

New (Observation Driven) Models for Dynamic Volatilities, Correlations, and Credit Risk

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Motivation

- ▶ Financial time series are often inherently non-stable: the model's parameters vary over time
- ▶ Financial time series may be continuous and/or discrete and be observed at different frequencies and share common (dynamic) features
- ▶ It is often difficult to find the observable variables for these shared features: shared dynamics may be due to *unobserved* components
- ▶ Conceptual and computational challenges are large: can we make a step forward?

Motivation (ctd)

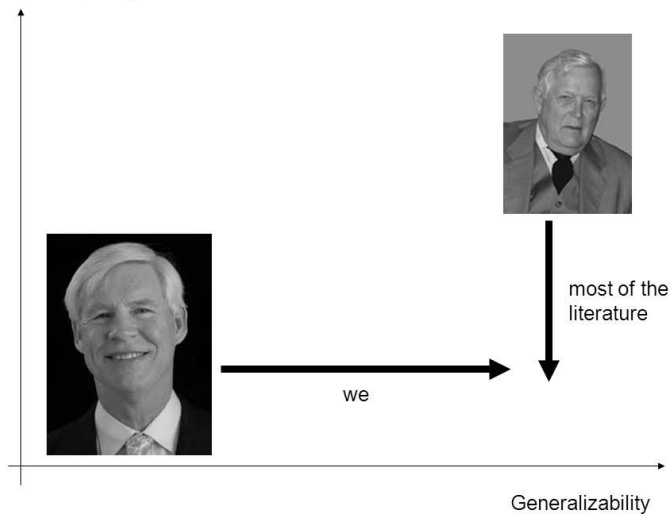
Computational complexity



Generalizability

Motivation (ctd)

Computational complexity



Contributions

- ▶ We introduce a new, general, computationally straightforward econometric framework to model time variation in parameters
- ▶ The framework is easily applied to complex models involving time-varying higher order moments for series with different distributions and different frequencies:
 - ▶ Modeling dynamic volatilities and correlations under skewness and fat tails
 - ▶ Modeling dynamic portfolio credit risk
- ▶ The framework is easily exploited for forecasting and risk analysis
- ▶ The framework gives rise to new models
- ▶ The framework has comparable performance to its (computationally involved) state-space counterparts

Time varying parameter models

Consider a sequence of observations y_1, \dots, y_T with density

$$y_t \sim p(y_t | F^t, Y^{t-1}, \psi),$$

which depends on a vector of time varying parameters f_t . These evolve through time as a linear process

$$f_{t+1} = \omega + Bf_t + As_t.$$

- ▶ $Y^t = \{y_1, \dots, y_t\}$, $F^t = \{f_1, \dots, f_t\}$.
- ▶ f_t contains all time varying parameters in observation density.
- ▶ ψ contains parameters in ω , A , B and “fixed” coefficients.
- ▶ s_t is the “driving” mechanism.
- ▶ Exogenous variables (covariates) can also be included.

Time varying parameter models

$$\begin{aligned}y_t &\sim p(y_t|F^t, Y^{t-1}, \psi), \\f_{t+1} &= \omega + Bf_t + As_t,\end{aligned}$$

Two options for dealing with s_t (Cox, 1981):

- ▶ let s_t be an i.i.d. random sequence \Rightarrow parameter driven
- ▶ s_t is a deterministic function of $Y^t \Rightarrow$ observation driven

In case of an “observation driven” approach, what is an appropriate function $g(\cdot)$ in $s_t = g(Y^t)$?

Time varying parameter models

In our work, we build observation driven models

$$\begin{aligned}y_t &\sim p(y_t|F^t, Y^{t-1}; \psi), \\f_{t+1} &= \omega + Bf_t + As_t,\end{aligned}$$

where the evolution of the factors are determined by the scaled score

$$\begin{aligned}s_t &= S_t \cdot \nabla_t, \\ \nabla_t &= \frac{\partial \ln p(y_t|F^t, Y^{t-1}; \psi)}{\partial f_t}, \\ S_t &= S(t, Y^t; \psi).\end{aligned}$$

- ▶ s_t is a m.d.s. which acts as a natural sequence of innovations.
- ▶ Creal, Koopman, Lucas (2008) propose this class of models and label them **Generalized Autoregressive Score** (GAS) models.
- ▶ Different choices for S_t : Fisher info. matrix, identity matrix,....

