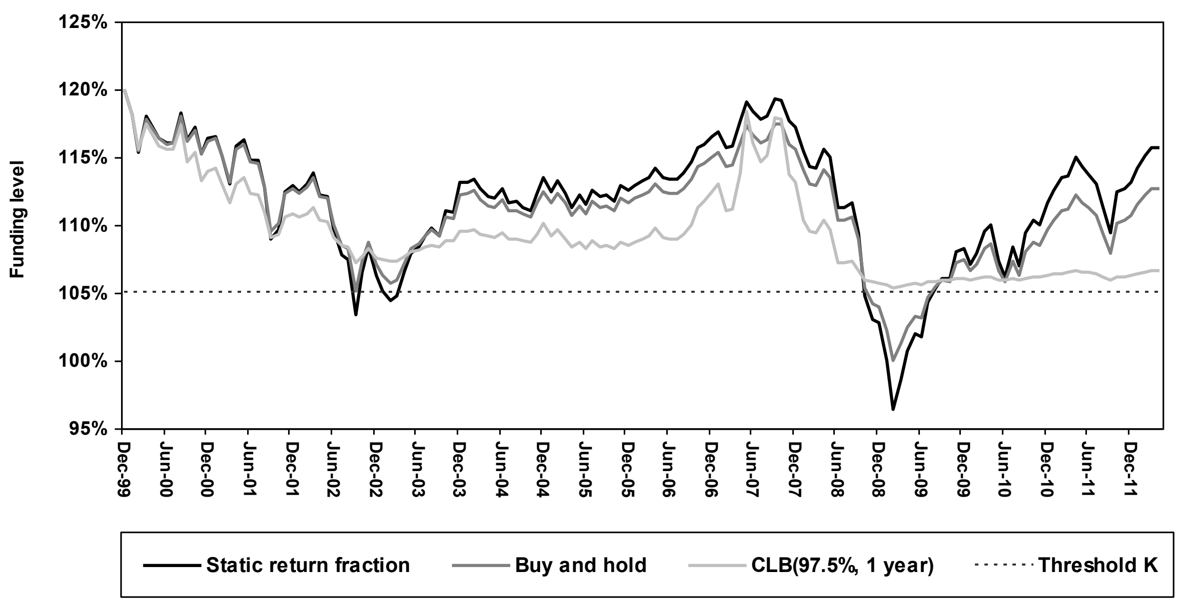

FIGURE 4: Development of funding level over time for a pension fund. The initial funding level in December 1999 is assumed to be 120%. The development of the funding level in case of a CLB strategy with 97.5% confidence level and one year horizon is compared with a static mix and a buy-and-hold strategy. The pension fund is regarded as underfunded when its funding level is below K = 105%. The figure reveals that the CLB strategy protects against under funding, whereas under funding occurs in case of a static mix or a buy-and-hold strategy.

but the protection against under funding is missing.

A potential weakness in the proposal is the assumption that the volatility in the return portfolio is constant. Recently, this model is extended by assuming stochastic volatility in the return portfolio, using techniques of differential geometry (Burgt 2010). The discipline of differential geometry is rather new in finance (see for example Labordere 2009, Paulot 2009) and out of scope for this short note. The interested reader is referred to this extended version, which has appeared in Insurance Risk Magazine of May 2013.

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Robust VaR estimation in operational risk modeling

– by Joris Chau (Financial Services Risk, Ernst & Young)

Abstract: In this report we consider estimating the operational risk regulatory capital charge (Value-at-Risk or VaR) via the Loss Distribution Approach. We argue that if we use standard estimation techniques, such as maximum likelihood, the estimated capital charge is highly sensitive to minor contamination of the operational loss data. This is a major issue in practice: large swings may be produced in the capital charge when a single or a few loss events are added to the database. In order to ensure a stable capital charge, we introduce the robust statistics framework, which is aimed at sacrificing some efficiency at the exact model, in order to gain robustness against minor deviations of the model. We conclude that using robust estimation techniques, the estimated capital charge maintains high efficiency at the exact model, while remaining stable under contamination of the operational loss data.

Initial model: A common method to calculate the operational risk regulatory capital charge is via the Loss Distribution Approach (LDA): we model a separate loss frequency and loss severity distribution, which are combined into a compound loss distribution. According to the Basel II framework, the regulatory capital charge can be calculated as 99.9%-VaR of the compound annual loss distribution. Below we estimate 99.9%-VaR for independent and identically (IID) non-truncated and truncated loss data samples of size 100 from the lognormal distribution, with location $\mu = 10.95$ and scale $\sigma = 1.75$ and log-gamma distribution, with shape $a = 34.5$ and rate $b = 3.5$. Furthermore, the annual loss frequencies are modeled by a Poisson distribution, with intensity parameter $\lambda = 25$. The results are the mean values of 100 VaR computations.

