doubleeffect

The Funding Value Adjustment | real or imaginary?

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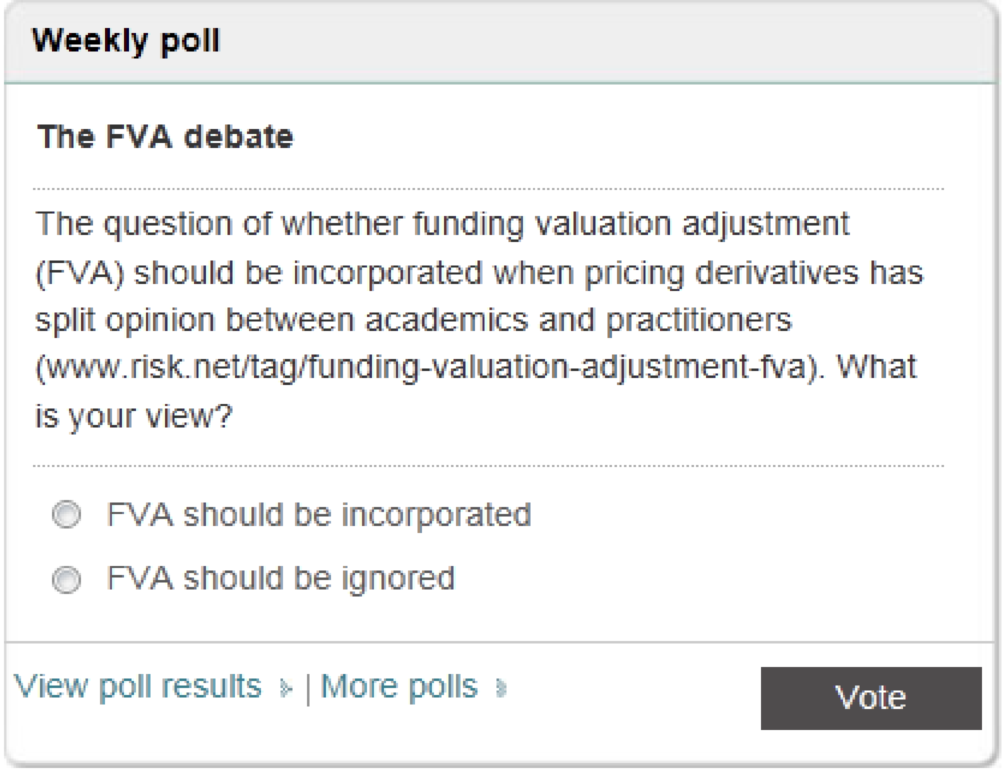
21 November 2012

The Funding Value Adjustment is topic of a heated debate

For example, Risk magazine (risk.net) had a poll on its website:



