CDOs and the Financial Crisis: Credit Ratings and Fair Premia

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Based on work of Marcin Wojtowicz

Top Quant 2012
Amsterdam, November 21
Motivation

- Spectacular rise of structured finance markets - annual issuance growing from $250 billion in 2000 to $2 trillion in 2006

- Followed by even more spectacular downfall in 2007-2008, see Benmelech and Dlugosz (2009)

- Investors shocked by collapse of some “risk-free” triple-A tranches

- “The story of the credit rating agencies is a story of colossal failure” - Henry Waxman (chairman of the House of Representatives Oversight and Government Reform Committee)
Collateralized debt obligation

Introduction

Portfolio of bonds

100 bonds

- bond 1
- bond 2
- bond 3

Collateralized Debt Obligation

- super-senior (AAA)
- senior
- mezzanine
- equity (not rated)

Risk transfer

Risk split into tranches

22%
12%
9%
6%
3%
Default occurs if the scaled asset value of an obligor, $V_i \sim N(0, 1)$, falls below a pre-specified threshold:

$$V_i < K_i = \Phi^{-1}(p_i)$$  \hspace{1cm} (1)

The one-factor Gaussian copula model:

$$V_i = \sqrt{\rho} Y + \sqrt{1 - \rho} X_i,$$  \hspace{1cm} (2)

where:

- $Y \sim N(0, 1)$ - common (systemic) factor
- $X_i \sim N(0, 1)$ - idiosyncratic (obligor-specific) component
- $\rho \in [0, 1]$ - correlation parameter

Default threshold $K_i = \Phi^{-1}(p_i)$ can be calibrated to historical or risk-neutral default probabilities.
Credit ratings measure pure default risk of a security

- S&P ratings are based on real-world default probability
- Moody’s ratings are based on real-world expected loss

Ratings play a central role in structured finance markets

- Risk management, see Crouhy et al. (2008), Krahnen and Wilde (2008)
- Pricing and investment decisions, see Brennan et al. (2009), Coval et al. (2009)

An S&P document states:

- “Our ratings represent a uniform measure of credit quality globally and across all types of debt instruments. In other words, an AAA rated corporate bond should exhibit the same degree of credit quality as an AAA rated securitized issue.”
Tranche payoffs depend on the portfolio loss rate

Distribution of portfolio losses is calculated by simulating default times and corresponding recoveries. Importance of correlation

The real-world portfolio loss distribution is used to determine tranche subordination levels and credit ratings
CDS (Credit default swap)

- A buys protection from B on bonds of company C:

- In case of default:

  Notional \times (1 - \text{Recovery Rate})
Expected premium:

\[ \pi_H \approx p_H (1 - RR) \]  

where \( p_H \) is the historical default probability

Risk premium:

\[ \pi_{MI} \approx p_{MI} (1 - RR) \]  

where \( p_{MI} \) is the market-implied default probability

Empirical studies:

\[ p_{MI} > 2p_H \]  

see Hull et al. (2005), Delianedis and Geske (2003), Berndt et al. (2005)
Consider a corporate bond with:

- real-world PD of 10% over a 10-year period (roughly 1% per annum)
- recovery rate of 50%

Consequently, this bond obtains a BBB- rating by S&P and Ba1 by Moodys

Assume that risk-neutral PD is 20 %

- It implies that CDS spread on the bond is 111.95 bps
Consider a pool of 100 BBB- bonds just described. Assume that pair-wise asset value correlations are 12.5%

Junior mezzanine tranche is tailored to have identical credit quality as the collateral bonds

- The tranche attachment point is chosen such that tranche default probability is 10% (‘BBB-’ rating by S&P)
- The tranche detachment point is chosen such that tranche expected loss is 5% (‘Ba1’ rating by Moody’s)

Other tranches:

- The super-senior tranche with AAA rating by S&P (PD = 0.36%)
- The senior tranche with AA rating by S&P (PD = 0.87%)
- We also obtain the equity and the senior mezzanine tranches
Table: CDO tranche risk statistics, ratings and premia