For non-truncated loss data we estimate the distribution parameters by means of maximum likelihood and for

		Truncation threshold H	
	true VaR	$H = 0$	$H = 25\,000$
lognormal	63 945 425	65 516 330	66 045 350
	Bias	1.02	1.03
log-gamma	62 290 900	64 859 110	63 476 660
	Bias	1.04	1.02

truncated loss data we apply the constrained maximum likelihood approach, which maximizes the likelyhood function of the truncated density functions (see Chernobai et al. [1] for further details). Furthermore, we apply an efficient method using Fourier inversion techniques to construct the compound loss distribution (see Shevchenko [2] for a detailed explanation).

The problem: Although we are able to produce fairly unbiased results in estimating 99.9%-VaR for perfectly IID operational loss data, when we introduce minor contamination, the estimated capital charge becomes almost useless. To illustrate this, we contaminate the non-truncated loss data samples in the previous simulation experiment according to the following procedures:

1. We add a single loss event equal to twice the largest in-sample loss to the data sample.

2. We add a single absolute loss event of € 10 to the data sample.

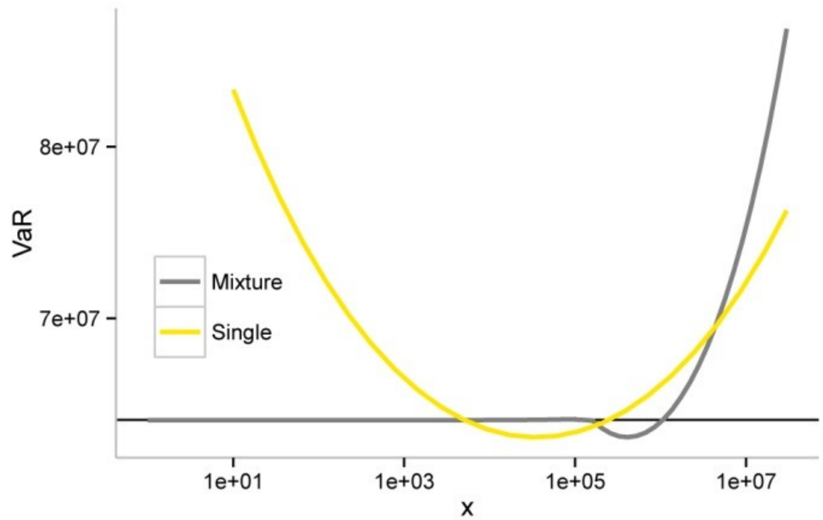
Below we estimate 99.9%-VaR for the contaminated non-truncated loss data samples in the same way
as before.

Essentially, lengthening the tails of the underlying severity distribution (the right tail by adding severe losses, but also the left tail by adding very small losses) explodes the variance of the severity distribution and it is seen that a single new recorded loss event can result in large swings in the estimated capital charge.

		Data contamination	
	true VaR	1.	2.
lognormal	63 945 425	96 109 134	132 971 124
	Bias	1.50	2.08
log-gamma	62 290 900	115 543 428	462 752 895
	Bias	1.85	7.43

Mixed severity distributions: In order to reduce the impact of very small loss events on the capital charge, we consider fitting a mixture of severity distributions to the loss data. That is, we separately fit a light-tailed severity distribution (e.g. exponential) to the body region of the loss data and a

heavy-tailed severity distribution (e.g. lognormal or log-gamma) to the tail region of the loss data. In the figure above we illustrate the impact of contamination due to a single loss observation in the point *x* on the estimated capital charge for a non-truncated loss data sample of size 100 from the lognormal distribution. The single fitted severity distribution corresponds to the approach in the initial model. The mixture of severity distributions corresponds to a fitted exponential distribution on the body region $[0, 1\times10^5)$ and a fitted lognormal distribution on the tail region $[1\times10^5, \infty)$. It is seen that the sensitivity of the capital charge to losses below the body-tail threshold is totally mitigated. However, this comes at the expense of a higher sensitivity to losses in the right tail, since the parameters of the tail distribution need to be estimated using only the loss observations above the body-tail threshold.



Robust estimation methods: In practice the loss data will always be truncated, which also reduces the impact of small loss events on the estimated capital charge. A more straightforward procedure might be to fit a single severity distribution to the loss data, but instead of standard estimation techniques (i.e. maximum likelihood and the constrained maximum likelihood approach) we apply *robust* estimation procedures. In this way, we do not have to split up the (usually scarce) loss data into a separate body and tail region. We consider two robust estimation procedures:

- *Optimal bias robust estimation(OBRE)*: the aim of the OBRE is to find robust estimators such that the efficiency loss is minimal. The estimators partially down weight outlying observations, where the measure of robustness is specified by the tuning parameter *c*. (If we choose $c = \infty$, the robust estimators correspond exactly to the maximum likelihood estimators). For a detailed explanation see Hampel et al. [3].