Weekly poll

The FVA debate

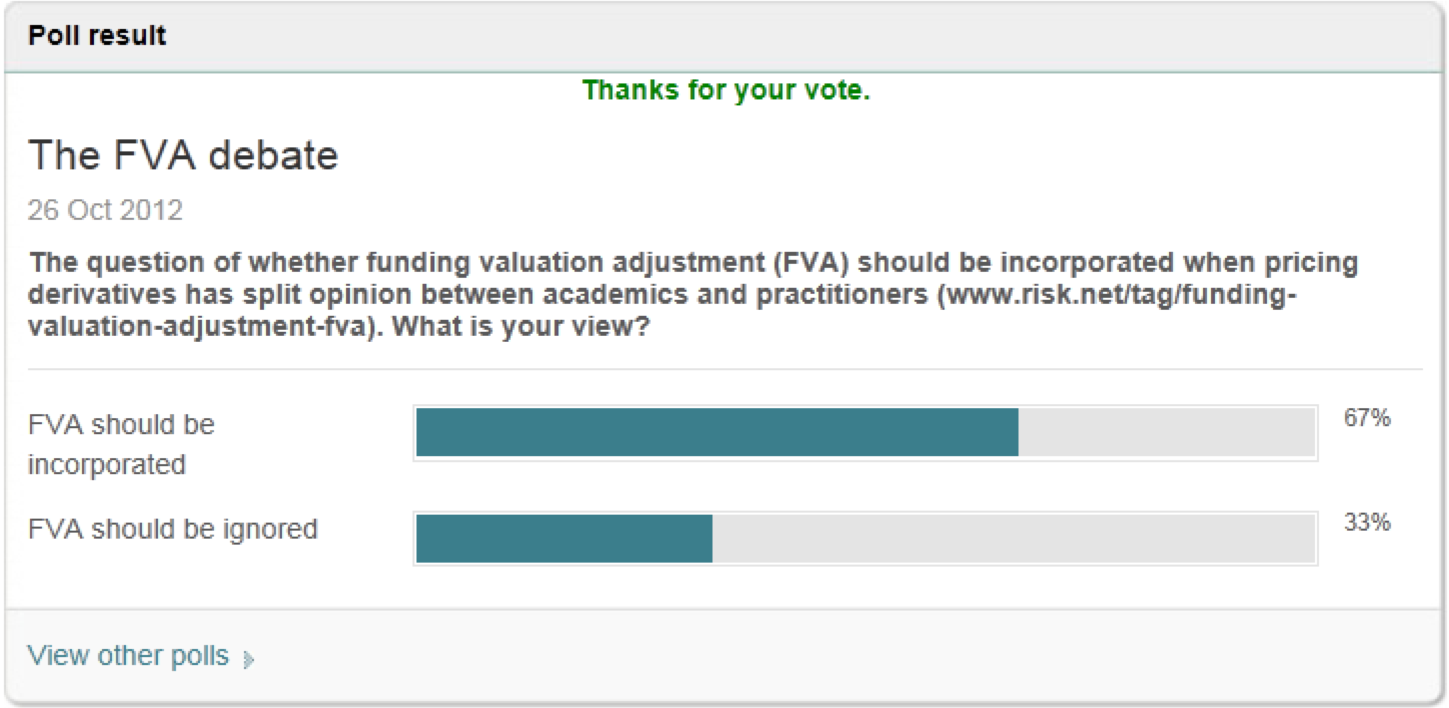
The question of whether funding valuation adjustment (FVA) should be incorporated when pricing derivatives has split opinion between academics and practitioners (www.risk.net/tag/funding-valuation-adjustment-fva). What is your view?

○ FVA should be incorporated

○ FVA should be ignored

The Funding Value Adjustment is topic of a heated debate

The result of the poll…



Poll result

Thanks for your vote.

The FVA debate

26 Oct 2012

The question of whether funding valuation adjustment (FVA) should be incorporated when pricing derivatives has split opinion between academics and practitioners (www.risk.net/tag/funding-valuation-adjustment-fva). What is your view?

FVA should be incorporated	67%
FVA should be ignored	33%

View other polls ›

The aim of this presentation is to understand both positions

What is the Funding Value Adjustment (FVA)?

Many banks cannot borrow (unsecured) at the risk free rate, since the crisis. This may be reflected in the valuation of derivatives.

⇩

The resulting adjustment is called FVA.

Pre-crisis: bank funding obtained at Libor rate

Pre - crisis

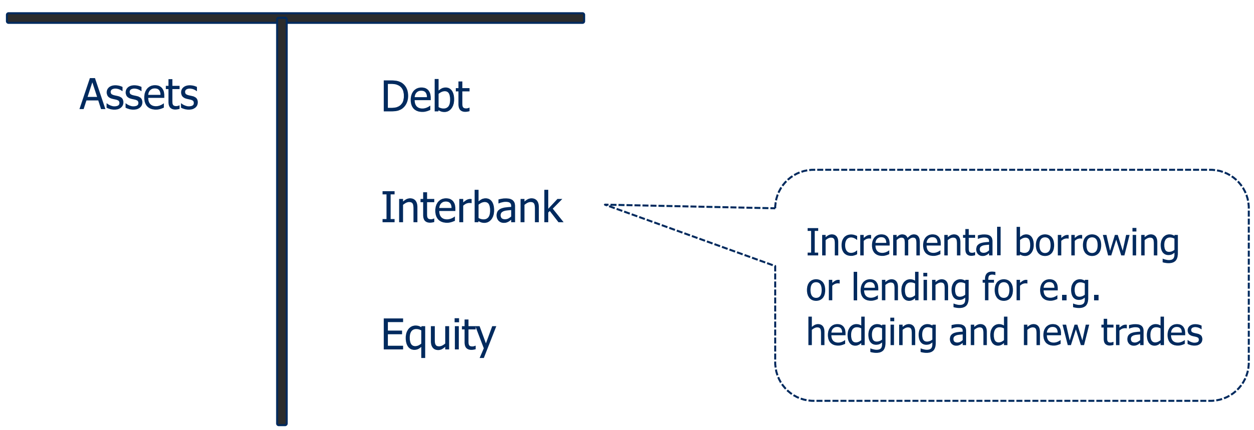
Interbank rates (OIS,1M,3M,…) almost equal