Example # 1: GARCH

Consider model $y_t \sim \mathcal{N}(0, \sigma_t^2)$:

$$y_t = \sigma_t \varepsilon_t \quad \varepsilon_t \sim \mathcal{N}(0, 1)$$

Let $f_t = \sigma_t^2$. The score and inverse of the information matrix are:

$$\begin{aligned}\nabla_t &= \frac{1}{2f_t^2} y_t^2 - \frac{1}{2f_t}, \\ S_t &= \mathcal{I}_{t-1}^{-1} = 2f_t^2.\end{aligned}$$

The GAS(1, 1) recursion reduces to the GARCH(1, 1) model:

$$f_{t+1} = \omega + A_1(y_t^2 - f_t) + B_1 f_t$$

($A_1 = \alpha$ and $B_1 = \alpha + \beta$ from standard GARCH parameterization)

Example # 2: E-ACD

Consider an exponential (\mathcal{E}) model,

$$y_t = \lambda_t \varepsilon_t, \quad \varepsilon_t \sim \mathcal{E}(1).$$

Let $f_t = \lambda_t$. The score and inverse of the information matrix are:

$$\begin{aligned}\nabla_t &= \frac{y_t}{f_t^2} - \frac{1}{f_t}, \\ S_t &= \mathcal{I}_{t-1}^{-1} = f_t^2.\end{aligned}$$

The GAS(1,1) recursion reduces to:

$$f_{t+1} = \omega + A_1(y_t - f_t) + B_1 f_t$$

E-ACD(1,1) model of Engle Russell (1998)

Special cases of this idea

- ▶ GARCH with normal dist.: Engle (1982), Bollerslev (1986)
- ▶ Exponential dist. (E-ACD and ACI): Engle & Russell (1998) and Russell (2001), respectively
- ▶ Gamma dist. (MEM): Engle (2002), Engle & Gallo (2006)
- ▶ Poisson dist.: Davis, Dunsmuir & Street (2003)
- ▶ Multinomial dist. (ACM): Russell & Engle (2005)
- ▶ Binomial dist.: Cox (1956), Rydberg & Shephard (2002)
- ▶ Vola and correl under Student's t dist. (Creal, Koopman, Lucas, 2011) or GH distributions (Zhang, Creal, Koopman, Lucas, 2011)
- ▶ Beta and ordered logit dist.: Creal, Schwaab, Koopman, Lucas (2011)
- ▶ Related to literature on approximations to state space models: Masreliez (1975), West (1981), Fahrmeir (1992), Nelson Foster (1994), Müller Petalas (2009)

Application I: volatilities under skewness and fat-tails

- ▶ If you have a fat-tailed density, you expect to observe large (absolute) realizations from time to time
- ▶ Such large values are due to the fat-tailed error density, and . . . not to recent increases in volatility
- ▶ Ergo, if you see a large realization but you have fat-tailed data, you should not immediately increase the volatility
- ▶ Similar: correlations
- ▶ Similar: if errors are skewed, you expect a different effect of a positive (compared to a negative) realization
- ▶ None of these features are present in (multivariate) GARCH models or the DCC

The GH (skewed t) multivariate distribution

$$p(y_t; \mu, \tilde{\Sigma}_t, \gamma, \nu) = \frac{\nu^{\frac{\nu}{2}} 2^{1-\frac{\nu+n}{2}}}{\Gamma(\frac{\nu}{2}) \pi^{\frac{n}{2}} |\tilde{\Sigma}_t|^{\frac{1}{2}}} \cdot \frac{K_{\frac{\nu+n}{2}} \left(\sqrt{d(y_t) \cdot (\gamma' \gamma)} \right) e^{\gamma' \tilde{L}_t^{-1} (y_t - \mu)}}{(d(y_t) \cdot (\gamma' \gamma))^{-\frac{\nu+n}{4}} d(y_t)^{\frac{\nu+n}{2}}}$$
$$d(y_t) = \nu + (y_t - \mu)' \tilde{\Sigma}_t^{-1} (y_t - \mu), \quad (1)$$

If $\gamma = 0$, the GH skewed t simplifies to a Student's t density.

Time variation in $\tilde{\Sigma}_t$ is driven by 1st and 2nd derivative of the pdf.