- *Method of trimmed moments (MTM)*: the MTM estimators are closely related to the standard method of moment estimators. We obtain robustness by specifying left and right trimming (a, b). The estimators are found by solving the system of equations resulting from matching the sample trimmed moments to the population trimmed moments. For a detailed explanation we refer to Brazauskas et al. [4].

Results: We assess the efficiency and stability of the capital charge under the introduced robust methods. Consider the following class of ε-contaminated distributions (gross-error model):

$$F_\varepsilon = (1-\varepsilon)F + \varepsilon G$$

With $\varepsilon \geq 0$ the level of contamination, *F* the assumed *true* model distribution and *G* a contaminating distribution, according to:

1. Random contamination: *G* randomly draws gross errors from the interval

[1, 3×10^7] on a logarithmic scale. Informally, we replace a fraction ε of the (IID) loss data by random points on a logarithmic scale.

2. *Left-tail contamination*: *G* randomly draws gross errors from the interval [1, 1000]. This corresponds to contamination in the left-tail of the loss distribution.

3. *Right-tail contamination*: *G* randomly draws gross errors from the interval [1×10^7, 3×10^7] on a logarithmic scale. This corresponds to contamination in the right-tail of the loss distribution.

We present the results of the following simulation experiment: we simulate 250 non-truncated loss data samples from the log-gamma distribution, with shape a = 34.5 and rate b = 3.5 of size 500. Each loss data sample is contaminated according to the previously described procedures. We model the severity distribution according to the following methods:

1. A single log-gamma severity distribution via maximum likelihood estimation. This method corresponds to the approach in the initial model.
2. A single log-gamma severity distribution via OBRE, with tuning parameter c = 2.5.

3. A single log-gamma severity distribution by MTM estimation, with trimming proportions (a, b) = (0.05, 0.05).

4. A mixture of severity distributions, with an exponential distribution on the body region [0, 25 000) and a log-gamma distribution on the tail region [25 000, ∞), where the parameters are estimated by the OBRE, with tuning c = 2.5.

5. A mixture of severity distributions, with an exponential distribution on the body region [0, 25 000) and a log-gamma distribution on the tail region [25 000, ∞), where the parameters are estimated by the MTM, with trimming b = 0.03.

	Random ε-contamination				Left-tail		Right-tail	
Method	$\varepsilon = 0$		$\varepsilon = 0.025$		$\varepsilon = 0.01$		$\varepsilon = 0.01$	
	VaR	Bias	VaR	Bias	VaR	Bias	VaR	Bias
1. MLE	64 114 710	1.03	688 214 835	11.05	597 019 390	9.58	115 524 255	1.85
2. OBRE	68 127 620	1.09	86 041 340	1.38	76 109 990	1.22	75 323 875	1.21
3. MTM	64 811 725	1.04	87 732 260	1.41	73 746 420	1.18	85 615 860	1.37
4. Mix. OBRE	40 469 935	0.65	56 673 100	0.91	41 028 020	0.66	73 643 350	1.18
5. Mix. MTM	49 228 740	0.79	66 606 925	1.07	49 839 460	0.80	78 274 020	1.26

The annual loss frequencies are modeled by a Poisson distribution with intensity λ = 25. The average estimated VaR measures of the 250 loss data samples and corresponding biases, with respect to the capital charge under the true parameters, can be found in the table above.

Conclusion: It is seen that we should not blindly apply standard estimation methods (MLE) when the operational loss data does not follow the assumed severity distribution exactly. If the loss data is slightly contaminated, robust estimated methods may be a viable alternative. The estimated capital charge stays close to the *true* VaR measure when there is no contamination of the loss data and it remains much more stable when the loss data becomes increasingly contaminated. In this report we have summarized the main topics and results of the Master's thesis: *Robust estimation in operational risk modeling*. We refer the interested reader to the TopQuants website for the complete thesis.

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Collateral haircuts

- by Toni Budimir (RBS - Quantitative Review team)

Abstract: Haircuts are discounts imposed on the value of non-cash collateral assets. The value of these assets change over time and the risk due to these changes is mitigated by discounting their notional value. Risk haircuts are proposed in CRD IV. It is shown here that simulations of the forward curve of bond issuers can be used to find appropriate discounts. We find that for shorter residual maturities supervisory haircuts are high while for longer dated bonds they are rather low.

