



The Ultimate Forward Rate - Background, Issues and Impact

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Presentation builds on work in the following two papers:

- ▶ KOCKEN, T., OLDENKAMP, B. AND J. POTTERS (2012). “An alternative model for extrapolation”, *Insurance Risk*, September 2012.
- ▶ KOCKEN, T., OLDENKAMP, B. AND J. POTTERS (2012). “Dangerous design flaws in the Ultimate Forward Rate: The impact on risk, stakeholders and hedging costs”, working paper.

All papers available at <http://www.cardano.com>.

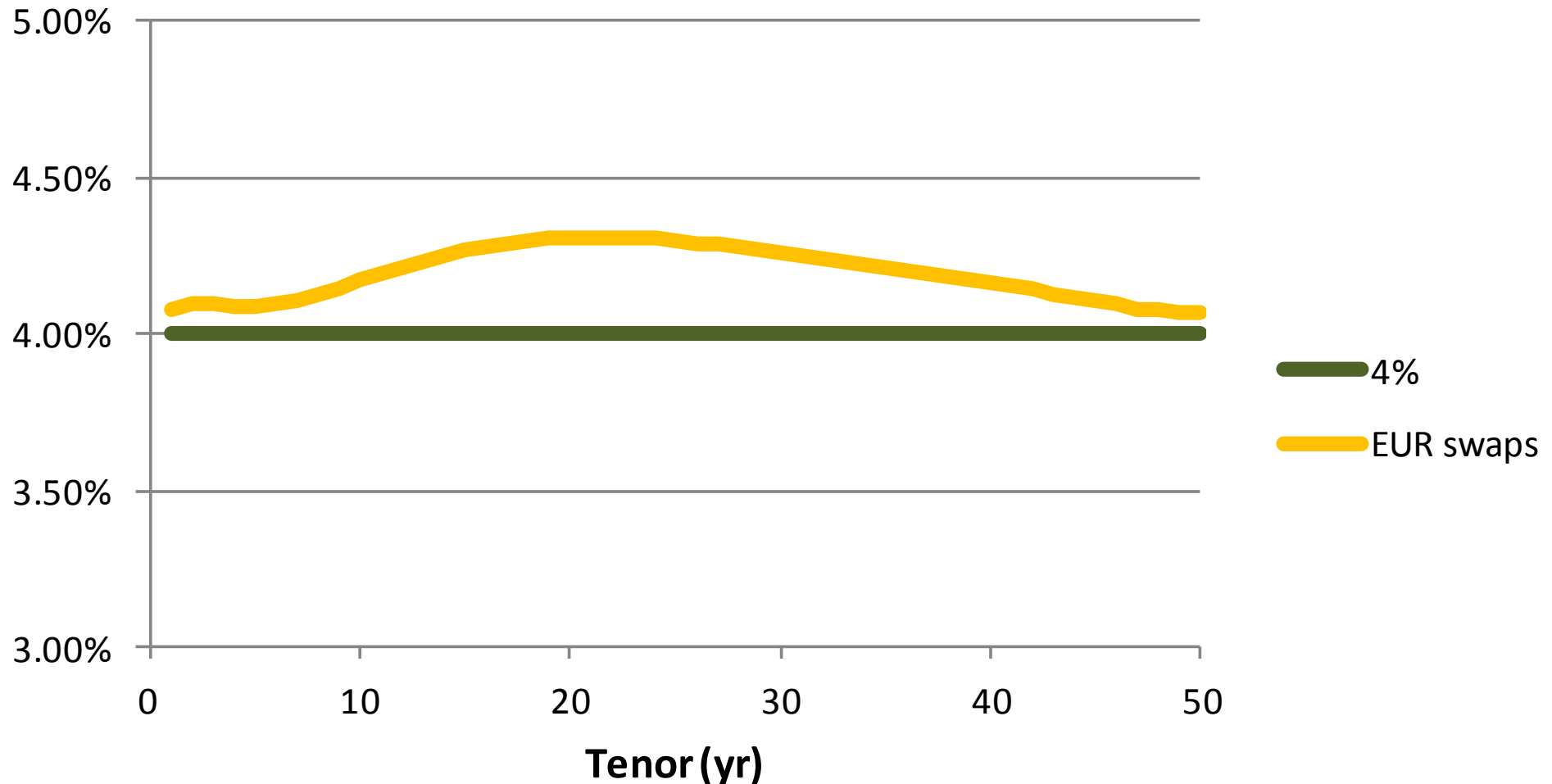
These slides and future paper will also be available via <http://www.rogerlord.com>.

- ▶ Ultimate Forward Rate – what is it?
- ▶ The Smith-Wilson curve
- ▶ Objections to the UFR
- ▶ Intermezzo: robust pricing
- ▶ Hedge implications
- ▶ Alternatives
- ▶ Conclusions



- ▶ Prior to 2007 Dutch pension funds among others had to discount liabilities at a fixed rate of 4%
- ▶ With the advent of the Financial Assessment Framework (FTK) in 2007, the market swap curve was to be used for discounting
- ▶ Market valuation made it transparent that interest rate risk is a serious, and often dominant source of risk
- ▶ This, and similar developments in other countries, led to a marked increase in the usage of interest rate derivatives among pension funds

Switch to market consistent valuation in 2007 (NL)