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Tranche subordination</th>
<th>Physical measure (PD=10%)</th>
<th></th>
<th></th>
<th>Risk-neutral measure (PD=20%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Default probability &amp; S&amp;P rating</td>
<td>Expected loss &amp; Moody's rating</td>
<td>Spread (bps)</td>
<td>Default probability</td>
<td>Expected loss</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>tranche 1 equity</td>
<td>0.00%</td>
<td>98.33%</td>
<td>47.50%</td>
<td>636.54</td>
<td>99.90%</td>
<td>78.53%</td>
</tr>
<tr>
<td>tranche 2 junior mezz.</td>
<td>9.90%</td>
<td>10.00%</td>
<td>5.00%</td>
<td>48.25</td>
<td>44.64%</td>
<td>30.24%</td>
</tr>
<tr>
<td>tranche 3 senior mezz.</td>
<td>14.75%</td>
<td>1.97%</td>
<td>1.35%</td>
<td>12.76</td>
<td>18.43%</td>
<td>14.55%</td>
</tr>
<tr>
<td>tranche 4 senior</td>
<td>17.08%</td>
<td>0.87%</td>
<td>0.58%</td>
<td>5.43</td>
<td>11.09%</td>
<td>8.46%</td>
</tr>
<tr>
<td>tranche 5 super-senior</td>
<td>19.45%</td>
<td>0.36%</td>
<td>0.01%</td>
<td>0.10</td>
<td>6.21%</td>
<td>0.27%</td>
</tr>
<tr>
<td>corporate bond</td>
<td>n.a</td>
<td>10.00%</td>
<td>5.00%</td>
<td>53.06</td>
<td>20.00%</td>
<td>10.00%</td>
</tr>
</tbody>
</table>

While the BBB- bonds have a market spread of 111.95 bps, the similarly-rated mezzanine tranche has a fair spread of 320.69 bps.
Table: Comparison of fair spreads on tranches and risk-equivalent bonds

<table>
<thead>
<tr>
<th>Tranche</th>
<th>S&amp;P rating</th>
<th>Moody's rating</th>
<th>Fair spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>tranche 2</td>
<td>‘BBB-’</td>
<td>‘Ba1’</td>
<td>320.69</td>
</tr>
<tr>
<td>junior mezz.</td>
<td></td>
<td></td>
<td>111.95</td>
</tr>
<tr>
<td>tranche 3</td>
<td>‘A-’</td>
<td>‘Baa1’</td>
<td>143.83</td>
</tr>
<tr>
<td>senior mezz.</td>
<td></td>
<td></td>
<td>28.03</td>
</tr>
<tr>
<td>tranche 4</td>
<td>‘AA’</td>
<td>‘A2’</td>
<td>81.81</td>
</tr>
<tr>
<td>senior</td>
<td></td>
<td></td>
<td>11.62</td>
</tr>
<tr>
<td>tranche 5</td>
<td>‘AAA’</td>
<td>‘Aa1’</td>
<td>2.52</td>
</tr>
<tr>
<td>super-senior</td>
<td></td>
<td></td>
<td>0.22</td>
</tr>
</tbody>
</table>

- Risk-equivalent bonds have the same real-world PD and real-world EL as the corresponding tranches
- Yield enhancement is obtained for all tranches
Relation between expected tranche losses and collateral default probability
High sensitivity of CDOs leads to lower rating stability

Consider a scenario such that:

- CDO is structured and rated under the baseline assumptions
- Soon after CDO origination credit conditions deteriorate
- 10-year default probability of the collateral bonds increases from 10% to 13% which corresponds to a one-notch downgrade from BBB- to BB+
**Table:** Impact of a deterioration in collateral credit quality on tranche ratings

