Multiple discount and forward curves

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This presentation reflects personal views and not necessarily the views of NIBC

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- The effects of credit crisis
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Curve building

For linear interest rate products (i.e. swaps, loans, futures, FRAs) the market value (or net present value NPV) is calculated by discounting the projected cash flows on the appropriate discount curve.

$$NPV = \sum_{i=1}^{N} CF_i * df(t_i, ccy)$$

- The discount function *df(t)* is the value of a unit cash flow (zero coupon bond) in the future for any particular currency.

- Alternative we use the concept yield curve *Y(t)* to describe the term structure of interest rates. However the precise value of the yield curve depends upon the day count convention (e.g. Act/365 or Act/Act), compounding frequency (annual, semi-annual, continuously) and business day convention (following, modified following). Most commonly used are the continuously compounding zero rate (z_{cc}) and annual zero rate (z_a):

$$df(t) = \exp(-z_{cc}(t)t)$$

$$df(t) = \frac{1}{(1 + z_a(t))^t}$$

Valuation and discounting in pre-credit crisis era

- Libor (Euribor) is assumed to be the risk free rate
- No arbitrage condition
- Discount function can be used for discounting and for calculating forwards:

$$df(t_0, t_2) = df(t_0, t_1)\, df(t_1, t_2)$$

$$E[Libor(t_{i-1}, t_i)] = \{\frac{df(t_{i-1})}{df(t_i)} - 1\} / \beta(t_{i-1}, t_i)$$

- Valuing an instrument reduces to determining the discount function at discrete term point and the interpolation between these points:
  - Select a number of financial instruments with known prices (deposits, FRAs and money market futures, interest rate swaps). For the short end of the curve we use Libor rates. For the middle part of the curve we use either futures or FRAs. For the long-end we use par swaps.

Valuation and discounting in pre-credit crisis era (cont)

- For deposits (Libor rates) and FRAs/futures the discount function can be calculated in directly:

$$df_i = \frac{df_{i-1}}{1+\alpha_{i-1,i} r}, \quad df_0 = 1$$

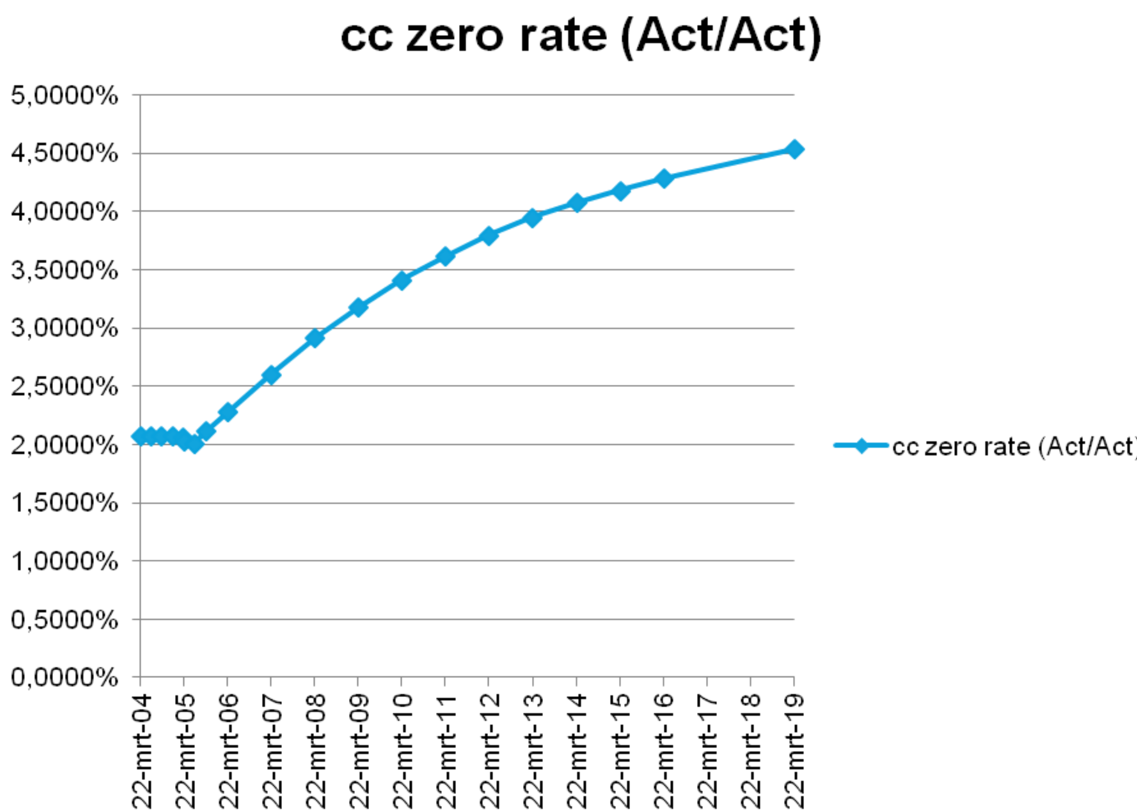
- For swaps the fixed rate is such that the value of the fixed and floating leg are identical:

$$C_N \sum_{i=1}^{N} \alpha_i df_i + df_N = \sum_{j=1}^{M} \beta_j E[L(j-1, j)] df_j + df_M$$

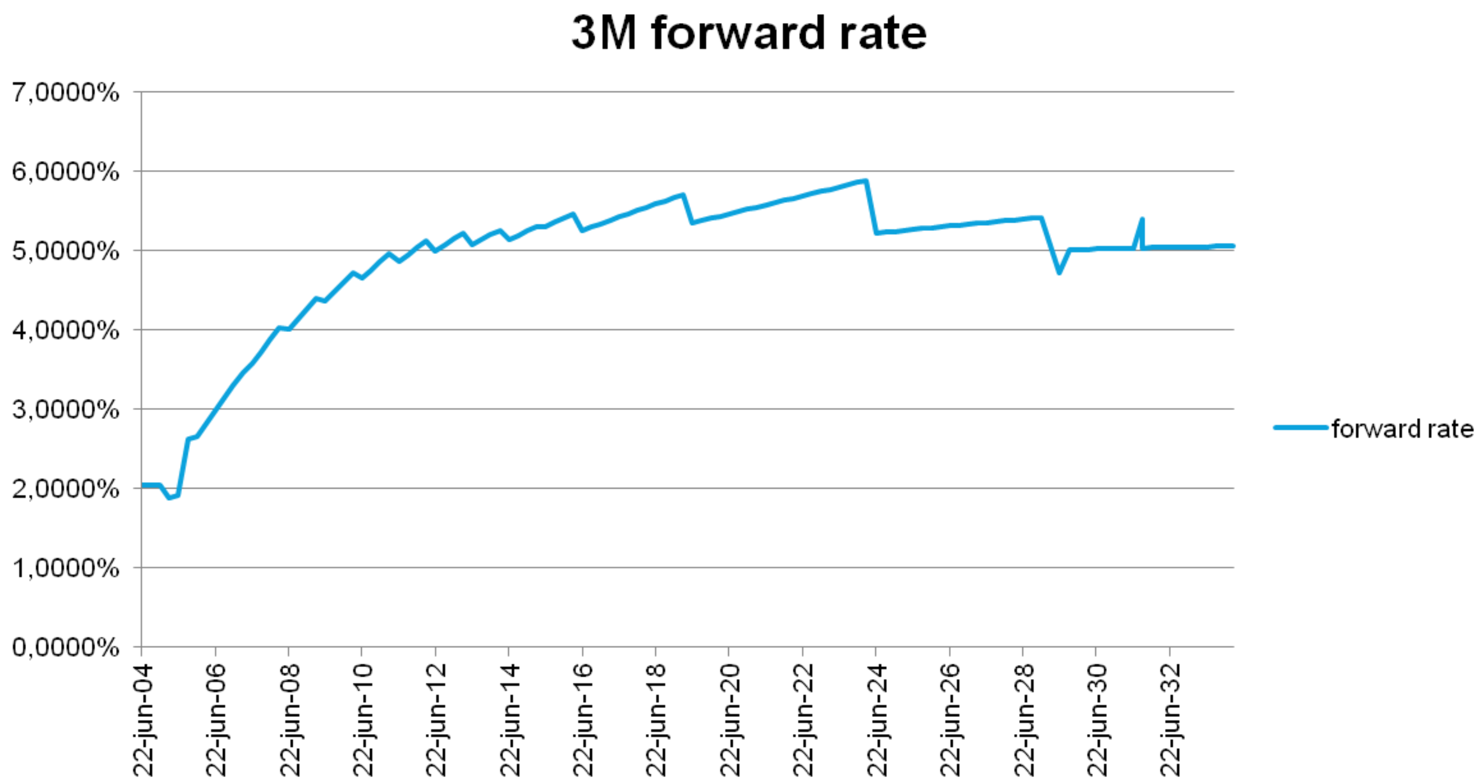
- The floating leg becomes par, because we use Libor both for discounting and for calculating forwards. The result is a closed form solution for the discount function, which can be solved iteratively. This process is called bootstrapping