Source: Cardano, zero yield curve shown as of 2nd of January 2007

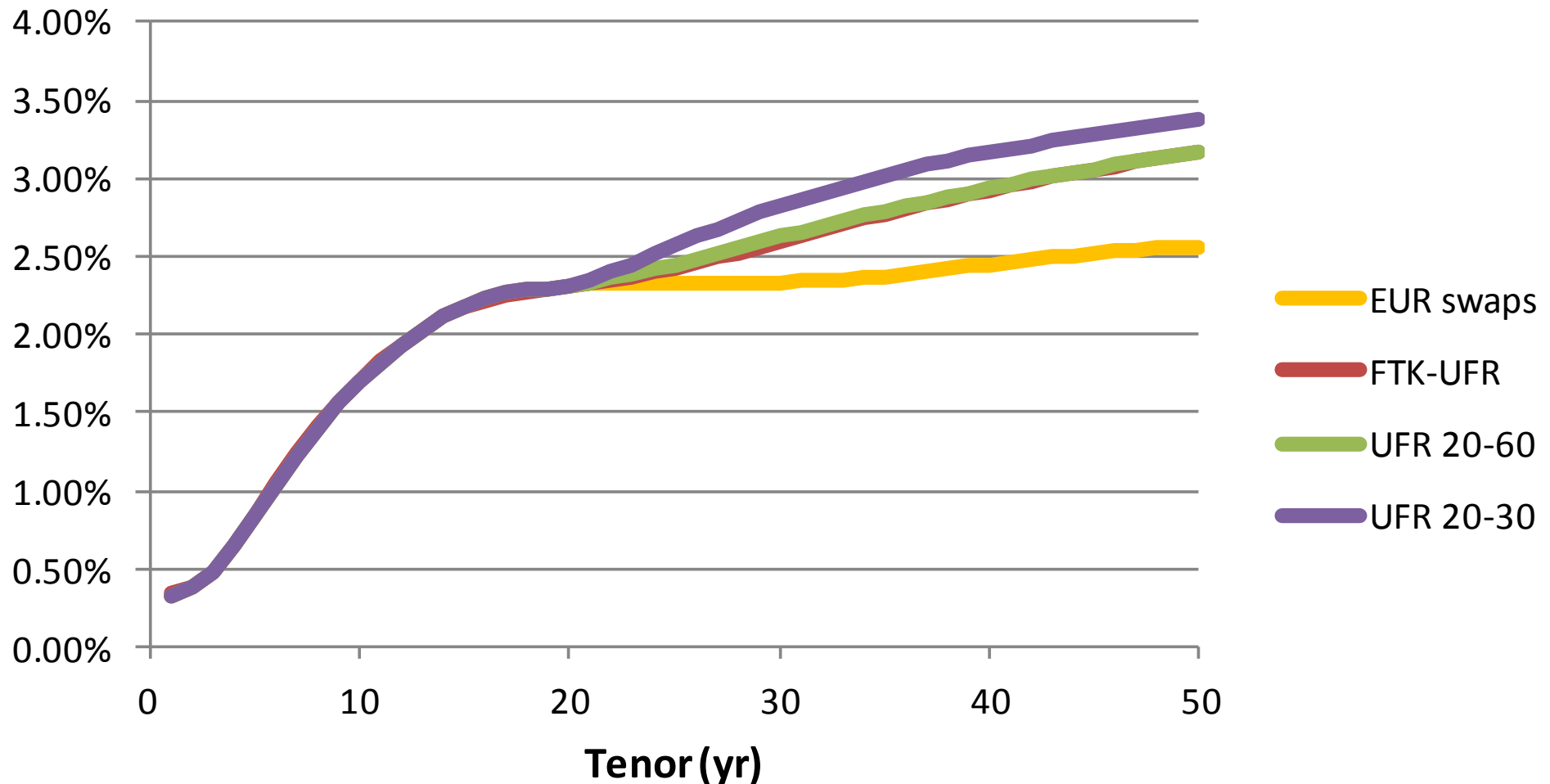
- ▶ Under Solvency II (Basel III for insurers) a method has been proposed where forward rates tend to a fixed forward rate (Ultimate Forward Rate, UFR)

- ▶ Reasons quoted for this have been:
 - ▶ A consistent approach to extrapolation has to be used
 - ▶ No market prices exist after a certain point (e.g. > 60y)
 - ▶ Transaction volumes for long-dated swaps are much lower than for maturities up to and including 30y
 - ▶ Unofficially: a fully market consistent approach would cause some major headaches for insurers...

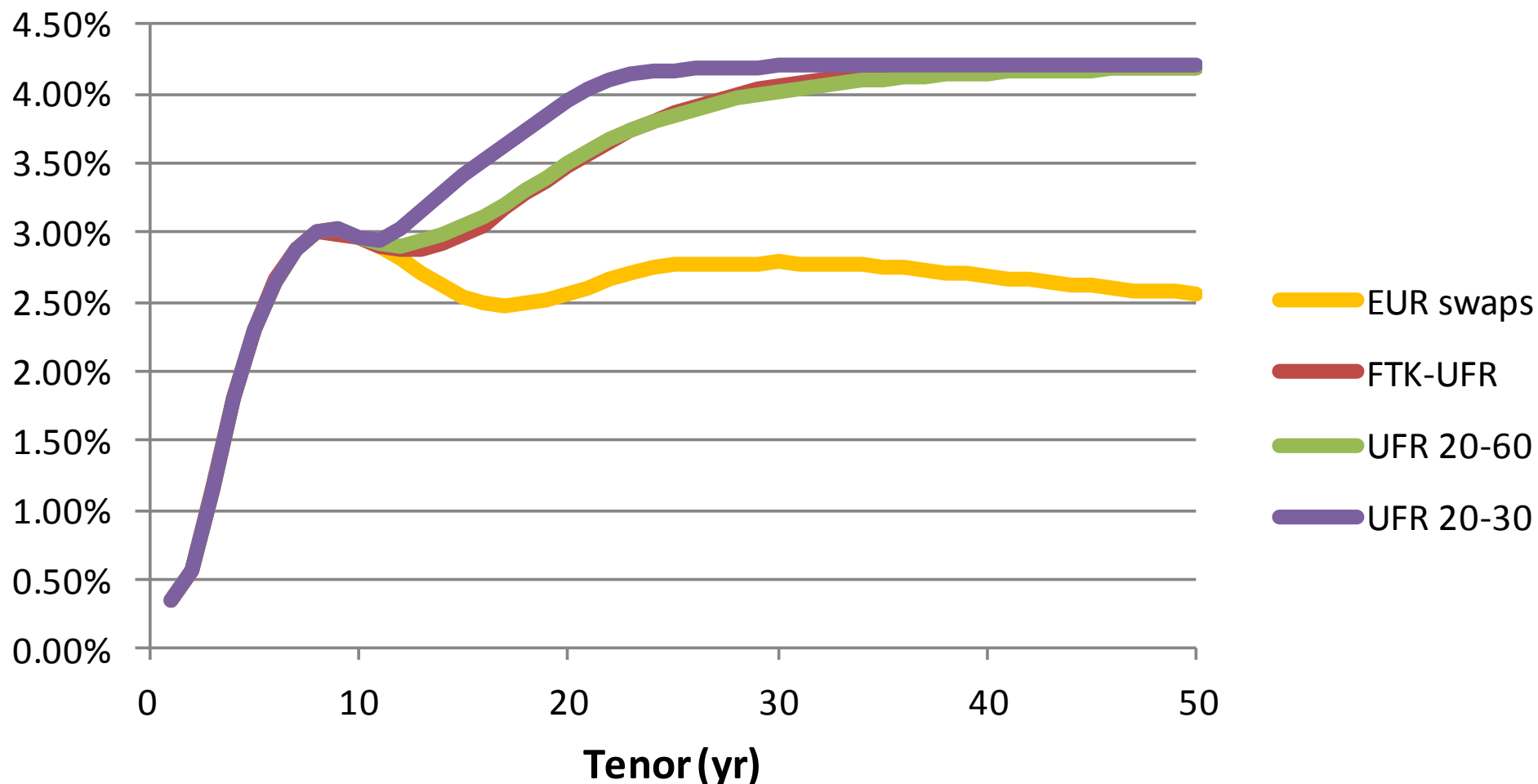
- ▶ The proposed approach in QIS5 is to:
 - ▶ Match liquid part of curve up to last liquid point (LLP)
 - ▶ Extrapolate using Smith-Wilson parametric curve, where the curve is reparameterised in terms of convergence
 - ▶ Fix the UFR as sum of a long-term real yield and inflation rate (2% and 2.2% for EUR, leading to a UFR of 4.2%)
 - ▶ Exact parameters (UFR, LLP, convergence period) are still being debated