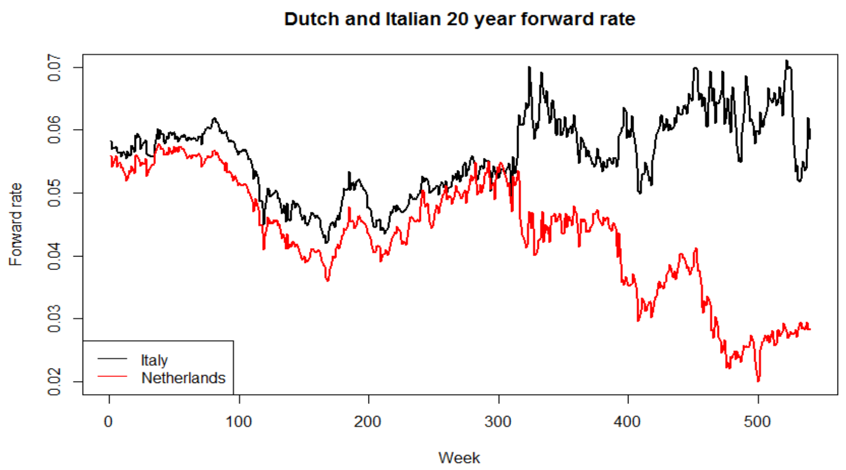

Figure 1: 20-year forward rate over a 10 year period for the Netherlands and Italy

Introduction to Collateral Haircuts:

Collateral is an often overlooked aspect of counterparty credit risk CCR) calculation. Yet, efficient collateral management leads to lower exposures, and careful inclusion of collateral in exposure models can help in lowering RWAs. Collateralisation can be a good alternative to clearing trades on exchanges, or can be done via a clearing house.

CRD IV distinguishes between funded and unfunded collateral. Funded collateral are for instance cash and bonds. Unfunded collateral are instruments like letters of credit and CDSs. When using collateral in a trade, or when risking collateralised trades, discounts are often used on the value of non-cash collateral. These discounts are called haircuts and they determine the cash equivalent value of the instrument. For instance, a bond with a haircut of 10% and valued at par, would have its' cash-equivalent value as 90.

Different bonds should have different haircuts. CRD IV provides generic distinctions in characteristics of issuers to determine haircuts. It is interesting however to calculate haircuts based on all the characteristics of the issuer which ensures that all the available bond prices are used. In fig 1, the 20-year forward rates of Italy and the Netherlands are shown as an example. The time ranges from end 2002 until March 2013. Up to week 300, i.e. August 2008, the forward curves behave similarly, but after 2008 the differences grow. The characteristics of the issuers differ greatly which warrants different haircuts.

There are risk-based haircuts and commercial haircuts. Commercial haircuts depend not only on the collateral but also on the counterparty. A commercial haircut is negotiated while a risk haircut is set by the risk department.

Commercial haircuts determine when a collateral call is made, i.e. they determine the value of the collateral posted compared to the exposure incurred by the derivatives. Commercial haircuts are important for collateral calls, but the final exposure calculation should be based on risk haircuts.

Risk haircuts can be used in exposure calculations for OTC derivatives and Repo style agreements. For OTC derivatives, haircuts apply to collateral that is in the margin account while for Repo transactions, haircuts are also applied to the underlying security that is posted against cash. Risk haircuts should satisfy the following ``consistency'' properties:

1. They should be a function of the length of the margin period of risk with a larger period implying a higher risk haircut.
2. For the same issuer a risk haircut for a bond with longer residual maturity should be higher than the risk haircut of a bond with a shorter residual maturity.
3. Risk haircuts should be non-negative.

The last requirement is controversial. In certain cases, it may happen that a negative haircut is warranted. Bonds can be scarce in the market, or there may be more faith in the issuer than in cash money. Governments may buy their own bonds, while parties that traded short need to deliver bonds. For example in the euro zone investors may prefer holding German government bonds over cash.

Bond Forecast and Haircut Computation:

The most obvious way of extracting a haircut for a bond is to generate a number of forward scenarios for the yield curve and to revalue the bond in each of these scenarios. The resulting distribution of value changes over the margin period of risk, here 14 days, can be used to obtain haircuts. The 95th percentile value change is our choice for haircuts on outgoing collateral, and the 5th percentile is the haircut for incoming collateral.

To extract the yield curve, we employ the most commonly used Nelson-Siegel model. Directly fitting Nelson-Siegel to bond data implies that the long term rate depends very much on the available data points. The available data points are usually heavily skewed towards the shorter bond maturities. Hence we have chosen to first interpolate the forward rate using splines, and to fit the forward rate to the interpolated rate, as suggested by Lesniewski (2013).

The interpolation should be such that it does not throw away any information. We choose to use a spline interpolation with 12 basis functions. Further, we condition the interpolation by imposing a penalty on the second derivative, i.e. on the arc length. Through the penalty term we can determine how sinuous the curve is.

Finally, using a uniformly spaced grid rather than a grid determined from data availability, we fit the Nelson Siegel curve. The uniformly spaced grid leads to quite different results than using the data directly, as seen in fig 2.

We use an AR(1) model to forecast the parameters of the Nelson Siegel model. The AR(1) residuals are correlated through a dynamic correlation model that reverts to the long term correlation of the coefficients.

