Fast MVA Calculations with Algorithmic Differentiation

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Outline

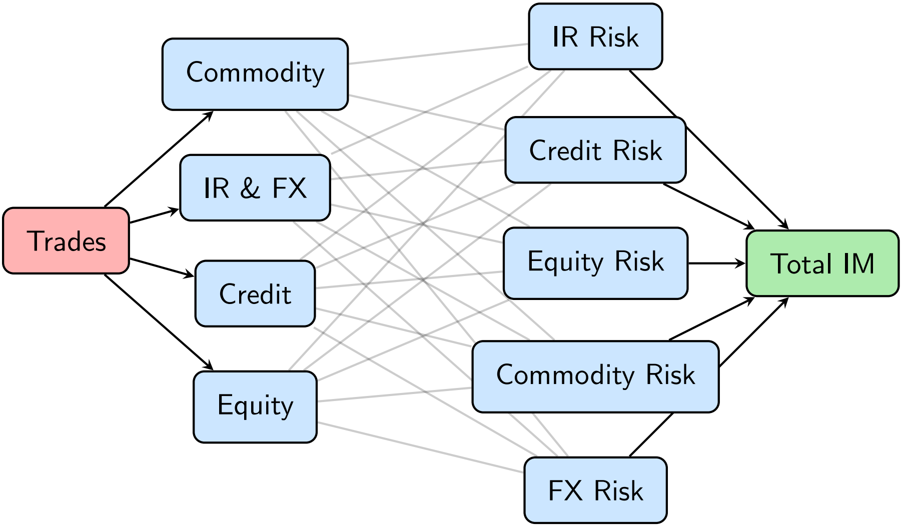
Initial Margin

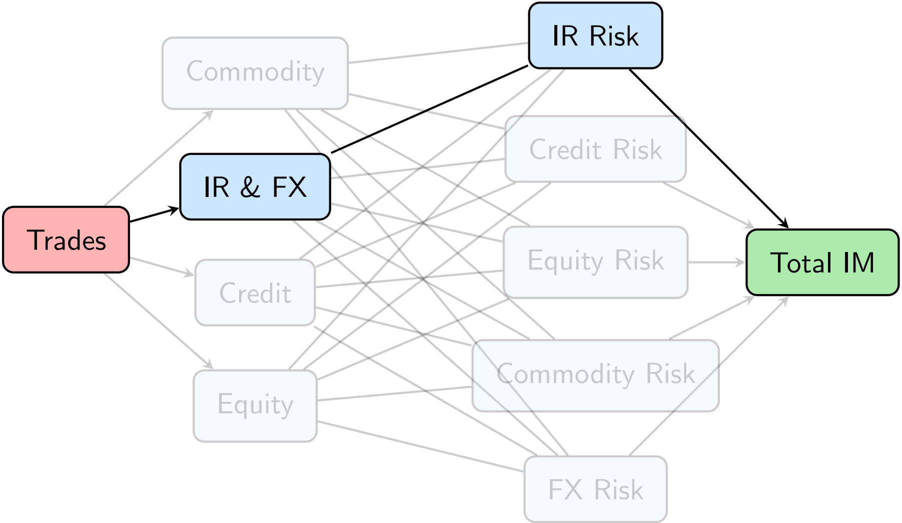
- BCBS and IOSCO regulation on initial margin
- Non-centrally cleared derivatives
- PFE of 99% VaR over 10-day margin period of risk

SIMM

- **S**tandard **I**nitial **M**argin **M**odel
- Proposed by ISDA
- Easy to implement
- Non-procyclical

- First-order delta and vega sensitivities **w.r.t market quantities**
- 6 Risk classes and 4 product classes
- Risk weights and correlations provided by ISDA
- IM corresponds to 99% 10-day VaR