- ▶ Although Solvency II will only come into effect as of 2014:
 - ▶ *12th of June 2012*: Danish regulator has chosen to impose the UFR 20-30 curve for pension funds and insurers;
 - ▶ *2nd of July 2012*: Dutch regulator imposes UFR 20-60 for insurers;
 - ▶ *3rd of October 2012*: Dutch regulator imposes altered UFR 20-60 approach (FTK-UFR) for pension funds;

Switch to variations of UFR curves in 2012



Switch to variations of UFR curves in 2012



The Smith-Wilson curve, see Smith and Wilson [2000], was constructed as a method where:

- ▶ an arbitrary number of quotes can be fit exactly
- ▶ the limiting forward rate is an input

Its original incantation specified the discount factor as:

$$DF(t) = \frac{1}{(1+UFR)^t} + \sum_{i=1}^N \zeta_i W(t, u_i)$$

with the spine points u_1 through u_N , and ζ_1 through ζ_N being parameters through which we can calibrate to our input instruments.

The symmetric Wilson functions are given by:

$$W(s, t) = \frac{1}{(1 + \text{UFR})^{t+s}} \cdot \left\{ \alpha \cdot \min(s, t) - \frac{1}{2} e^{-\alpha \max(s, t)} \cdot \left(e^{\alpha \min(s, t)} - e^{-\alpha \min(s, t)} \right) \right\}$$

In their original paper Smith and Wilson demonstrate that the proposed discount function minimises:

$$\frac{1}{2\alpha^3} \int_0^\infty \left(\frac{\partial^2}{\partial t^2} f(t) \right)^2 dt + \frac{1}{2\alpha} \int_0^\infty \left(\frac{\partial}{\partial t} f(t) \right)^2 dt$$

where $f(t) = (1 + \text{UFR})^t \cdot \text{DF}(t)$.

The objective function combines smoothness (first part) with ensuring that the forward rate is close to the UFR (second part). The speed of convergence α plays a role akin to mean reversion. 12

Fitting the Smith-Wilson curve to zero-coupon bonds at maturities u_1 through u_N with market prices m_1 through m_N can easily be done as it leads to the following system of equations:

$$\underline{m} = \underline{\mu} + W \underline{\zeta}$$

where $\mu_i = \frac{1}{(1+UFR)^{u_i}}$, and $W_{ij} = W(u_i, u_j)$. Hence we can solve:

$$\underline{\zeta} = W^{-1}(\underline{m} - \underline{\mu})$$

This can easily be extended to any type of vanilla interest rate instrument if we view it as an affine combination of discount factors, as one typically does when bootstrapping a curve.

In QIS5 the Smith-Wilson curve is reparameterised in terms of the convergence period (T) to the UFR. Specifically:

- ▶ Convergence speed α starts at an initial value of 0.1
- ▶ Is increased by 0.001 until the (T-1) x 1 year forward is within 3 basis points (ε) of the UFR

We can speed up the search procedure by defining:

$$f(x) = \frac{DF(T-1)}{DF(T)} - 1 \Bigg|_{\alpha=x}$$

and finding the root (if $\alpha = 0.1$ is not satisfactory) of:

$$g(x) = (f(x) - \varepsilon)(f(x) + \varepsilon)$$

Many papers mention that the Wilson function can be interpreted as the covariance function of an integrated Ornstein-Uhlenbeck (IOU) process. Let us consider an OU process:

$$dX(t) = -\alpha X(t) dt + \sigma dW(t)$$

with the following solution:

$$X(t) = e^{-\alpha t} X(0) + \sigma \int_0^t e^{-\alpha(t-u)} dW(u)$$

It has the following autocovariance function if X is stationary:

$$\lim_{t+s \rightarrow \infty} \text{Cov}(X(t), X(s)) = \lim_{t+s \rightarrow \infty} \frac{\sigma^2}{2\alpha} \left(e^{-\alpha|t-s|} - e^{-\alpha(t+s)} \right) = \frac{\sigma^2}{2\alpha} e^{-\alpha|t-s|}$$

We can define the IOU process as:

$$Y(t) = \int_0^t X(u) du$$

leading to the following autocovariance function in case X is stationary:

$$\begin{aligned} \text{Cov}(Y(t), Y(s)) &= \int_0^t \int_0^s \text{Cov}(X(u), X(v)) dv du \\ &= \frac{\sigma^2}{2a^3} \left(2\alpha \cdot \min(t, s) - 1 + e^{-\alpha t} + e^{-\alpha s} - e^{-\alpha|t-s|} \right) \\ &\propto (1 + \text{UFR})^{t+s} \cdot W(t, s) - \frac{1}{2} \left(1 + e^{-\alpha t} + e^{-\alpha s} + e^{-\alpha(t+s)} \right) \end{aligned}$$

so unfortunately we cannot directly use a probabilistic interpretation to figure out properties of the discount factor.

We can however analyse the analytical properties of the discount function. An important question to ask is: can we obtain negative discount factors (in our domain of interest)?