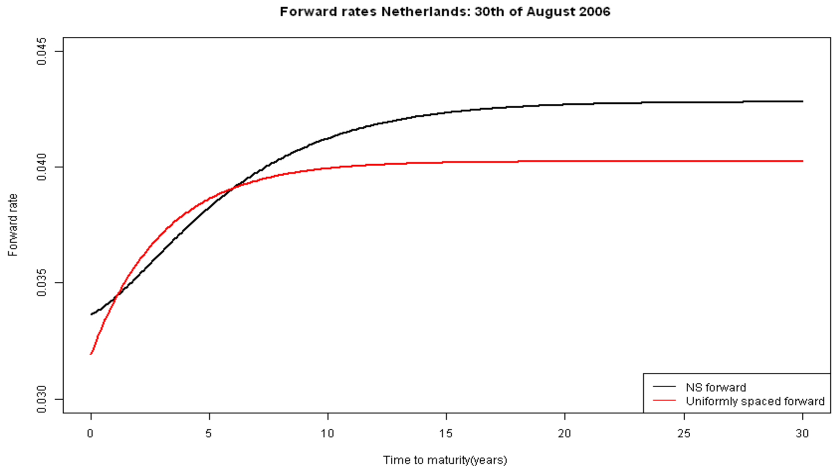

Figure 2: Forward curves for the two methods

From the forward curve, we can revalue the cash flows of the bonds and hence can compute the 14-day of the value changes. Finally, we compute the haircuts from these 14-day value changes lead to haircuts.

The forecast haircuts are markedly different from the haircuts that are given in the CRD IV regulations. We observe that within the generic subdivisions of the characteristics of government bonds there are large differences for the calculated haircuts.

Time-to-maturity	0.86	1.3	4	7	10	20
CRD IV	0.5	2	2	4	4	4
Netherlands	0.401	0.524	1.314	2.594	2.933	5.175
CRD IV	1	3	3	6	6	6
Italy	0.733	1.084	2.541	4.246	4.848	8.517

The haircuts for bonds with a very long maturity seem to show an underestimation of the risk that is involved with using such bonds in collateral agreements. The haircuts need not to be based on the benchmarks given by the regulator, but can be based on percentages obtained from simulations. Further research can be done for the calculation of haircuts on other bond types, e.g. corporate bonds, asset-backed bonds.

The models used to forecast the coefficients of the forward curve are all mean-reverting models. In the analysis of the bond prices we observe that the time to mean reversion is high which means that within one year of forecasting the influence of the initial parameters has decreased significantly. After this period, the forecasts of the forward curves and thus the bond prices are of limited use. The haircut is here the maximum 95th percentile of the price change distributions over one year into the future. By using mean-reverting models, it is not possible to have longer dated forecasts of the bond prices.

Conclusion

The haircuts found using simulations are different from the benchmark rates given by the regulator. The simulated forward curves tend to show fast convergence to the mean. The haircuts can be determined only over a small time interval since the influence of the data decreases rapidly.

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Econometric Game 2013: Forecasting Spanish GDP in a Rich Data Environment

- *by Guillermo Carlomagno, Andrés García-Suaza, Salvatore Lobello, Michelle Sánchez, Pedro Sant' Anna*

(Universidad Carlos III de Madrid)

Econometric Game 2013: In 1999, the Econometric Game started as a one-day national competition between VU University of Amsterdam and University of Amsterdam. Over the last couple of years, it has evolved into an international three-day event which has participants of econometrics background from 30 well known universities all around the globe. This event is highly recommended by distinguished professors who regard it as an ideal event to show that econometrics reaches beyond economics and finance with regard to its applicability to solve a variety of problems that affect the global economy. The year 2013 marked the fourteenth Econometric Game event which focused on the topic, "The effects of fiscal policy on economic growth". This article presents the case study of the winning team of 2013 from *the Universidad Carlos III de Madrid.*

Abstract: Recently, there is some evidence that the effectiveness of fiscal policies are not independent of the economic situations. Hence, being able to provide real GDP growth forecasts using all the available information is crucially important for economic authorities. In this paper, using a dataset of 70 different variables for the period 1970-2012 at quarterly frequency, we employ dynamic factors models and LASSO regression techniques to provide different forecasts for the Spanish quarterly GDP growth in 2013. We conclude that these techniques provide superior predictive ability than a simple AR(4) model. Nonetheless, we find superiority of combined forecasts over single model based predictions. With our preferred model, we forecast for Spanish 2013 yearly GDP growth to be 0.08%.

Introduction

Forecasting the real GDP growth rate is crucially important for economic authorities in order to take efficient policy decisions. This is in general a very challenging task. It becomes even more relevant in times of crisis, when governments tend to make key interventions to correct the adverse situation of the economy.