$$df(t_N) = \frac{1 - C_N \sum_{i=1}^{N-1} \alpha_{i-1,i} df_i}{1 + C_N \alpha_{N-1,N}}$$

EUR Yield curve – 18 March 2004



EUR 3M forward curve – 18 March 2004



Development of 3M vs 6M EUR 5Y basis swap

Valuation and discounting in post-credit crisis era

- The assumption that you can always borrow and invest at Libor is no longer valid. In the market you can notice that:
  - Basis swap spreads are no longer near zero
  - The fixed rate of a par swap now depends upon the repricing frequency of the floating leg
  - The price of e.g. a 3-9 FRA is no longer close to the average of a 3-6 and 6-9 FRA
  - The market quotes refer to collateralised transactions. There are no unambiguous quotes for non-collateralised transactions.

- This results in different curves for different repricing frequencies. In constructing these curves we should consistently use instruments with the same <u>repricing</u> frequency. We can still assume that the value of a floating leg equals the value of the fixed leg. However we do not assume anymore that the value of the floating leg is par.
  - You should use the terms "yield curve" and "discount function" with care. If we use market instruments to construct the discount function, then this can only refer to the o/n curve. The other curves are also based upon collateralised instruments and can in principle only be used for calculating forwards

- The simple no-arbitrage condition and relationship between forward rates and the discount function no longer holds

Curve building (single currency, post-crisis)

- Only the o/n rate (EONIA, OIS, SONIA, etc) is regarded as the risk-free rate. We assume that this is the collateral rate.

- The bootstrapping process now requires a specific order
  - First we construct the overnight discount curve. This curve is used for discounting collateralised cash flows and for calculating forward o/n rates.
  - Then we construct the curves (mostly 3M and 6M), for which direct quotes are available. These curve are only used to calculate forward rates
  - Finally we build the curves based upon basis swaps (mostly 1M). Also these curves are only used for calculating forward rates.

- For discounting non-collateralised transactions, each institution should choose his “natural” repricing frequency. For most institutions this will be 3M or 6M.
  - The discount curve is now created the other way round. From the collateralised bootstrapping process the forwards are known and the discount function is calculated by using the simple pre-crisis relationship between the forward rates and discount function

Curve building (single currency, post-crisis)

o/n curve

- For the o/n curve we can still use the traditional bootstrapping method. We only use the superscript *c* for the discount function to denote this as the discount function for collateralised transactions:

$$df^{c}(t_N) = \frac{1 - C_N \sum_{i=1}^{N-1} \alpha_{i-1,i} df_i^{c}}{1 + C_N \alpha_{N-1,N}}$$

- We neglect the fact that the o/n swaps have a 1-day settlement leg.

3M curve

- The other curves are only used for calculating forwards. The first points on the 3M and 6M curves are straightforward, as these are already forwards (Libor rates or futures / FRAs). For convenience we define the 3M discount function as derived from the 3M forwards as:

$$df^{3M}(t_N) = \frac{1}{\prod_{i=1}^{N} (1 + \beta(t_{i-1}, t_i)\, fr^{3M}(t_{i-1}, t_i))}$$

- However this is not the discount function, which is used for discounting. It is only a tool to express the 3M yield curve

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Curve building (single currency, post-crisis)

3M curve

- For the long end of these curves the value of the fixed leg equals the value of the floating leg, if discounted on the o/n curve:

$$\sum_{j=1}^{M} \beta_{j,j-1} fr_{j,j-1} df_j^c = C_N \sum_{i=1}^{N} \alpha_{i,i-1} df_i^c$$

- However this requires a different kind of bootstrapping:
  - The discount function on all points is known and only the last forward rate(s) is (are) unknown.
  - In case there is only 1 unknown forward rate, this equation can be solved directly
  - In case there is more than 1 unknown forward rate (which is mostly the case) an assumption about the interpolation of these unknown forward rates have to be made. Unless you choose a linear interpolation of these forward rates, this equation has to be solved iteratively.