The model with time varying parameters

Assuming the time varying parameters follow

$$f_{t+1} = Bf_t + As_t, \quad (2)$$

where

$$s_t = S_t \nabla_t, \quad (3)$$

$$\nabla_t = \partial \ln p(y_t | Y_{t-1}; f_t, \theta) / \partial f_t. \quad (4)$$

In the dynamic GH model, the steps s_t are

$$s_t = S_t \cdot \frac{1}{2} \Psi_t' \tilde{\Sigma}_{t \otimes}^{-1} \text{vec} \left(w_{1t} y_t y_t' - \tilde{\Sigma}_t - w_{2t} \gamma y_t' \right), \quad (5)$$

where S_t is the inverse information matrix.

The Steps and The Parametric Assumptions

In the dynamic GH-Gaussian model, the steps s_t are:

$$s_t = S_t \cdot \frac{1}{2} \Psi_t' \tilde{\Sigma}_{t \otimes}^{-1} \text{vec} \left(\begin{array}{c} y_t y_t' - \tilde{\Sigma}_t \end{array} \right).$$

The Steps and The Parametric Assumptions

In the dynamic GH-Student's t model, the steps s_t are:

$$s_t = S_t \cdot \frac{1}{2} \Psi'_t \tilde{\Sigma}_t^{-1} \text{vec} \left(w_{1t} y_t y'_t - \tilde{\Sigma}_t \right).$$

$$w_{1t} \propto \frac{1}{1 + (\nu - 2)^{-1} y'_t \tilde{\Sigma}_t^{-1} y_t}.$$

The Steps and The Parametric Assumptions

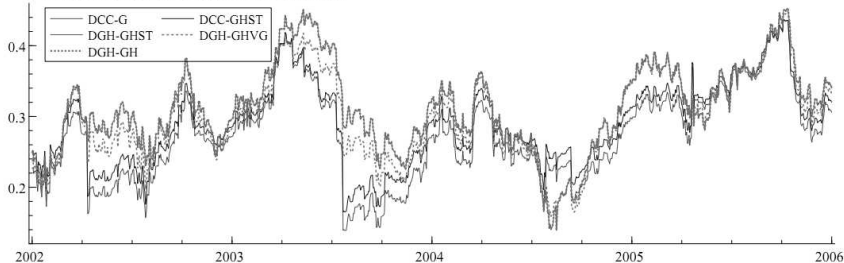
In the dynamic GH model, the steps s_t are:

$$s_t = S_t \cdot \frac{1}{2} \Psi'_t \tilde{\Sigma}_t^{-1} \text{vec} \left(w_{1t} y_t y'_t - \tilde{\Sigma}_t - w_{2t} \gamma y'_t \right).$$

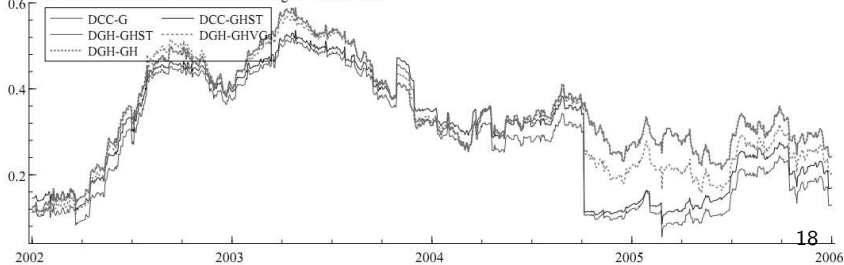
w_{1t} and w_{2t} more complicated expressions, but same intuition:
smaller weights for discordant observations

The quadrivariate GH in operation

A Zoom-in of Correlation Coca-IBM: 2002 to 2006



A Zoom-in of Correlation Merck-JP Morgan: 2002 to 2006



Example 2: Mixed measurement panel data models

We introduce mixed measurement observation driven models

$$\begin{aligned}y_{it} &\sim p_i(y_{it}|F^t, Y^{t-1}; \psi), \quad i = 1, \dots, N, \\f_{t+1} &= \omega + B_1 f_t + A_1 s_t\end{aligned}$$

The score function is

$$\begin{aligned}s_t &= S_t \nabla_t \\ \nabla_t &= \sum_{i=1}^N \delta_{it} \nabla_{i,t} = \sum_{i=1}^N \delta_{it} \frac{\partial \log p_i(y_{it}|F^t, Y^{t-1}; \psi)}{\partial f_t},\end{aligned}$$

- ▶ The observations y_{it} may come from different distributions.
- ▶ The factors f_t may be common across distributions.
- ▶ KEY: The score function allows us to pool information from different observations to estimate the common factor f_t .
- ▶ δ_{it} is an indicator function equal to 1 if y_{it} is observed and zero otherwise. Missing values are naturally taken into account.