+

Funding could be obtained at interbank rates (LIBOR)

=

Risk-free rate to be used in replication is LIBOR

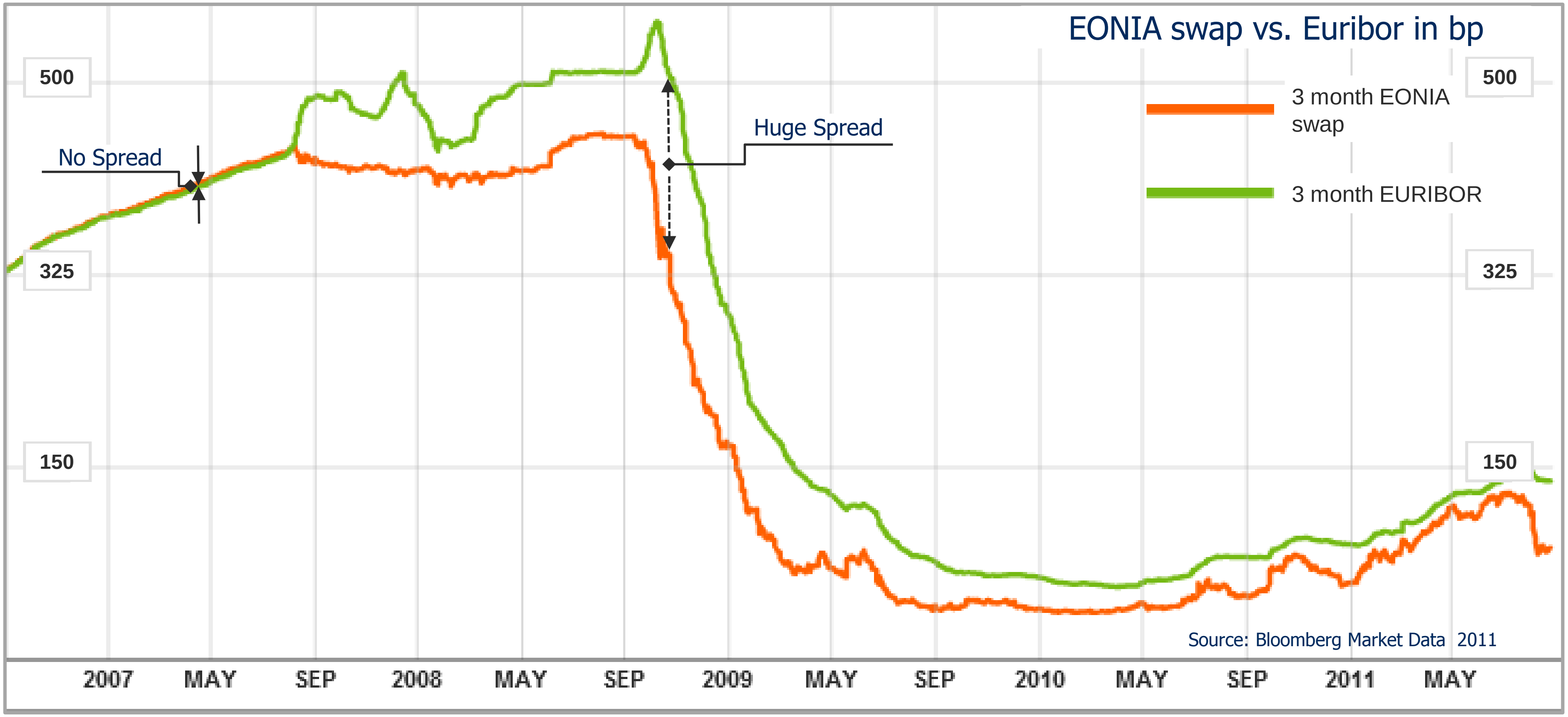


Incremental borrowing or lending for e.g. hedging and new trades

Basis Spreads before, during and after crisis

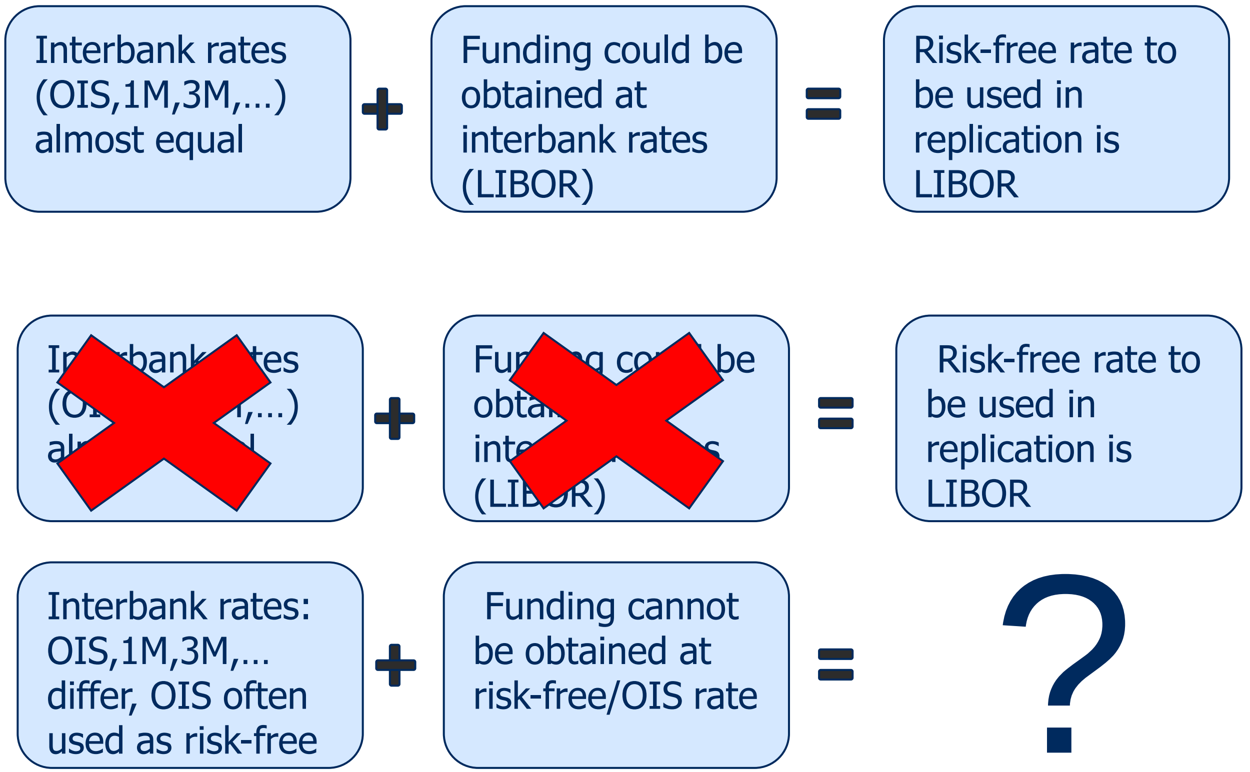
3M EONIA (overnight) swap curve and 3M EURIBOR

Same trend as LIBOR – OIS; spread picked at 194 bp in October 2008



Source: Bloomberg Market Data 2011

Post-crisis: bank funding more expensive



Pre crisis

Interbank rates (OIS,1M,3M,…) almost equal + Funding could be obtained at interbank rates (LIBOR) = Risk-free rate to be used in replication is LIBOR

Post crisis

~~In rbank tes (O ,…) al l~~ + ~~Fun g co l be obta inte s (LIB R)~~ = Risk-free rate to be used in replication is LIBOR

Interbank rates: OIS,1M,3M,… differ, OIS often used as risk-free + Funding cannot be obtained at risk-free/OIS rate = ?