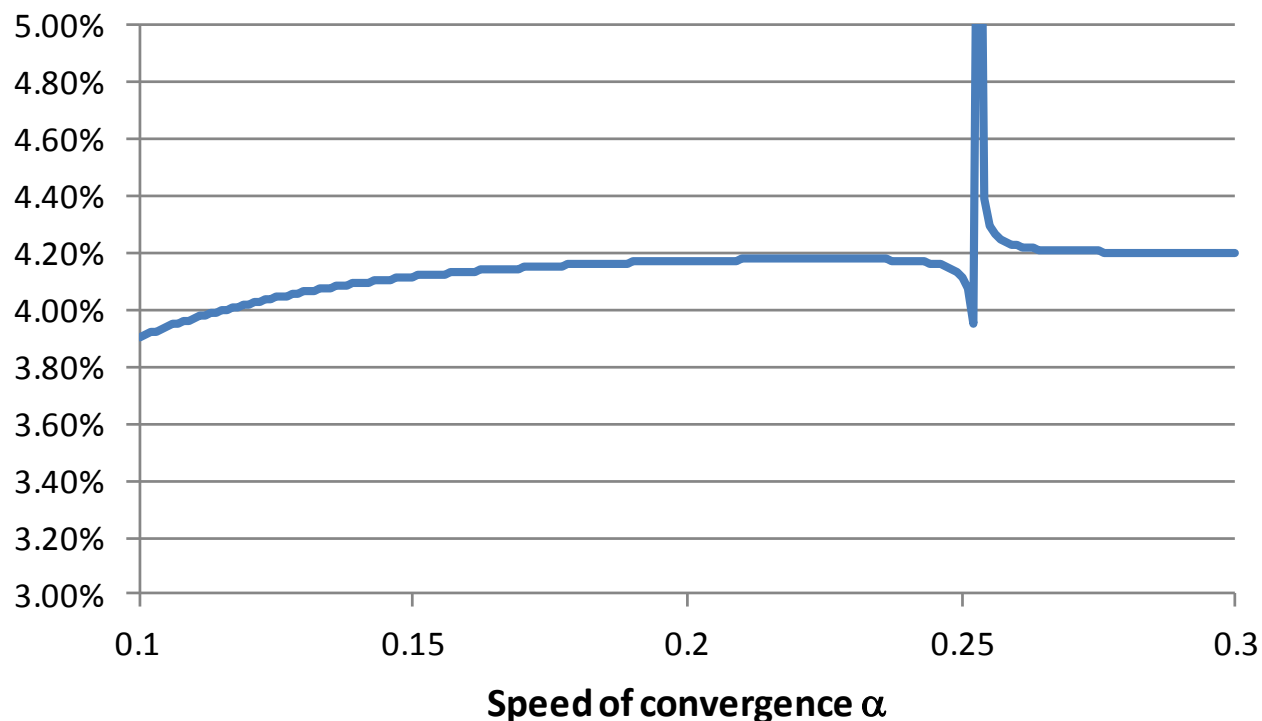
If we define $ufr = \ln(1+UFR)$ we can write $DF(t)$ as:

$$DF(t) = Ae^{-ufr \cdot t} + Be^{-(ufr + \alpha)t} = e^{-ufr \cdot t} (A + Be^{-\alpha t})$$

Since $DF(LLP) > 0$, only if $A < 0$ and $B > 0$ will $DF(t)$ become negative for some $t > LLP$.

Can this be a problem if we start at $\alpha = 0.1$ and increment it until we converge to within 3 bp of the UFR?

Indeed it can... consider 15y and 20y swap rates at 4.2% and 6.3%, a UFR of 4.2%, LLP of 20y and convergence period of 40y.



With the proposed algorithm we would find $\alpha = 0.197$, whereas that would lead to 27y discount factors already being negative.

Luckily that is an extreme example. We can express the instantaneous forward rate as:

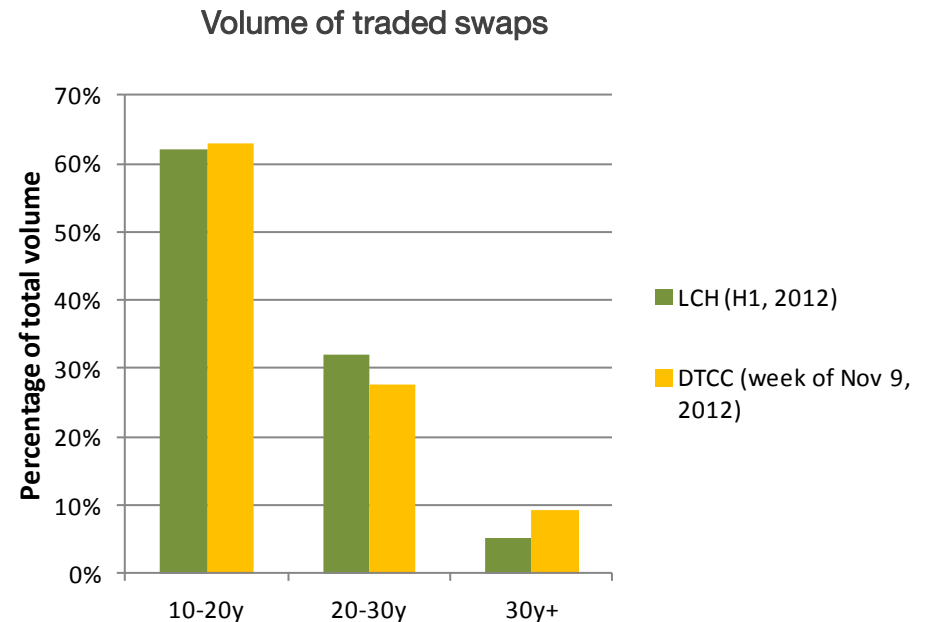
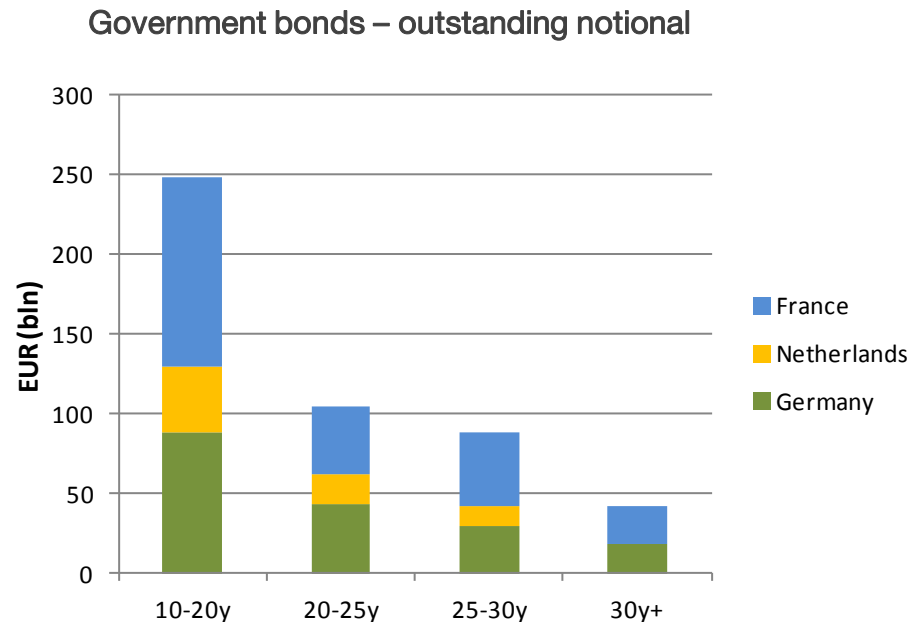
$$-\frac{\partial \ln DF(t)}{\partial t} = \text{ufr} + \alpha \cdot \frac{Be^{-(\text{ufr} + \alpha)t}}{DF(t)}$$