Scaling matrix

Consider the eigenvalue-eigenvector decomposition of Fisher's (conditional) information matrix

$$\mathcal{I}_t = \mathbb{E}_{t-1}[\nabla_t \nabla_t'] = U_t \Sigma_t U_t',$$

The scaling matrix is then defined as

$$S_t = U_t \Sigma_t^{-1/2} U_t'$$

- ▶ S_t is then the “square root” of a generalized inverse.
- ▶ The innovations s_t driving f_t have an identity covariance matrix, when the info. matrix is non-singular.
- ▶ The conditional information matrix is additive for our models:

$$\mathcal{I}_t = \mathbb{E}_{t-1}[\nabla_t \nabla_t'] = \sum_{i=1}^N \delta_{it} \mathbb{E}_{i,t-1}[\nabla_{it} \nabla_{it}'].$$

Log-likelihood function and ML estimation

- ▶ The log-likelihood function for an observation-driven model can easily be computed.
- ▶ The ML estimator is

$$\hat{\psi} = \arg \max_{\psi} \sum_{t=1}^T \sum_{i=1}^N \delta_{it} \log p_i(y_{it} | F^t, Y^{t-1}; \psi),$$

- ▶ Estimation is similar to a GARCH model.

Credit risk

- ▶ Growing econometrics literature on models for credit risk: McNeil et al. (2005), Bauwens and Hautsch (JFEct, 2006), Gagliardini and Gourieroux (JFEct, 2005), Koopman Lucas and Monteiro (JEct, 2008), Duffie et al. (JFE, JoF 2008).
- ▶ Basic observations:
 1. Probability of default varies over time with the business cycle.
 2. Conditional on default, the loss (recovery rate) varies with the business cycle.
 3. We observe excess clustering of defaults and ratings transitions beyond what can be explained by simply adding covariates.
 4. The literature focuses on a credit risk or frailty factor.
- ▶ Industry standard models are too simple to capture these features.
- ▶ New models in the literature are parameter driven models requiring simulation methods for estimation.
- ▶ We provide observation driven alternatives.

Data: Moody's and FRED

- ▶ We observe data from Jan. 1980 to March 2010.
- ▶ 7,505 companies are rated by Moody's.
- ▶ We pool these into 5 ratings categories (IG, BB, B, C, D).
- ▶ We observe transitions, e.g. IG \rightarrow BB or C \rightarrow D
- ▶ There are $J = 16$ total types of transitions.
- ▶ 19,450 total credit rating transitions.
- ▶ 1,342 transitions are defaults.
- ▶ 1,125 measurements of loss-given default (LGD).
- ▶ LGD is the fraction of principal an investor loses when a firm defaults.
- ▶ We also observe six macroeconomic variables: industrial production growth, credit spread, unemployment, annual S&P500 returns, realized volatility, real GDP growth (qtly).

Models

- ▶ Credit ratings can be modeled using the (static) ordered probit model of CreditMetrics; one of the current industry standards, see Gupton Stein (2005).
- ▶ LGD's are often modeled by (static) beta distributions.
- ▶ GOAL: Build models that improve on current industry standards and are (relatively) easy to implement and estimate.
 1. Time-varying ordered logit
 2. Time-varying beta distribution
- ▶ Forecasting credit risk.
- ▶ Simulation of loss distributions and scenario analysis.
- ▶ Bank executives and regulators and can use them for “stress testing.”

Mixed measurement model for credit risk

$$\begin{aligned}y_t^m &\sim N(\mu_t, \Sigma_m) \\y_{i,t}^c &\sim \text{Ordered Logit}(\pi_{ijt}, j \in \{\text{IG, BB, B, C, D}\}), \\y_{k,t}^r &\sim \text{Beta}(a_{kt}, b_{kt}), \quad k = 1, \dots, K_t,\end{aligned}$$

- ▶ y_t^m are the macro variables.
- ▶ $y_{i,t}^c$ are indicator variables for each credit rating j for firm i .
- ▶ $y_{k,t}^r$ are the LGDs for the k -th default.
- ▶ K_t are the number of defaults in period t .
- ▶ μ_t , π_{ijt} , and (a_{kt}, b_{kt}) are functions of an $M \times 1$ vector of factors f_t .