Question: how to include the higher (than risk-free) Funding Costs in Derivatives pricing

Pre-crisis: Black and Scholes derivation

- Buy option and set-up Delta hedge, and borrow/lend required cash (on MM account)
  - Option: V
  - Hedge: $-\Delta S$
  - MM account: $-V + \Delta S$ accrues at risk-free/interbank rate r

$$dV - \Delta dS + r(-V + \Delta S)dt = 0$$

$$\Downarrow$$

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV$$

Post-crisis: Black and Scholes including funding costs

- Buy option and set-up Delta hedge, and borrow/lend required cash (on MM account)
  - Option: V
  - Hedge: $-\Delta S$
  - MM account: $-V + \Delta S$ accrues at the funding rate r_F

$$dV - \Delta dS + r_F(-V + \Delta S)dt = 0$$

$$\Downarrow$$

$$\frac{\partial V}{\partial t} + r_F S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = r_F V$$

Hidden assumption: inelastic Funding Costs

However there is an important (hidden) assumption made, when funding costs are included in this way

Inelastic Funding assumption:
Funding costs are fixed. They do not change when new transactions are added.

Although this assumption may seem reasonable, funding costs of a bank do not change (much) when a transaction is added, it is also clear that adding many bad assets to a balance sheet of a bank will increase the funding costs.

The opposite assumption: elastic funding costs

To investigate the impact of the assumption on the funding costs, consider the opposite assumption: Elastic funding assumption.

Elastic Funding assumption:
Funding costs adjust immediately to new transactions and other changes in the asset composition.

Elastic Funding Assumption: Zero-Coupon bond

- Simple Balance sheet: ZC bond funded by equity

ZC_1	E

- Funding rate r_F determined by:

$$\mathbb{E}[E(T)] = e^{rT} E(0)$$

$$(1 - PD_1) E(0) e^{r_F T} + PD_1 \times 0 = e^{rT} E(0)$$

$$e^{r_F T} = \frac{e^{rT}}{1 - PD_1}$$

Elastic Funding Assumption: Zero-Coupon bond

- Simple Balance sheet: ZC bond funded by equity

ZC_1	E

- Fair spread on the ZC bond s_1 is determined by:

$$\mathbb{E}[ZC_1(T)] = \mathbb{E}[E(T)]$$

$$(1 - PD_1)ZC_1(0)e^{(r+s_1)T} + PD_1 \times 0 = (1 - PD_1)E(0)e^{r_F T}$$
$$e^{(r+s_1)T} = e^{r_F T}$$

$$e^{s_1 T} = \frac{1}{1 - PD_1}$$

Note that the fair spread is determined by the PD alone, without additional funding spread

Elastic Funding Assumption: Zero-Coupon bond

- Add a second ZC bond

ZC_1	E_1
ZC_2	E_2

- The new Funding rate r_F is determined by:

$$\mathbb{E}[E_1(T) + E_2(T)] = e^{rT}(E_1(0) + E_2(0))$$

$$\begin{gathered}(1 - PD_1 - PD_2 + PD_{12})(E_1(0) + E_2(0))e^{r_F T} + \\ (PD_1 - PD_{12})E_2(0)e^{r_F T} + \\ (PD_2 - PD_{12})E_1(0)e^{r_F T} + \\ (PD_1 + PD_{12}) \times 0 \\ = e^{rT}(E_1(0) + E_2(0))\end{gathered}$$

Elastic Funding Assumption: Black-Scholes

- The result for the funding rate is:

$$e^{r_F T} = e^{rT} \frac{ZC_1(0) + ZC_2(0)}{(1 - PD_1)ZC_1(0) + (1 - PD_2)ZC_2(0)}$$

- This may be compared to the old Funding rate

$$e^{r_F{}^{old} T} = e^{rT} \frac{1}{1 - PD_1}$$

Note that:

If $PD_2 < PD_1 \rightarrow \quad r_F < r_F{}^{old}$

If $PD_2 > PD_1 \rightarrow \quad r_F > r_F{}^{old}$

Elastic Funding Assumption: Zero-Coupon bond

- Although it seems that adding a new transaction to a large balance sheet ($ZC_1 \gg ZC_2$) will not change the funding rate much, it is still sufficient to make the fair spread independent of the funding rate

The fair spread s_2 is determined by:

$$\mathrm{E}[ZC_1(T) + ZC_2(T)] = \mathrm{E}[E_1(T) + E_2(T)]$$

$$\begin{gathered}(1 - PD_1 - PD_2 + PD_{12})(ZC_1(0)e^{(r+s_1)T} + ZC_2(0)e^{(r+s_2)T}) + \\ (PD_1 - PD_{12})ZC_2(0)e^{(r+s_2)T} + \\ (PD_2 - PD_{12})ZC_1(0)e^{(r+s_1)T} + \\ (PD_1 - PD_{12}) \times 0 \\ = e^{rT}(ZC_1(0) + ZC_2(0))\end{gathered}$$