Curve building using basis swaps

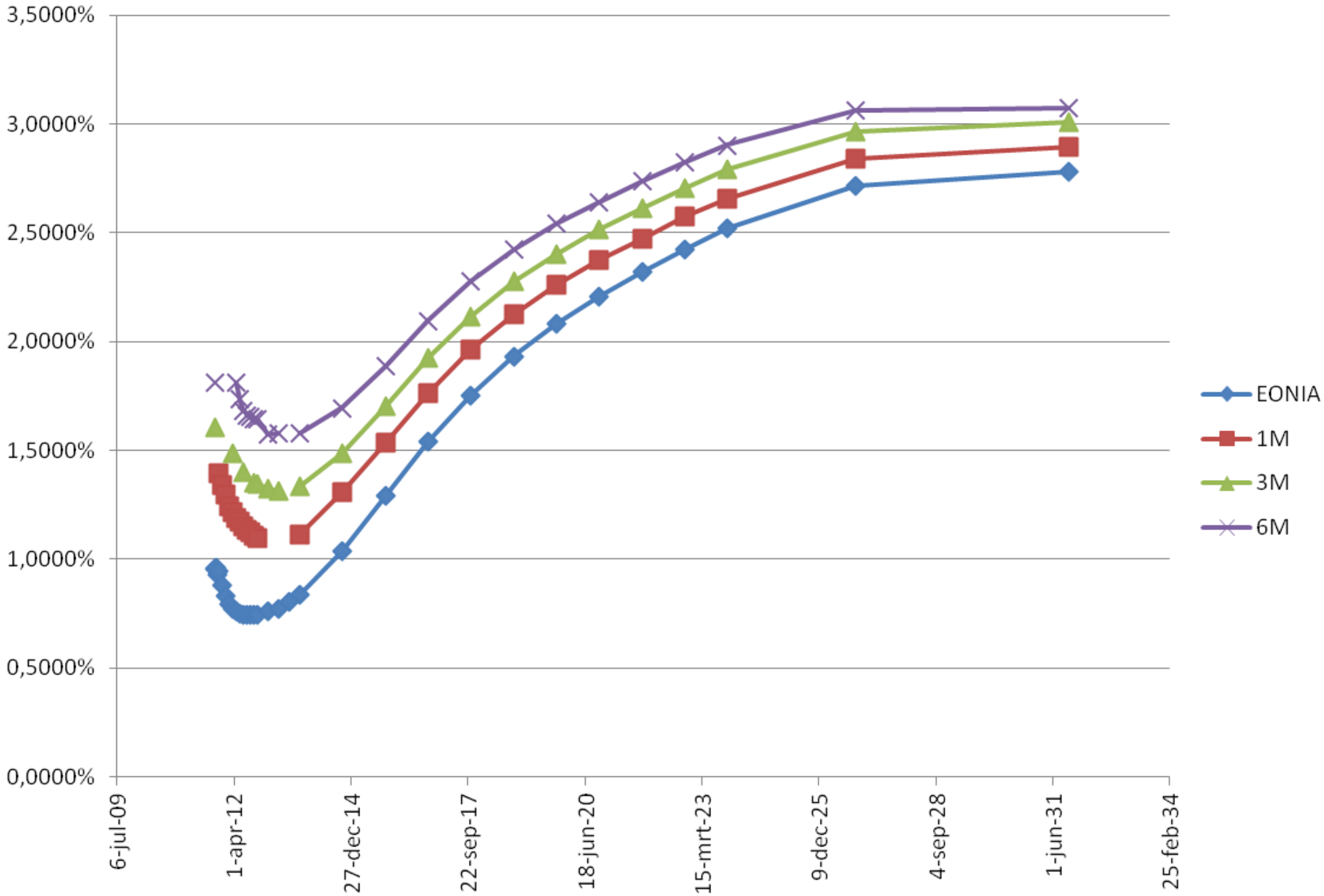
1M curve

- Basis swaps are quoted as a spread rate on the shortest repricing frequency for a specific tenor, e.g. 1M Euribor + 15 bps versus 3M Euribor for a 5Y period. To compute the 1M forward curve, first the 3M forward curve and o/n discounting curve have to be calculated. The bootstrapping process for the 1M forward curve then is similar to the bootstrapping process for the 3M forward curve:

$$\sum_{j=1}^{M} \beta_{j,j-1}^{1M}(b_N + fr_{j,j-1}^{1M})df_j^c = \sum_{i=1}^{K} \beta_{i,i-1}^{3M} fr_{i,i-1}^{3M} df_i^c$$