Time varying Gaussian model for macro data

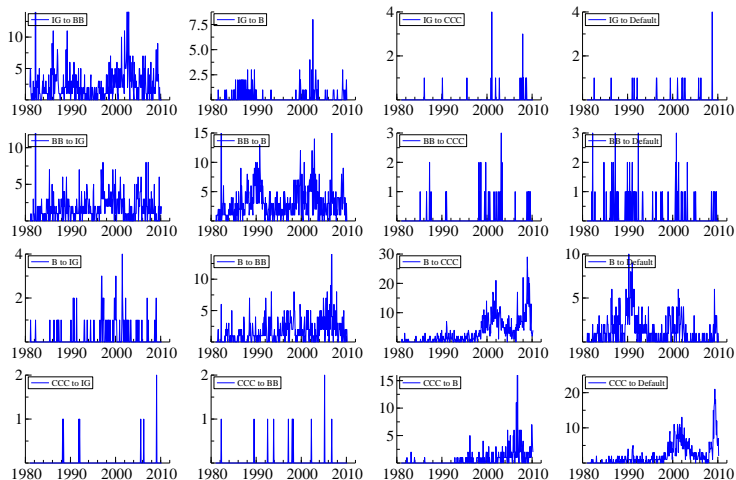
$$\begin{aligned}y_t^m &\sim \text{N}(\mu_t, \Sigma_m), \\ \mu_t &= Z^m f_t.\end{aligned}$$

- ▶ Z^m is a $(6 \times M)$ matrix of factor loadings.
- ▶ Σ_m is a (6×6) diagonal covariance matrix.
- ▶ \tilde{S}_t is a selection matrix indicating which macro variables are observed at time t .

$$\begin{aligned}\nabla_t^m &= (\tilde{S}_t Z^m)' (\tilde{S}_t \Sigma_m \tilde{S}_t')^{-1} \tilde{S}_t (y_t^m - \mu_t), \\ \mathcal{I}_t^m &= (\tilde{S}_t Z^m)' (\tilde{S}_t \Sigma_m \tilde{S}_t')^{-1} \tilde{S}_t Z^m.\end{aligned}$$

Moody's monthly credit ratings transitions

The data have been pooled together each month.



Time-varying ordered logit

$$y_{i,t}^c \sim \text{Ordered Logit}(\pi_{ijt}, j \in \{\text{IG}, \text{BB}, \text{B}, \text{C}, \text{D}\}),$$

$$\pi_{ijt} = P[R_{i,t+1} = j] = \tilde{\pi}_{ijt} - \tilde{\pi}_{i,j-1,t},$$

$$\tilde{\pi}_{ijt} = P[R_{i,t+1} \leq j] = \frac{\exp(\theta_{ijt})}{1 + \exp(\theta_{ijt})},$$

$$\theta_{ijt} = z_{ijt}^c - Z_{it}^{c'} f_t.$$

- ▶ $J^c = 5$ categories $j \in \{\text{IG}, \text{BB}, \text{B}, \text{C}, \text{D}\}$.
- ▶ R_{it} is the rating for firm i at the start of month t .
- ▶ y_{it}^c is an indicator variable for each rating type.
- ▶ π_{ijt} is the probability that firm i is in category j .
- ▶ $\tilde{\pi}_{i,D,t} = 0$ and $\tilde{\pi}_{i,IG,t} = 1$.
- ▶ To our knowledge, a time-varying ordered logit model is new.

Time-varying ordered logit

The contribution to the log-likelihood at time t is

$$\ln p_i(y_{it}^c | F^t, Y^{t-1}; \psi) = \sum_{i=1}^{N_t} \sum_{j=1}^{J^c} y_{ijt}^c \log(\pi_{ijt})$$

The score and information matrices are

$$\begin{aligned}\nabla_t^c &= -\sum_{i=1}^{N_t} \sum_{j=1}^{J^c} \frac{y_{ijt}^c}{\pi_{ijt}} \cdot \dot{\pi}_{ijt} \cdot Z_{it}^c, \\ \mathcal{I}_t^c &= \sum_{i=1}^{N_t} n_{it} \left(\sum_j \frac{\dot{\pi}_{ij,t}^2}{\pi_{ij,t}} \right) Z_{it}^c Z_{it}^{c'}\end{aligned}$$