Since we can only end up in this situation if $B > 0$ and naturally $DF(\text{LLP}) > 0$, it is clear that:

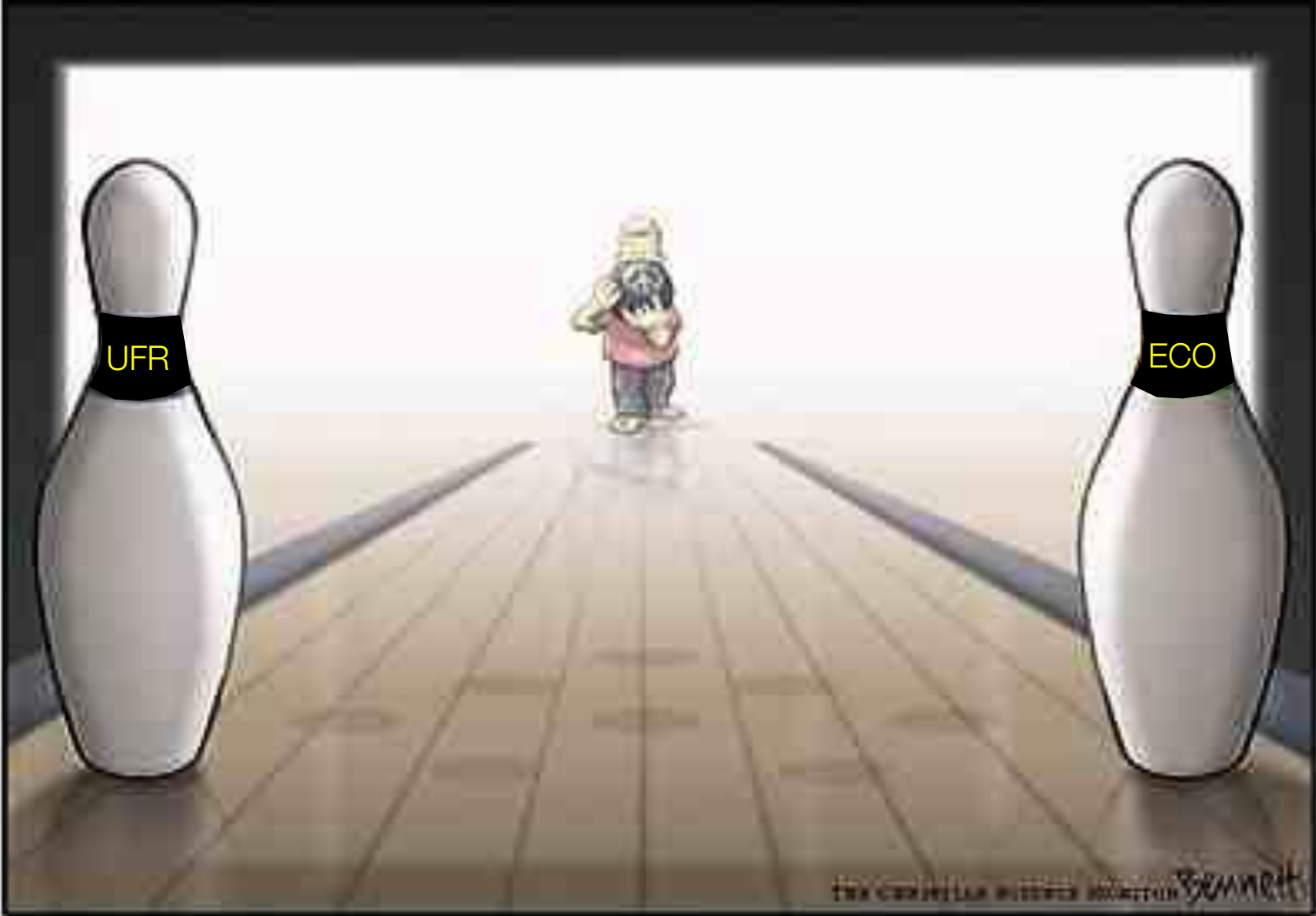
$$-\frac{\partial \ln DF(t)}{\partial t} \Big|_{t=\text{LLP}} > \text{ufr} + \alpha$$

which at least at present in EUR is not an issue. This condition is mentioned in Alexander and Smith [2012] without proof.

Is the market really that illiquid for long-dated transactions?



An LLP of 30y seems quite defensible. Moreover, bid/ask spreads for long-dated swaps are not such that we consider it impossible to trade at these maturities.



Introduces inconsistency in the balance sheet

Assets

Thank you
interest
rate



Liabilities

Naughty
interest
rate



Summarising:

- ▶ Enough liquidity between 20 – 30 years maturity
- ▶ Dilemma of economic versus regulatory hedging
- ▶ Inconsistency between both sides of the balance sheet
- ▶ Transfer of money from young participants to the older ones
- ▶ Controversial choice of 4.2% - how will UFR be updated?