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Tranche subordination</th>
<th>Default prob. Bond</th>
<th>Default prob. CDO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>tranche 1</td>
<td>0%</td>
<td>98.33%</td>
<td>‘NR’</td>
</tr>
<tr>
<td>equity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tranche 2</td>
<td>9.90%</td>
<td>10.00% ‘BBB-’</td>
<td>13.00% ‘BB+’</td>
</tr>
<tr>
<td>junior mezz.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tranche 3</td>
<td>14.75%</td>
<td>1.97% ‘A-’</td>
<td>4.96% ‘BBB’</td>
</tr>
<tr>
<td>senior mezz.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tranche 4</td>
<td>17.08%</td>
<td>0.87% ‘AA’</td>
<td>2.40% ‘A-’</td>
</tr>
<tr>
<td>senior</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tranche 5</td>
<td>19.45%</td>
<td>0.36% ‘AAA’</td>
<td>1.06% ‘AA-’</td>
</tr>
<tr>
<td>super-senior</td>
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</tr>
</tbody>
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**Standard market conditions (PD = 10%)**

**Deteriorated market conditions (PD = 13%)**

"Stability of ratings"
Is a one-notch downgrade of the collateral pool realistic?

- Similar deterioration in credit conditions can occur if a portion of bonds is downgraded by more than one notch
- A large increase in default probabilities can be explained by default contagion
- Consider a scenario when a single default within the collateral portfolio occurs soon after CDO origination. This default signals a low realization of the common economic factor Y
Impact of an early default within collateral on default probabilities of surviving bonds

- If the first default occurs after 1, 3 and 6 months, then conditional default probabilities of the surviving bonds jump to 16.19%, 13.31%, 11.14%, respectively.
Default risks of CDO tranches are concentrated in systematically adverse economic states (Coval et al. 2009)

If collateral default probability is underestimated, then CDO tranches might turn out to be dramatically more risky

CDO tranches are exposed to correlation risk

Modeling CDO tranches is prone to large model error
■ CDO-squared collateral pool is composed of 30 junior mezzanine CDO tranches just described. While these tranches have identical properties, we assume there is no overlap among their collateral portfolios.

■ Asset value correlation is 12.5% between obligors within the same CDO collateral pool and 3.5% between obligors belonging to collaterals of different underlying tranches.

■ The structuring process is analogous to the CDO case. In particular, the junior mezzanine tranche is tailored to have a real-world PD of 10% and expected loss of 5% implying the same credit ratings.
Sensitivity analysis
The case of CDO-squared

Relation between expected tranche losses and collateral default probability

- CDO-squareds are more sensitive than CDOs
- CDO-squared tranches are at the critical points: tranche PDs and ELs are still low at the 10% collateral PD level, but they blow-up if collateral PD rises
Table: CDO-squared tranche risk statistics, ratings and premia.

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Tranche subordination</th>
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<td>Expected loss &amp; Moody's rating</td>
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<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>equity</td>
<td>0.00%</td>
<td>77.88% 'CCC-'</td>
<td>32.46%</td>
</tr>
<tr>
<td>tranche 2 junior mezz.</td>
<td>13.27%</td>
<td>10.00% 'BBB-'</td>
<td>5.00%</td>
</tr>
<tr>
<td>tranche 3 senior mezz.</td>
<td>24.92%</td>
<td>2.07% 'A-'</td>
<td>1.38%</td>
</tr>
<tr>
<td>tranche 4 senior</td>
<td>31.25%</td>
<td>0.87% 'AA'</td>
<td>0.58%</td>
</tr>
<tr>
<td>tranche 5 super-senior</td>
<td>37.50%</td>
<td>0.36% 'AAA'</td>
<td>0.04%</td>
</tr>
<tr>
<td>corporate bond</td>
<td>n.a</td>
<td>10.00% 'BBB-'</td>
<td>5.00%</td>
</tr>
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The junior mezzanine CDO-squared tranche has a fair spread of 795.71 bps compared to 320.69 bps in the CDO case.
Conclusions:

• Fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds. Rating arbitrage possibilities.

• Tranche yield enhancement is attributed to concentration of risk premia. Systemic risk.

• Even if the rating agencies provide accurate and unbiased credit ratings, then ratings are still not sufficiently informative about fair spreads.

• CDO tranches are likely to perform poorly during unfavorable market conditions. Rating stability of tranches lower than of bonds.