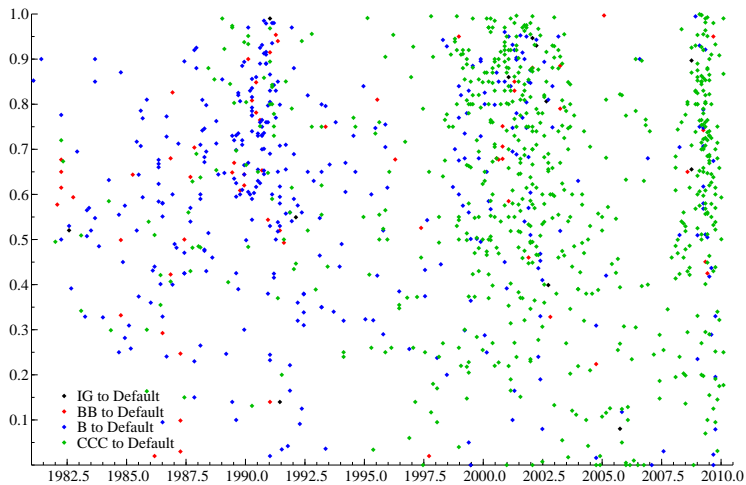
where

$$\dot{\pi}_{ijt} = \tilde{\pi}_{ijt} (1 - \tilde{\pi}_{ijt}) - \tilde{\pi}_{i,j-1,t} (1 - \tilde{\pi}_{i,j-1,t}).$$

Loss given default

- ▶ When a firm defaults, investors typically lose a fraction of their investment (alternatively, they recover a fraction of their investment).
- ▶ The fraction of losses experienced by investors also varies with the business cycle.
- ▶ We develop a new model for a time-varying beta distribution.
- ▶ See McNeil and Wendin (2007 JEmpFin) for Bayesian inference in a state space model.

Loss given default by transition type



Time-varying beta distribution

$$y_{k,t}^r \sim \text{Beta}(a_{kt}, b_{kt}), \quad k = 1, \dots, K_t,$$

$$a_{kt} = \beta_r \cdot \mu_{kt}^r$$

$$b_{kt} = \beta_r \cdot (1 - \mu_{kt}^r)$$

$$\log(\mu_{kt}^r / (1 - \mu_{kt}^r)) = z^r + Z^r f_t.$$

- ▶ We observe $K_t \geq 0$ defaults at time t .
- ▶ $0 < y_{k,t}^r < 1$ is the amount lost conditional on the k -th default.
- ▶ μ_{kt}^r is the mean of the beta distribution.
- ▶ z^r is the unconditional level of LGDs.
- ▶ Z^r is a $(1 \times M)$ vector of factor loadings.
- ▶ β_r is a scalar parameter

Time-varying beta distribution

The contribution to the log-likelihood at time t is

$$\ln p_i(y_{kt}^r | F^t, Y^{t-1}; \psi) = \sum_{k=1}^{K_t} (a_{kt} - 1) \log(y_{kt}^r) + (b_{kt} - 1) \log(1 - y_{kt}^r) - \log[B(a_{kt}, b_{kt})]$$

The score and information matrices are

$$\nabla_t^r = \beta_r \sum_{k=1}^{K_t} \mu_{kt}^r (1 - \mu_{kt}^r) (Z^r)' (1, -1) \left((\log(y_{kt}^r), \log(1 - y_{kt}^r))' - \dot{B}(a_{kt}, b_{kt}) \right)$$

$$\mathcal{I}_t^r = \beta_r \sum_{k=1}^{K_t} (\mu_{kt}^r (1 - \mu_{kt}^r))^2 (Z^r)' (1, -1) \left(\ddot{B}(a_{kt}, b_{kt}) \right) (1, -1)' Z^r$$

where

$$\sigma_{kt}^2 = \mu_{kt}^r \cdot (1 - \mu_{kt}^r) / (1 + \beta_r).$$

Estimation details

- ▶ The macro data y_t^m has been standardized.
- ▶ We consider models with $p = 1$ and $q = 1$ factor dynamics.
- ▶ For identification of the level parameters, we set $\omega = 0$ in the factor recursion:

$$f_{t+1} = A_1 s_t + B_1 f_t$$

- ▶ For identification of the factors, we also impose restrictions on Z^m , Z^c , and Z^r .
- ▶ Some parameters have been pooled for “rare” transitions; e.g., $IG \rightarrow D$ and $BB \rightarrow D$.
- ▶ Moody’s re-defined several categories in April 1982 and Oct. 1999 causing incidental re-ratings (outliers), which we handle via dummy variables for these dates.