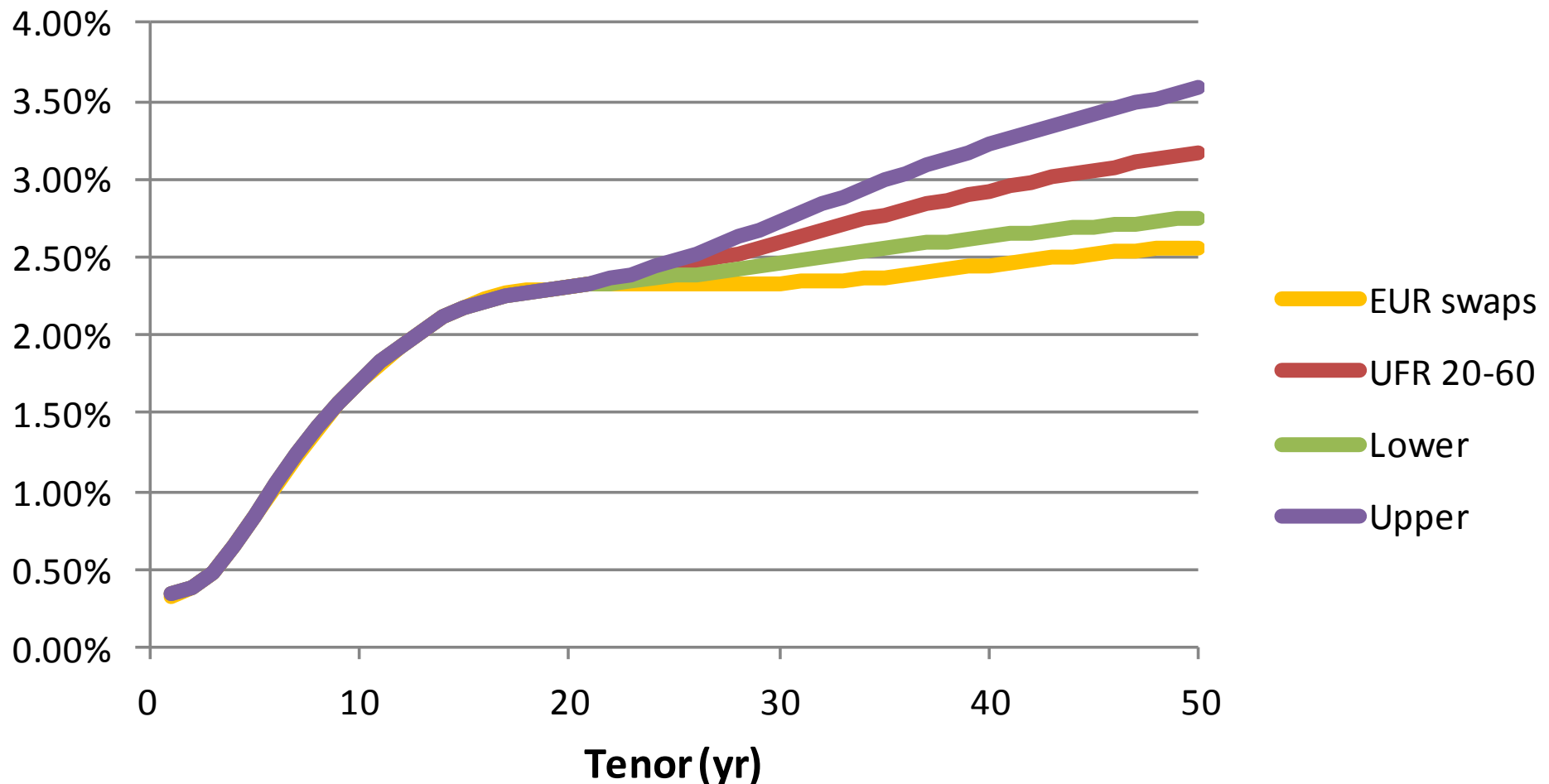
Thinking of the TopQuants Spring keynote by professor Pelsser [2012], robust pricing could have been an alternative: taking into account the uncertainty in the estimate of the UFR.

If we assume the UFR is in $[UFR - 2 * \sigma, UFR + 2 * \sigma]$, robust pricing would value a discount factor as:

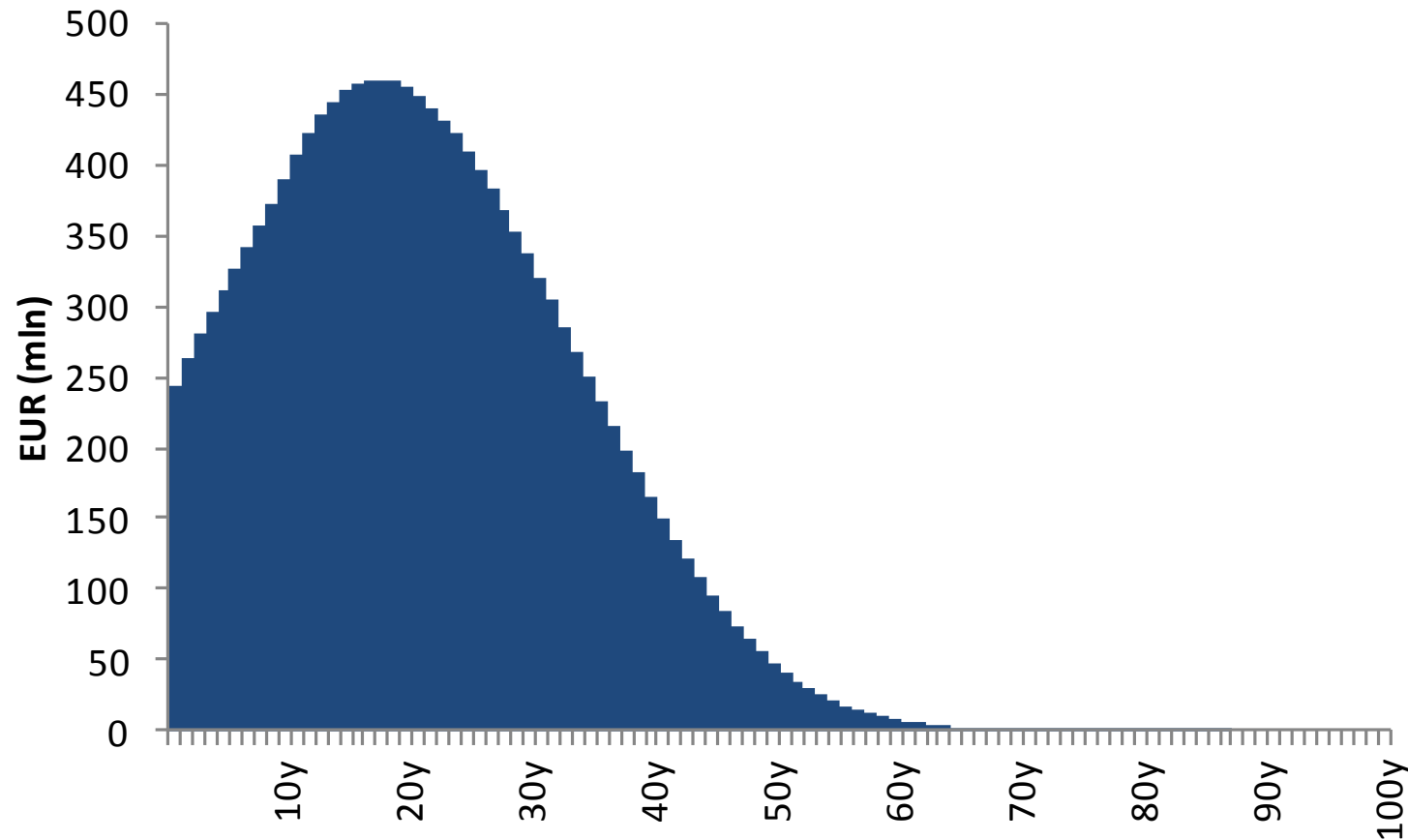
$$DF(t) = \min_{UFR \in [UFR - 2\sigma, UFR + 2\sigma]} E \left[\exp \left(- \int_0^t r(u) du \right) \right]$$

Alternatively, bid/ask spreads could have been used as a measure of liquidity.