- It is noted that:
  - For the short end of the curve we use standard fixed-floating swaps, discounted on the o/n curve. The solution for the forward rate is straightforward
  - In general the coupon payments coincide with the repricing frequency, so
    - *M=3K* and the accrual factors are different on both sides
    - Quotes are available for annual basis swaps

EUR Yield curves – 17 October 2011



Curve building (non-collateralised, post-crisis)

NC curve

- For discounting non-collateralised transactions, each institution should choose his “natural” repricing frequency. For most institutions this will be 3M or 6M.
  - The discount curve is now created the other way round. From the collateralised bootstrapping process the forwards are known and the discount function is calculated by using the simple pre-crisis relationship between the forward rates and discount function

$$df_i^{NC} = \frac{df_{i-1}^{NC}}{1 + \beta_{i-1,i}\, fr_{i-1,i}^{3M}}, \quad df_0^{NC} = 1$$

- As this curve is also used for discounting credit instruments (loans,bonds, deposits) it is common practise to add a credit spread to this curve. Be aware that:
  - Most systems add the credit spread to the zero curve, whereas it should be added to the forward curve
  - Calculating the interest sensitivity by shifting the zero curve can create significant non-zero sensitivities for a portfolio of floating loans and lead to incorrect hedges

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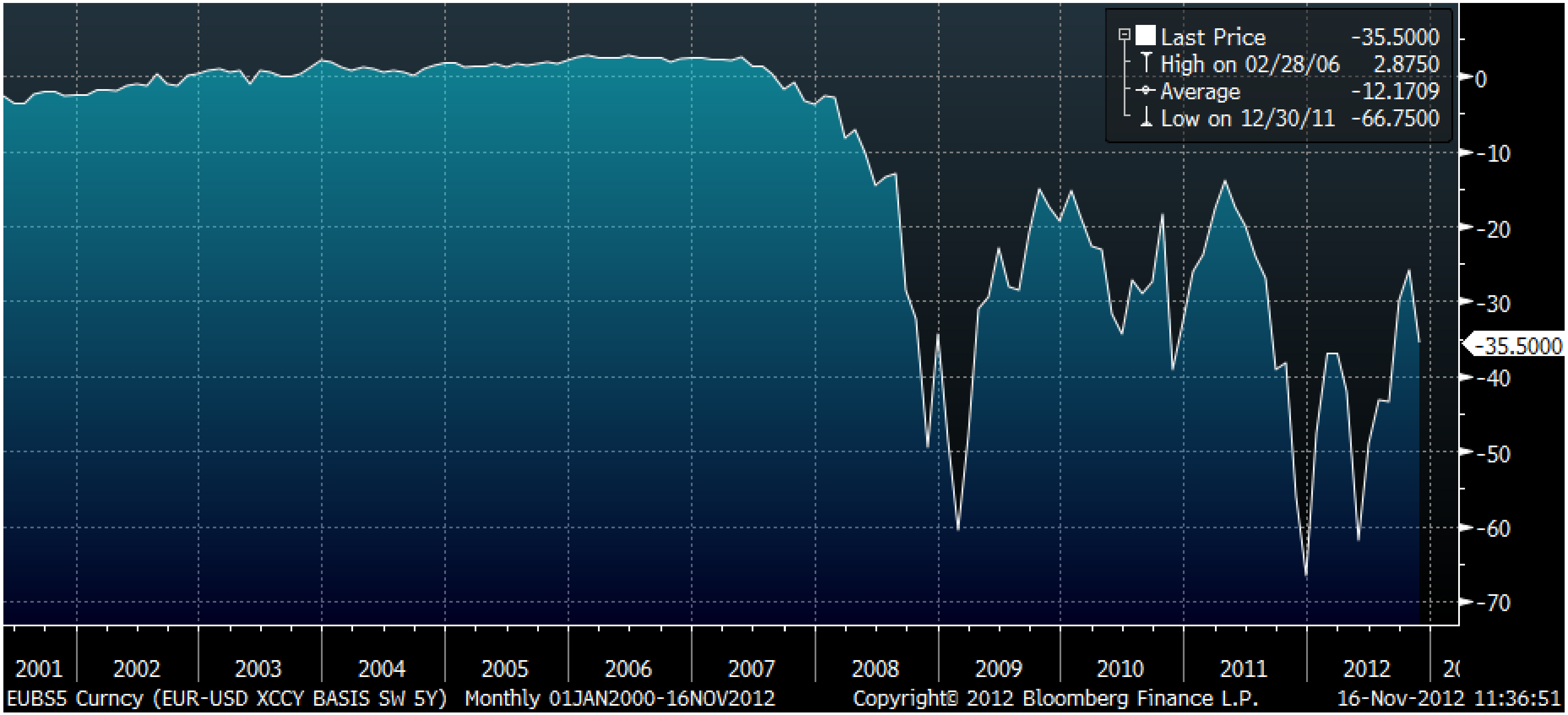
Development of EUR/USD 5Y basis swap