AIC, BIC, and log-likelihoods for different models

	(2,0,0)	(2,1,0)	(2,2,0)	(3,0,0)
log-Like	-40447.9	-40199.1	-40162.8	-40056.2
AIC	81005.9	80520.1	80457.0	80242.4
BIC	81640.0	81223.0	81218.0	80991.0
	(3,1,0)	(3,2,0)	(3,1,1)	(3,2,1)
log-Like	-39817.1	-39780.8	-39812.6	-39780.0
AIC	79776.2	79713.6	79771.2	79716.0
BIC	80594.0	80589.0	80612.0	80615.0

The number of factors for each data type are represented by (m, c, r) .

Parameter estimates for the (3,2,0) model

Macro loadings Z^m

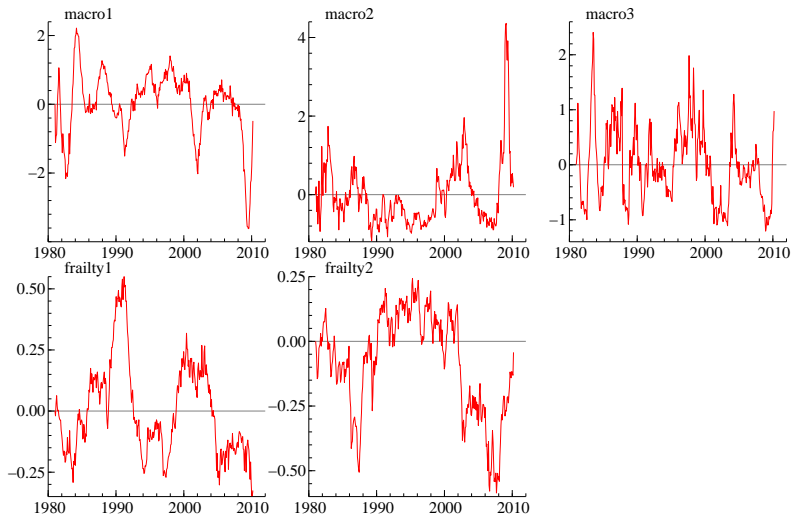
	macro ₁	macro ₂	macro ₃	frailty ₁	frailty ₂
IP	1.000	0.000	0.000	0.000	0.000
UR	-0.892*** (0.037)	0.122*** (0.041)	-0.062* (0.040)	0.000	0.000
RGDP	0.811*** (0.066)	0.072 (0.079)	0.336*** (0.074)	0.000	0.000
Cr.Spr.	-0.169** (0.085)	1.000	0.000	0.000	0.000
$r_{S\&P}$	0.049 (0.093)	-0.268*** (0.081)	1.223*** (0.093)	0.000	0.000
$\sigma_{S\&P}$	-0.007 (0.107)	0.648*** (0.084)	1.000	0.000	0.000

Parameter estimates for the (3,2,0) model

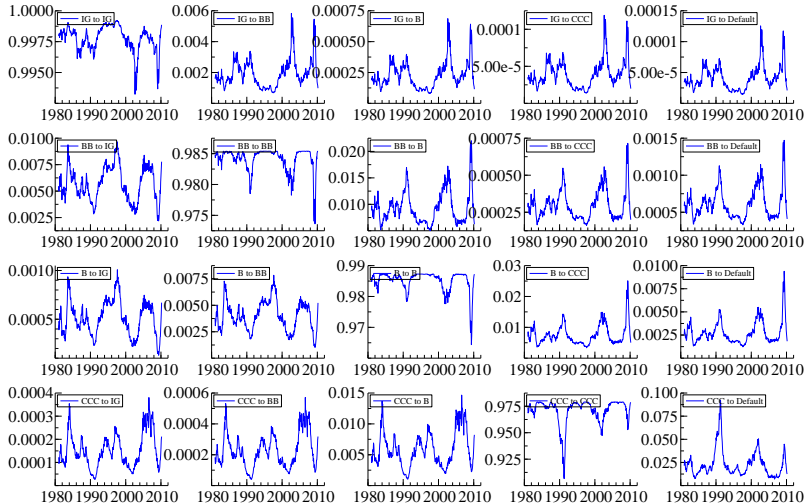
Credit rating and LGD loadings Z^c and Z^r

	macro ₁	macro ₂	macro ₃	frailty ₁	frailty ₂
Z^c					
IG	-0.052 (0.059)	0.202*** (0.055)	-0.123** (0.069)	1.475*** (0.371)	-1.165** (0.555)
BB	-0.078** (0.037)	0.172*** (0.037)	-0.102*** (0.040)	1.000 —	0.000 —
B	-0.184*** (0.035)	0.162*** (0.031)	-0.142*** (0.040)	0.970*** (0.156)	-0.016 (0.158)
CCC	-0.262*** (0.057)	0.073* (0.050)	-0.018 (0.075)	1.936*** (0.465)	1.000 —
Z^r	0.018 (0.049)	0.276*** (0.046)	-0.082* (0.062)	1.212*** (0.376)	1.065*** (0.301)