Nowadays countries in Southern Europe, such as Spain, are experiencing the biggest fiscal imbalances in recent economic history. The current situation seems to call for fiscal stabilization policies, which have to be properly assessed by taking into account also the growth perspective of the countries. Forcing a fiscal adjustment during a crisis might generate vicious circles difficult to escape from. Hence, selecting the inappropriate mechanism can have severe economic and social costs that may last for long periods.

The choice of most appealing fiscal policy is not independent of the economic situations. As we have seen in the first day of the Econometric Game 2013, there is indeed some empirical evidence in the literature on non-linearities in the way fiscal policy affects the economy, that is, the fiscal multipliers changes depending if the economy is in a boom or in a burst period. Therefore, to this extent it is of crucial importance to be able to forecast properly GDP growth.

In order to forecast GDP growth, economist traditionally use classical time series models such as ARMA, ARMAX and VAR models, where in ARMAX and VAR models one can take advantage of additional information throw economic variables apart from the GDP. Nonetheless, economist and practitioners tend to use small scale models, in order to avoid in-sample overfitting. This way, an underlying primitive of these models is that the economist knows which variables have strong predictability effect on GDP growth, which might not be too realistic in a time where we have access to hundreds of different macroeconomic and financial variables.

Given the advance of Econometrics and Statistics techniques, recent approaches implemented to forecast key macroeconomic variables take advantage of today's rich data bases. The possibility of extracting value from the additional information available can significantly improve the forecasting. Because of the nature of these datasets, classical techniques, such as ARMA, ARMAX or VAR, are not feasible for estimation and forecasting, as the number of regressors (therefore of parameters) is usually bigger than the number of observations. Nonetheless, different techniques have been proposed in the literature to deal with the dimensionality problem, e.g. Dynamic Factor Models, LASSO Models, Factor Augmented VAR, among others.

Our goal in this paper is to forecast the real GDP growth rate for Spain in 2013 by using a rich dataset from the OECD Economic Outlook. In order to do so, we use different models as a classical ARMA, Dynamic Factor Models, LASSO Models, Factor Augmented VAR. Being agnostic about which model is the "true" one, we also consider a forecasting combination of all the methods. In order to assess the performance of the methods, we perform several forecasting evaluations. In general, the forecast combination provides the most accurate forecasting in terms of Mean Square Predicted Errors. In line with this model, we predict that Spanish real GDP will present a 0.08 % growth in 2013.

The rest of the paper is organized as follows. In the next section we perform a detailed data description. Section 3 presents the models and the estimation results. In section 4 we perform forecasts comparison and evaluation. Section 5 concludes.

Data Description:

We use a rich dataset for Spain coming from the OECD Economic Outlook. It includes all relevant macroeconomic variables for the period 1970-2012 at quarterly frequency. The information includes time series of GDP, prices, expenditures, current accounts, exports, imports, exchange rates, prices, deflators, employment and interest rates. We have a total of 70 variables. Several variables are though repeated at different price levels, or both in value and volume. Whenever possible, we decide to keep the variables at 2005 prices in USD and in volume rather than in value. We also add the appropriate deflators. As our target is to forecast the volume of GDP, the strategy that
we use does not generate any information loss and at the same time it prevents us from overfitting the model with redundant variables. After deleting the observations with missing values we are left with a balanced panel spanning the period 1977-2012, with 143 observations. We work with a total of 45 variables.

Plotting all the series, we can observe that almost all of them are clearly trending over time, while many present a dynamic evolution which could be consistent with a white noise process. Before using these variables in our forecasting model, we therefore need to test for their stationarity. We run on each time series the Augmented Dickey Fuller (ADF) test, whose null hypothesis is that the process has a unit root. Given that the data is quarterly we use 4 lags to take into account the likely high correlation between the variables within the same year. The result of ADF tests suggest that we cannot reject the null hypothesis for any of the variables. This is an expected result when dealing with macroeconomic variables and we can easily tackle this difficulty. We compute growth rates of all the series rather than log-differences (as we have several negative values), solving the non-stationarity in this way. Price levels and deflators are the only problematic variables, as the plot of their growth rate makes us still doubtful about their non-stationarity. We follow the common practice of taking growth rates again, in order to make sure to have stationary series.

The presence of large outliers in the time series might distort the inference of our analysis. In order to control for this potential threat, we decide to replace the extreme values (over the 97.5 and before the 2.5 percentile) of each time series by the mean of the neighboring values (linear interpolation). To this extent, we simply follow the strategy of Beck, Marcelino and Banerjee (2011).

Methodology:

In order to exploit the data rich environment, we use different approaches. In particular, we use diffusion Indexes via estimation of a dynamic factor models a-la Stock and Watson (2002) and LASSO model initially proposed by Tibshirani (1996). We prefer to use different models in order to have some room for making comparisons. In this section we provide a brief introduction to Dynamic Factor Models and Lasso.