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Cross currency basis spreads

- We also notice that
  - Cross currency basis swap spreads in the major markets (EUR, USD, GBP, JPY) are non-zero. e.g, You pay 3M USD Libor and receive 3M Euribor minus 50 bps
  - Under some CSA agreements USD collateral is posted for EUR swaps and vice versa

- These spreads reflect a market, which is not in equilibrium
  - Assuming the same cost of funding in their home market and local currency a USD based bank has a competitive advantage over a EUR based bank.
  - It is no longer possible to use both EONIA and OIS as risk-free rate
  - Identical cash flows have different values for different market participants

A possible approach

- Step 1: construct EUR curves using the single currency approach as described before.
  - The EUR o/n curve will be used to discount all EUR cash flows with EUR collateral
  - The EUR o/n, 1M, 3M and 6M forward curves
- Step 2: construct USD curves using the single currency approach as described before.
  - The USD o/n curve will be used to discount all USD cash flows with USD collateral
  - The USD o/n, 1M, 3M and 6M forward curves
- Step 3: use 3M EUR vs 3M USD cross currency basis swaps for:
  - The USD discount function with EUR as collateral currency
  - The EUR discount function with USD as collateral currency

USD discount function with EUR collateral

- Assume the XCCY quotes will get EUR collateral
- This yields (spread b_N (currently negative) on the EUR leg):

$$N_{EUR}\left[-1+\sum_{j=1}^{M}\{\beta_{j,j-1}^{EUR,3M}(b_N + fr_{j,j-1}^{EUR,3M})df_j^{c(EUR)}(EUR)\} + df_N^{c(EUR)}(EUR)\right] =$$

$$fx_{USD}\left[-1+\sum_{j=1}^{M}\{\beta_{j,j-1}^{USD,3M}\ fr_{j,j-1}^{USD,3M}\ df_j^{c(EUR)}(USD)\} + df_N^{c(EUR)}(USD)\right]$$

- The only unknown is the USD discount function under EUR collateral, thus this can be solved by the traditional bootstrapping methods

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EUR discount function with USD collateral

- Assume the XCCY quotes will get USD collateral
- Same EUR/USD basis swap with spread $b_N{}^*$: (which is not necessarily the same as the previous basis spread)

$$N_{EUR}\left[-1+\sum_{j=1}^{M}\{\beta_{j,j-1}^{EUR,3M}(b_N^* + fr_{j,j-1}^{EUR,3M})df_j^{c(USD)}(EUR)\} + df_N^{c(USD)}(EUR)\right] =$$

$$fx_{USD}\left[-1+\sum_{j=1}^{M}\{\beta_{j,j-1}^{USD,3M}\ fr_{j,j-1}^{USD,3M}\ df_j^{c(USD)}(USD)\} + df_N^{c(USD)}(USD)\right]$$

- The only unknown is the EUR discount function under USD collateral, thus this can be solved by the traditional bootstrapping methods

- Using other XCCY spreads this results in for e.g. all EUR cash flows: one discounting curve per collateral currency.

Cross currency basis spreads

- This approach:
  - values all par swaps at par
  - neglects the cross currency spread in the cost of funding in foreign currencies:
  e.g. the XCCY spread has no influence on the discounting of a standard USD swap.

- Alternative: Choose EUR as base currency: Include the cross currency basis spread in the valuation of all non-EUR based financial instruments. However a the value of non-EUR par swap will not be par anymore. How? Would this be preferred?