Estimated factors for the (3,2,0) model



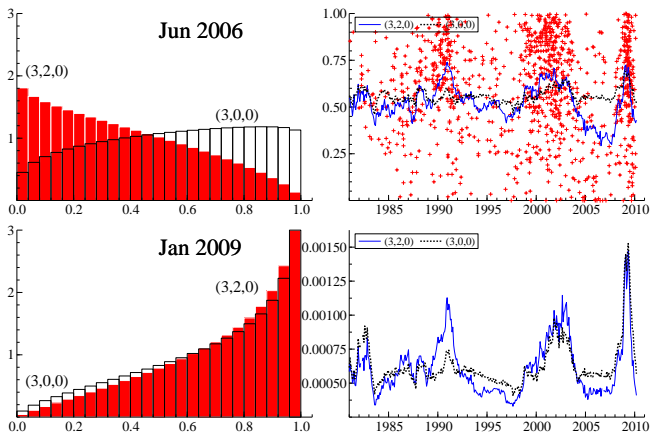
Time-varying transition probabilities



Simulating cumulative loss distributions

- ▶ Most financial institutions carry a large portfolio of credit related securities.
- ▶ Given a portfolio at time T , we can use the models to simulate different possible risk scenarios.
- ▶ GOAL: determine the amount of capital banks may need in the future.
- ▶ What happens if we do not include time-varying parameters f_t in the model?
- ▶ Scenario analysis:
 1. What happens if there is a negative shock to RGDP?
 2. What happens if there is an increase to credit spreads?

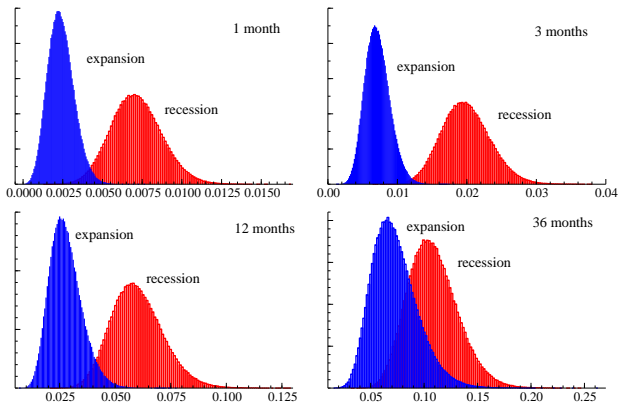
Loss given default results



Top and bottom left are loss distributions. Top right is a plot of the mean through time. Bottom right are transition probabilities from BB \rightarrow D.

Simulating cumulative loss distributions

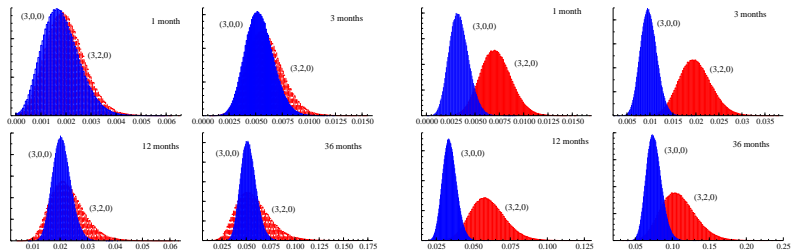
Cumulative losses on a portfolio of bonds at different horizons.



Comparison between a recession and expansion for (3,2,0) model.

Simulating cumulative loss distributions

Comparison of cumulative loss distributions with/without factors.



Left: starting at $f_T = 0$. Right: starting in a recession.

Conclusion and future work

- ▶ We introduce a new class of observation-driven models for mixed-measurement data which share exposure to common factors.
- ▶ Missing values and mixed frequencies are handled in a natural way.
- ▶ Using this approach, we develop new models for credit risk.
- ▶ The models can be used for simulating loss distributions, stress testing, and scenario analysis.
- ▶ Future work:
 - ▶ When computing loss distributions, current models do not account for changes in market prices of bonds or loans.
 - ▶ Current models depend on industry credit ratings by Moody's, Fitch, Standard & Poors.
 - ▶ Potential to use alternative sources of data.