Dynamic Factor Models:

Dynamic factor models (DFMs) where initially proposed by Geweke (1977) as the time-series extension of factor models previously designed for cross-sectional data. The starting point of DFMs is that the dynamics of a high dimensional (n) time-series vector
X(t) are driven by few (q) common factors f(it) and an idiosyncratic n-vector of disturbances e(t). The use of DFMs in economics became widespread after Geweke (1977) and Sims and Sargent (1977) who allowed both the factors and the idiosyncratic errors to be serially correlated. The factors f(t) are usually assumed to follow a VAR process whereas the idiosyncratic disturbances e(t) are assumed to follow univariate autoregressive processes. Thus, DFMs can be written as:

$$X(t) = \lambda(L)f(t) + e(t) \qquad (1)$$
$$\Gamma(L)f(t) = \eta(t) \qquad (2)$$

where the lag polynomials $\lambda(L)$ are the dynamics factor loadings of each series in X(t). Assume initially that both equations (1) and (2) are stationary. The idiosyncratic error e(t) is assumed to be uncorrelated with factors' innovations at all leads and lags (E(e(t), η(t-k))=0: for all k). In

the exact dynamic factor model it is also assumed that idiosyncratic disturbances are mutually uncorrelated at all leads and lags, that is, E(e(it)e(js) =0)for all s if i ≠ j.
As noted by Stock and Watson (2011), when the factors are known and the errors (e(t) and ηt)) Gaussian, an individual variable can be efficiently forecasted regressing it on the lagged factors and lags of the variable itself, so that we do not need to include all the (n) variables in the regression. Thus, in words of Stock and Watson (2006) DFMs allow to turn dimensionality from a curse into a blessing. However, not only the factors are unknown but also we do not know how many of them are driving the data. In order to select the number of factors, Bai and Ng (2008) highlight three possible information criteria for determining the number of factors, which are asymptotically consistent.

Lasso Regression Method

Another solution to deal with the dimensionality problem in forecasting is to use the Least Absolute Shrinkage and Selection Operator (Lasso), proposed by Tibshirani (1996) . The Lasso method is a regularized version of the least squares, which adds the constraint that the L(1)-norm of the parameter
vector, ||β||, is no greater than a given threshold. As it is well known, one can write the constrained problem as an unconstrained one using the Lagrange form of the problem. Hence, the Lasso estimator can be seen as the solution of the least-squares problem with the penalty λ||β|| added, where λ is a given constant. More formally, the Lasso estimate is the solution to

$$\min_{\beta} (1/n)(Y-X'\beta)'(Y-X'\beta) + \lambda \sum_{i=1}^{k} ||\beta(i)||$$

where $0 \le \lambda < \infty$. If $\lambda = 0$, we have the OLS problem, and as λ gets bigger, more parameters are shrunk to 0, and hence more regressors are excluded from the model. Knight and Fu (2000) studied the asymptotic properties of Lasso-type estimators. They showed that under appropriate conditions, the Lasso estimators are consistent for estimating the regression coefficients. Moreover, it has been demonstrated in Tibshirani (1996) that the Lasso is more stable and accurate than traditional variable selection methods such as best subset selection.

Results and Forecasts Evaluation

ARMA Model

We consider the ARMA model as a benchmark. Based on information criteria (AIC and BIC) and Box-Pierce test, we selected a AR(4) model. The model was estimated for the period 1977.3 - 2002.4, and use to forecast the period 2003.1 - 2012.4.

DFM Results

We estimate the space spanned by the factors using the principal components approach. For estimating the factors we used the usual normalization criteria, described in Bai and Ng (2008). We select 1, 4 or 5 factors depending on the information criteria used. We consider this three possibilities for the forecasting exercise. In the three cases we consider the following model for producing one step ahead forecast of GDP's growth rate where

$$\hat{y}(t+h) = c+\prod(L)F(t)+\Phi(L)y(t),$$

where $\prod(L)$ and $\Phi(L)$ are polynomials in the lag operator.

Lasso Results

Given the above mentioned advantages of the Lasso methodology, we also use it to forecast the GDP growth. We consider 8 lags of both the GDP and the other macroeconomic variables as covariates, summing to 360 regressors, more than 2 times the number of observations available. We normalize all the variables to have mean 0 and variance 1, and hence, we do not consider an intercept in the model. A crucial step to enjoy the nice properties of the Lasso estimator is to choose optimally the tuning parameter λ. We follow two approaches: first, we set it to the value 0.5, arbitrarily. Alternatively, we use cross-validation and it sets λ to 0.1, approximately.