Elastic Funding Assumption: Zero-Coupon bond

- The result for the fair spread is:

$$e^{s_2 T} = \frac{1}{1 - PD_2}$$

Note again that the fair spread is determined by the PD alone, without additional funding spread

- This may be compared to the result under the inelastic assumption

$$e^{s_2 T} = e^{(r_F{}^{old} - r)T} \frac{1}{1 - PD_1}$$

Pre-crisis: Black and Scholes derivation

- Buy option and set-up Delta hedge, and borrow/lend required cash (on MM account)
  - Option: V
  - Hedge: $-\Delta S$
  - MM account: $-V + \Delta S$

Since option + hedge is equivalent to a risk-free ZC bond on a each infinitesimal time interval t to t+dt, the fair spread is zero. The MM account fair rate is the risk-free rate r_{rf}

$$dV - \Delta dS + r_{rf}(-V + \Delta S)dt = 0$$

$$\Downarrow$$

$$\frac{\partial V}{\partial t} + r_{rf} S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = r_{rf} V$$

Under the elastic assumption the Funding Costs do not affect the value of derivatives

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- Inelastic assumption $\frac{\partial V}{\partial t} + r_F S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = r_F V$

- Elastic assumption $\frac{\partial V}{\partial t} + r_{rf} S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = r_{rf} V$

The FVA depends on the inelastic assumption. Under the elastic assumption the FVA=0.

Interpretation Elastic versus Inelastic

Inelastic — **reality** — **Elastic**

Inelastic

- investors do not have full insight in assets
- funding is mostly not ON

Elastic

- adding bad assets will increase funding costs

Note that the elastic assumption is somewhat similar to the assumption of continuous hedging, it cannot be realized exactly in reality, but nevertheless provides a fair value.

Arguments in favour of the Elastic and Inelastic assumption

Inelastic Funding

- Even would funding costs adjust, most funding is not ON (if only for liquidity risk). Therefore most funding cannot react to (new) transactions.
- For a trading desk the funding costs are simply given, and do not change when the desk does a transaction.
- Funding costs are not (solely) determined by asset composition, but also by other factors such as government support and liquidity.

Elastic Funding

- Funding costs do increase in the long term when the quality of the assets of the bank deteriorates.
- The elastic funding allows for the correct/consistent valuation of tradeable credit bonds.
- It allows for banks to invest in other banks (without adding funding costs twice).
- The elastic funding assumption sets the right incentives for trading/management decisions.

Two main points

The FVA is a consequence of the assumption of inelastic funding. Under the elastic funding assumption the FVA=0.

In my view the elastic assumption is preferred, since it leads to consistent valuation (e.g. of credit bonds) and better incentives for trading.

Let’s discuss!

More information can be found here

This presentation is based on

- On Funding Costs and Valuation of Derivatives, Bert-Jan Nauta, http://ssrn.com/abstract=2143979

Papers that explore the FVA under the inelastic assumption

- Funding Valuation Adjustment: a consistent framework including CVA, DVA, collateral, netting rules and re-hypothecation, Pallavicini, A., D. Perini, and D. Brigo, http://ssrn.com/abstract=1969114
- Funding beyond discounting: collateral agreements and derivatives pricing, V. Piterbarg, Risk magazine, Feb. 2010.
- Partial Differential Equation Representations of Derivatives with Bilateral Counterparty Risk and Funding Costs, C. Burgard and M. Kjaer, http://ssrn.com/abstract=1605307
- Funding, Liquidity, Credit and Counterparty Risk: Links and Implications, A. Castagna, http://ssrn.com/abstract=1605307

More information can be found here

Articles in the FVA debate

- The FVA debate, J. Hull and A. White, Risk magazine, Aug. 2012
- In defence of FVA - a response to Hull and White, S. Laughton and A. Vaisbrot, www.risk.net
- Yes, FVA is a Cost for Derivatives Desks - A Note on 'Is FVA a Cost for Derivatives Desks?' by Prof. Hull and Prof. White, A. Castagna, http://ssrn.com/abstract=2141663
- The FVA debate continues: Hull and White respond to their critics, J. Hull and A. White, Risk magazine, Oct. 2012