Trades
Commodity
IR & FX
Credit
Equity
IR Risk
Credit Risk
Equity Risk
Commodity Risk
FX Risk
Total IM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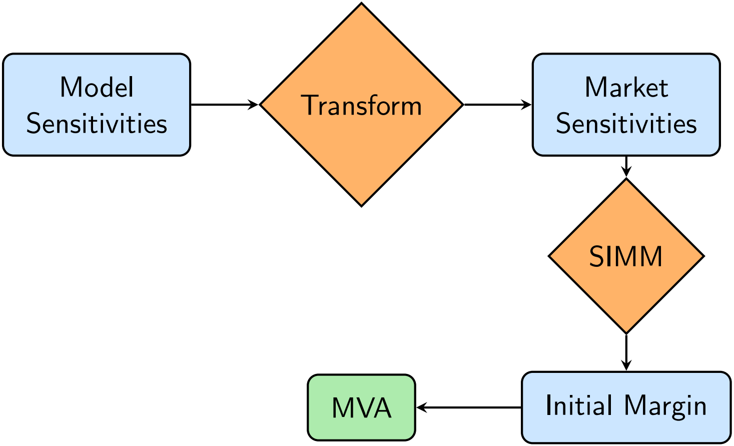
MVA

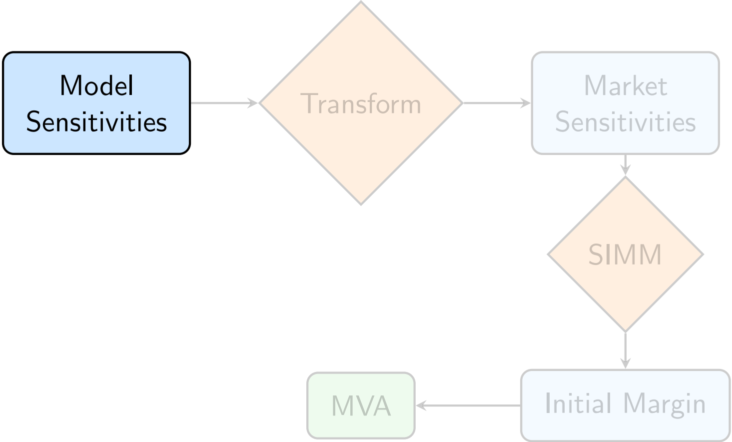
- **M**argin **V**aluation **A**djustment
- Initial margin cannot be rehypothecated
- Initial margin must be funded over the lifetime of a trade
- Charge counterparty for these funding costs

- $MVA = Expectation\left[IM\right]$
- $IM =$ sum of initial margin over the lifetime of a trade
    - Discounted and weighted by funding costs

Problem: Involves simulation of future initial margin (sensitivities)

Outline


Model Sensitivities
Transform
Market Sensitivities
SIMM
Initial Margin
MVA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Computing Sensitivities

- Numerical differentiation (bump-and-revalue)
  - $f'(x) \approx \frac{f(x+h)-f(x)}{h}$
  - Inefficient and inaccurate
- Symbolic differentiation
  - Inefficient but exact
- **Algorithmic differentiation**
  - Efficient and exact

Algorithmic Differentiation

- Decompose a function into a sequence of basic operations
  - Binary: $+, -, /, \ldots$ and unary: $\log, \exp, \ldots$
- $\omega_i = BasicOperation(\omega_j)$
- $\omega_1, \ldots, \omega_n$ $, \omega_{n+1}, \ldots, \omega_{l-m}, \ldots,$ $\omega_{l-m+1}, \ldots, \omega_l$
- Apply chain rule of differentiation
  - Forward mode
  - **Backward/Adjoint mode (AAD)**

AAD Algorithm

- Forward sweep
  1. Execute intermediate operations $\omega_i = BasicOperation(\omega_j)$
  2. Store instructions on **tape**
- Backward sweep
  1. Accumulate adjoint variables