It is important to notice that, regardless of the two choices of the tuning parameter, we only select first lag

Table 1: Diebold-Mariano forecast comparison test

	Factor Model 1	Factor Model 4	Factor Model 5	LASSO 2	LASSO 4	Combination
AR(4)	(+)**	(+)**	(+)**	(+)*	(+)**	(+)***
Factor Model 1		(−)***	(−)**	(−)**	(−)	(+)**
Factor Model 4			(+)***	(+)***	(−)	(+)***
Factor Model 5				(+)***	(−)	(+)***
LASSO 2					(+)***	(+)***
LASSO 4						(+)***

Note: *,** and *** denote statistically different from 0 at 10, 5 or 1% level.

variables. With the ad-hoc value of λ, we select only two variables (in first lag): total employment (National Accounts basis) and private final consumption expenditure (volume). As expected, once we reduce the threshold the optimal number of variables decreases: on the top of the aforementioned variables, export market for goods and services (volume, USD, 2005 prices) and inflation (GDP deflator with market prices). All the estimates have the expected signs: higher employment, inflation, exports and consumption lead to higher GDP.

An interesting feature is that the Lasso constrains the lags of GDP to zero. Nonetheless, we expect that this variable would improve the forecast accuracy of the model, and hence we introduce the extra restriction that the first lag of GDP must be different from zero.

Forecast Evaluation

In this subsection we discuss the forecasting power of the aforementioned models. Moreover, we consider a combination of all forecast since different studies find superiority of combined forecasts over single-model based predictions. Moreover, we consider the simple case of equal weights, since equal weighted forecast combination often outperforms estimated optimal forecast combinations - see e.g. Stock and Watson (2004).

For comparing the models we consider one-step ahead forecast errors for the period 2003.1-2012.4. Note that in this exercise we are producing true out of sample forecast given that models are estimated using data up to 2002.4. In order to be able to statistically compare the models via Mean Squared Predicted Error, we considered the Diebold-Mariano test.

Table 1 shows the sign of the difference between the mean squared errors across the different models the asterisk makes reference to the statistical significance. For reading the table, (+) means that the model in the row has a higher mean squared error than the one in the corresponding column. From Table 1 we conclude that the model with one factor and the Lasso model with 4 variables are the best options when comparing with the other models, but we cannot reject that the Factor 1 and Lasso 4 have the same forecasting power. Additionally, the combination of forecasts seems to be the best option overall.

Table 2: Point estimates for forecast of GDP growth. All models.

	Factor Model 1	LASSO 4	Combination
2013 Q1	0.09***	0.28***	0.11
2013 Q2	−0.22***	0.07***	−0.11
2013 Q3	−0.10***	0.20***	0.01*
2013 Q4	−0.01	0.14*	0.09
2013	−0.24***	0.88***	0.08

Note: *,** and *** denote statistically different from 0 at 10, 5 or 1% level.

2013 Forecasts

We now consider the forecasts of GDP growth produced by our models. The four periods ahead forecast with the Lasso model presents an additional difficulty since we need to forecast the "explanatory" variables. In order to do this without losing the rich information contained in the dataset, we do it in a Factor Augmented VAR (see Beranke, Boivin and Eliasz, 2005).

In Table 2 we present the forecasts of Spanish GDP growth for all 2013 quarters, using the best three models. All the values are in percentage points. On overall, all models present different forecasts. The Factor model forecast an overall contraction of 0.24 % , the Lasso an expansion of 0.88 % and the linear combination an expansion of 0.08 %. The forecasts of both Factor and Lasso model are statistically significant. Nonetheless, since the Forecast combination has been shown to has superior predictability than the others two, we believe that the Spanish GDP will present growth very close to zero in 2013.

Conclusions

Forecasting the real GDP growth rate becomes even more important in times of crisis, when governments need to choose public interventions with much more care to restore the macroeconomic equilibrium. For instance, economies in Southern Europe are currently experiencing an historical peak in debt to GDP ratios. Hence, public measures have to be properly implemented by taking into account the growth perspective of the countries.

In this report we have analyzed some of the possible models to forecast GDP growth for Spain. We provide some theoretical background on the different specifications and provide our own forecasts for 2013.

Our forecast results for the GDP growth rate in 2013 are very close to zero. This means that Spain is still not recovering from the crisis. Even in the absence of growth, one should find comforting that the models don't forecast any further recession. Regarding the fiscal crisis, this result supports the arguments for a smoother adjustment that can also be found in the IMFs country report for Spain of July 2012.

From a methodological point of view, we find that using the high dimensional models is important. This allows a more efficient use of all the information contained in large dataset and this is reflected in significantly more accurate forecasts.

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Upcoming Events

1. The next event is the 2013 TopQuants Autumn/Winter workshop on November 7th. The event will be held at the ABN AMRO Dialogues House in Amsterdam Zuid Oost. The official invitation will be mailed soon and further details of the event will be posted in due course on the TopQuants homepage .

2. The next issue of the TopQuants newsletter will follow in March 2014. Contributions for it are already welcome. Kindly contact Aneesh Venkatraman, (newsletter@topquants.nl).