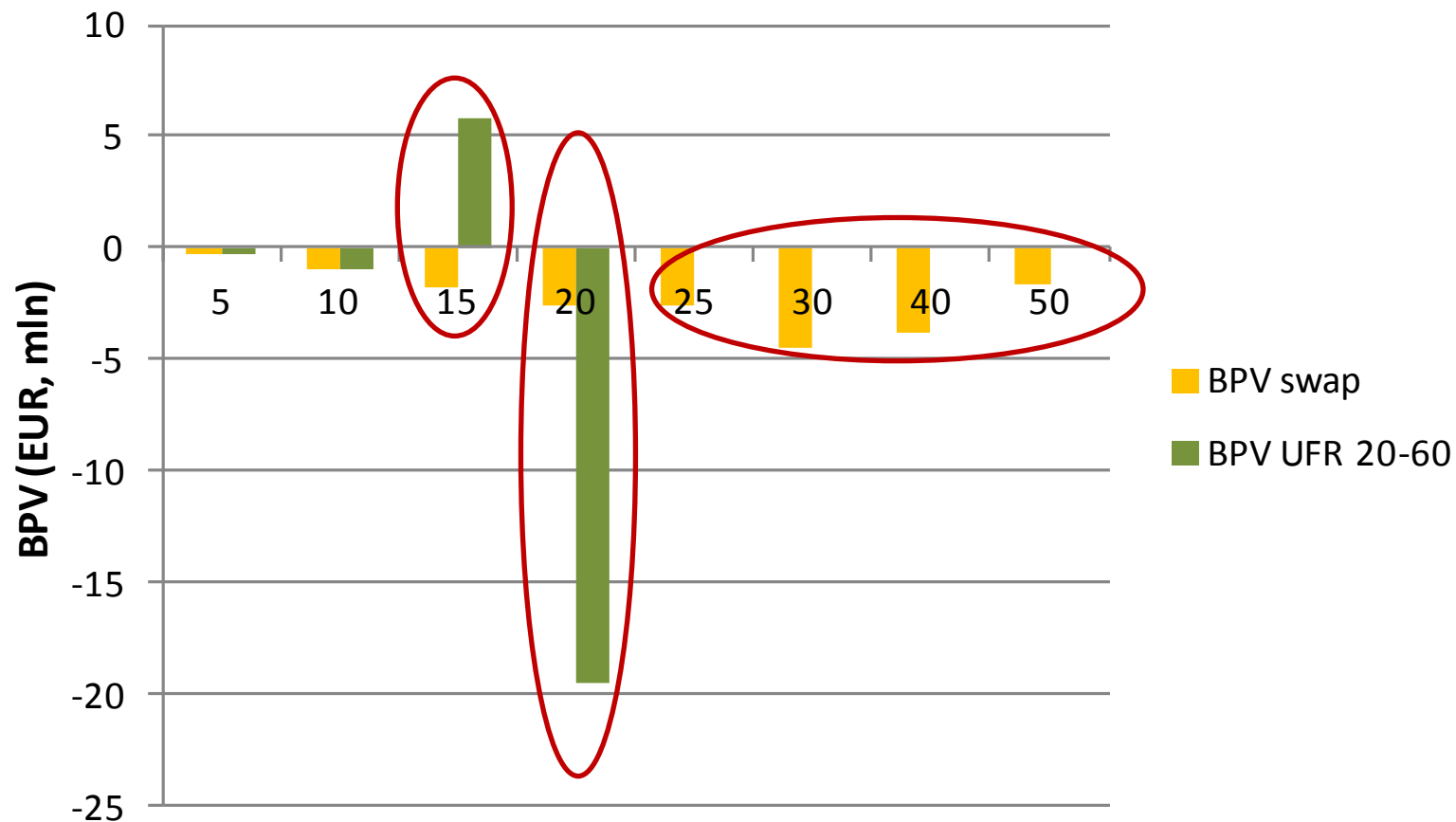
Using daily yield curves since 2005 to proxy the variability of the UFR by the variability of the 49x1 forward rate, leads to:



But there is more. Consider the liabilities of an average DB pension fund...



and the hedge implications of a switch to UFR 20-60...



This could have potentially disastrous consequences!

There has already been some visible market impact:



Source: Bloomberg

12/06/2012: Danish regulator imposes UFR

03/10/2012: Dutch regulator imposes UFR on pension funds

02/07/2012: Dutch regulator imposes UFR on insurers

Why is this happening? Easier to consider in zero rate space.
First start with a regular zero curve:

$$DF(i) = \frac{1}{(1 + z_i)^i}$$

Then clearly there is no spill-over:

$$\frac{\partial DF(i)}{\partial z_j} = 0 \quad \text{for } j \neq i$$

Now if we switch to the Solvency II UFR curve, we roughly have:

$$\begin{aligned} DF(i) &= DF(LLP) \cdot \prod_{j=LLP+1}^i \frac{1}{1 + w_j \cdot UFR + (1 - w_j) \cdot f_{LLP-1,LLP}} \\ &= DF(LLP) \cdot \prod_{j=LLP+1}^i DF_{j,j+1} \end{aligned}$$

where the forward rates are defined as:

$$f_{i,i+1} = \frac{DF(i)}{DF(i+1)} - 1$$

The interesting sensitivities are:

$$\frac{\partial \text{DF} (i)}{\partial z_{\text{LLP}-1}} = \text{DF} (i) \cdot \sum_{j=\text{LLP}+1}^N \frac{1}{\text{DF}_{j,j+1}} \cdot \left| \frac{\partial \text{DF}_{j,j+1}}{\partial z_{\text{LLP}-1}} \right|$$

$$\frac{\partial \text{DF} (i)}{\partial z_{\text{LLP}}} = \text{DF} (i) \cdot \frac{1}{\text{DF} (\text{LLP})} \cdot \left| \frac{\partial \text{DF} (\text{LLP})}{\partial z_{\text{LLP}}} \right|$$

$$- \text{DF} (i) \cdot \sum_{j=\text{LLP}+1}^N \frac{1}{\text{DF}_{j,j+1}} \cdot \left| \frac{\partial \text{DF}_{j,j+1}}{\partial z_{\text{LLP}}} \right|$$

Since all rates after the LLP depend on the (LLP-1) x 1 year forward, large offsetting delta's are being created.