Adjoint Operation

$$\bar{\omega}_j = \sum_{j \prec i} \bar{\omega}_i \frac{\partial BasicOperation_i(u_i)}{\partial \omega_j}$$

$$u_i = (\omega_j)_{j \prec i}$$

Computing Swap Greeks

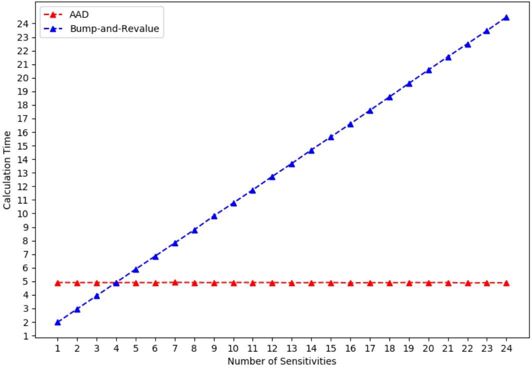


Figure: Performance of AAD and bump-and-revalue

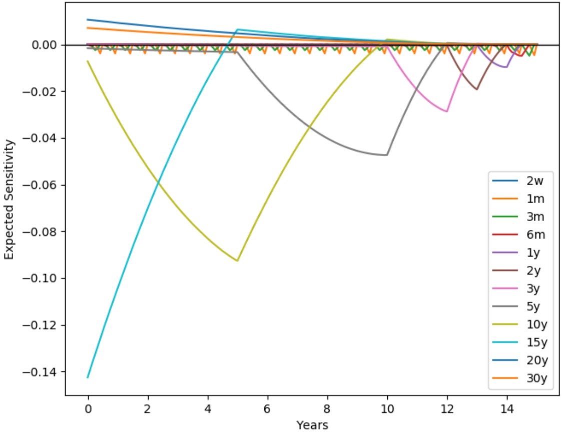
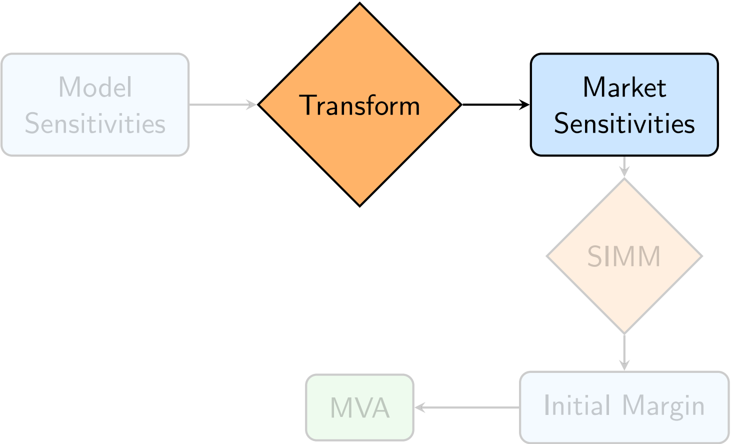


Figure: Expected sensitivity of receiver swap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Sensitivity Transformation

$$\frac{\partial V}{\partial R_{swap}} = \frac{\partial V}{\partial R_{zero}} \frac{\partial R_{zero}}{\partial R_{swap}}$$

- $\partial R_{zero} / \partial R_{swap}$
  - Constant
  - Piecewise constant
  - Interpolate
- $\partial V / \partial R_{swap}$
  - Melt linearly
  - Interpolate
  - Approximate with machine learning