Cardano (and many well-known academics) have been lobbying for an alternative extrapolation method that does not have similar disastrous consequences as the UFR.

Cardano's proposal boils down to:

$$\begin{aligned} DF(i) &= DF(LLP) \cdot \prod_{j=LLP+1}^i \frac{1}{1 + w_j \cdot UFR + (1 - w_j) \cdot f_{j-1,j}^{\text{market}}} \\ &= DF(LLP) \cdot \prod_{j=LLP+1}^i DF_{j,j+1} \end{aligned}$$

The original objective was to ameliorate the consequences, whilst staying as close to the SW curve as possible. Illiquidity is taken into account by moving towards the UFR more slowly.

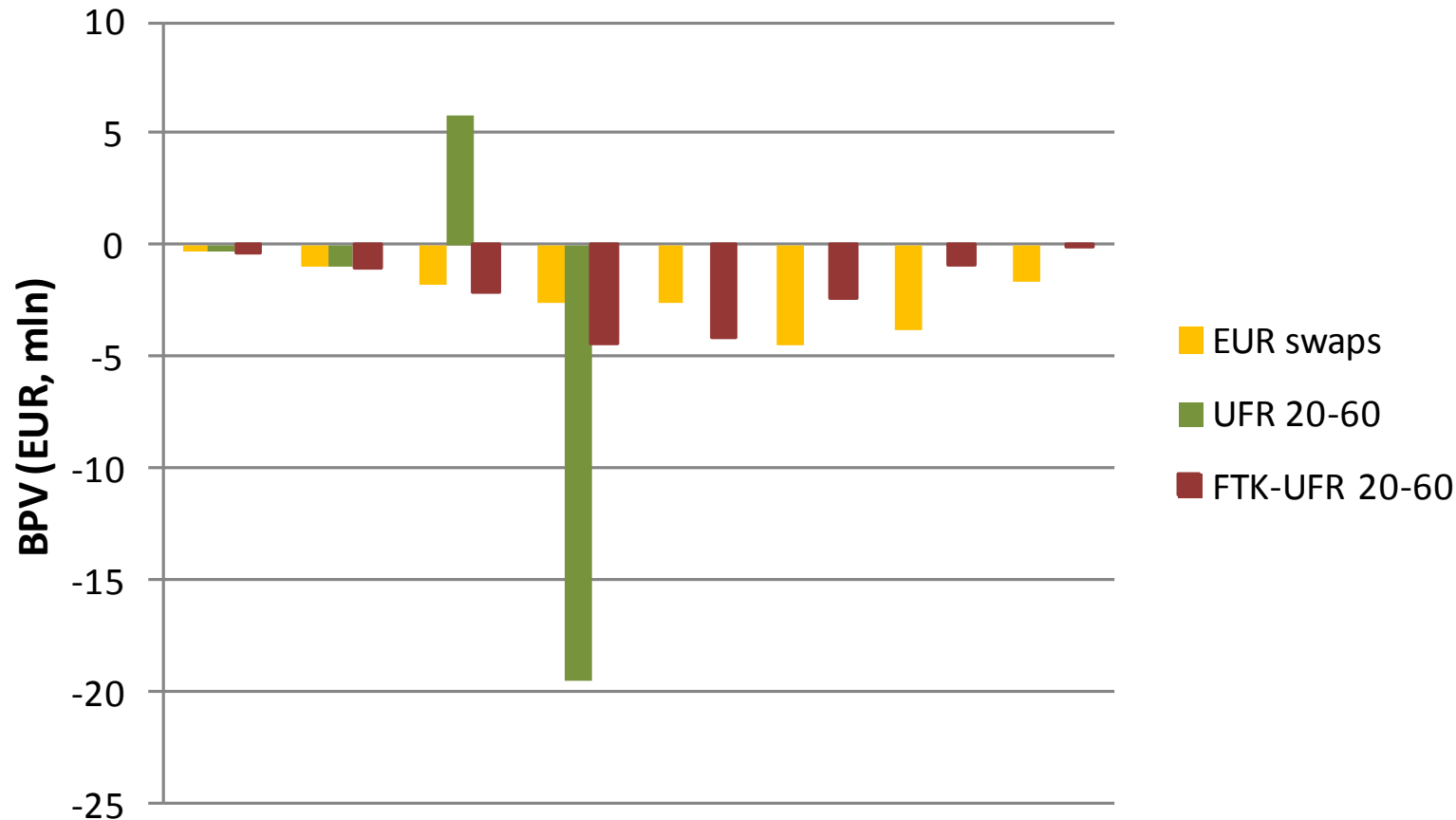
Sensitivities to the zero rates around the last liquid point are clearly much smaller:

$$\frac{\partial DF(i)}{\partial z_{LLP-1}} = 0$$

$$\frac{\partial DF(i)}{\partial z_{LLP}} = -\frac{DF(i)}{DF(LLP)} \cdot \left| \frac{\partial DF(LLP)}{\partial z_{LLP}} \right| + \frac{DF(i)}{DF_{LLP,LLP+1}} \cdot \left| \frac{\partial DF_{LLP,LLP+1}}{\partial z_{LLP}} \right|$$

but of course $\frac{\partial DF(i)}{\partial z_j} \neq 0$ for $j > LLP$, although this effect decays as j increases.

The impact is visibly smaller, and has been implemented by DNB



albeit with fixed weights. This is even easier to implement.

The Solvency II UFR methodology has several shortcomings:

- ▶ Expensive to hedge interest rate risks
- ▶ Could have a significant market impact
- ▶ Transfer of money from young to old participants
- ▶ Political risks (adjusting parameters)
- ▶ Adds a problem: to hedge economically or not?

The UFR curve imposed by DNB for pension funds overcomes the first two, and ensures hedge sensitivities are more in line with the economics ones. Several shortcomings still remain however.

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