Machine Learning for Swap Greeks

$$\widehat{sensitivity} = f\left(T - t, T, d_{payer}, V_t\right)$$

1. Simulate training set
2. Train/Fit $f(.)$ to training set
3. Generate explanatory variables
4. Predict sensitivities

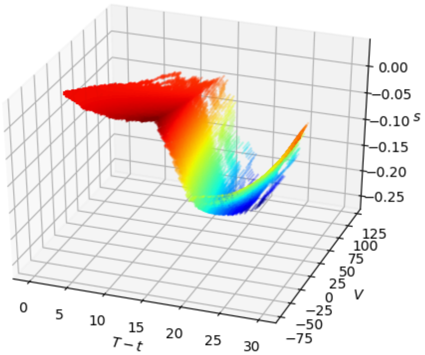
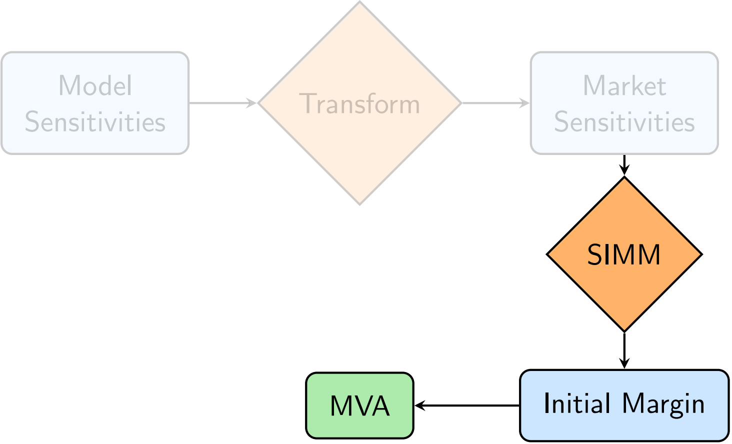


Figure: Training set for sensitivity w.r.t 20-year Euribor-based rate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Computing Initial Margin

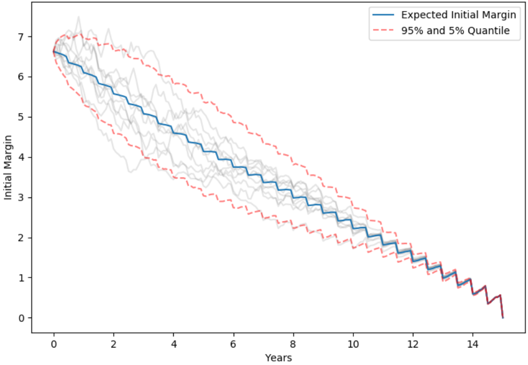

Figure: Expected initial margin profile for an IR swap

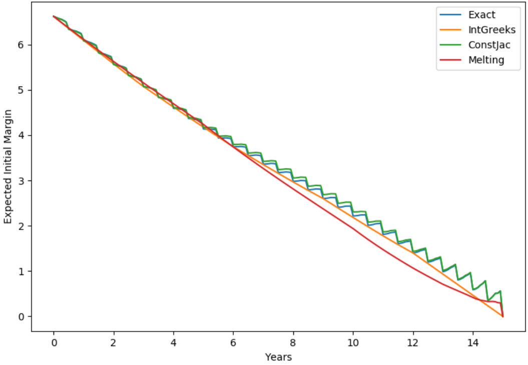

Figure: Different expected initial margin profiles

Outline

Simulation Setup

- Consider a single 15-year interest rate swap
  - Double-curve framework
- 2,000 simulation paths
- Monthly time spacing
- Funding spread of 20 bps

MVA

- MVA: 10.16 bps of swap notional

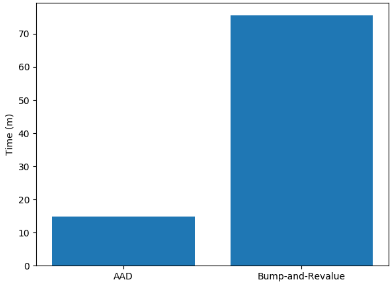

Figure: Performance of MVA Calculation

MVA

Method	Time (m)	MVA (bps)
Exact	251.57	10.16
ConstJac	14.92	10.28
PCJac	21.39	10.15
IntJac	20.00	10.16
IntGreeks	9.86	9.78
Melting	3.49	9.56
ML	2.77	10.13

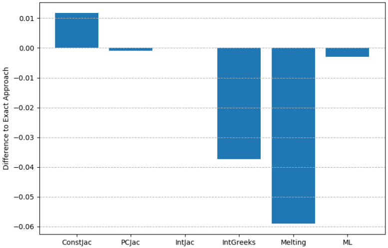

Figure: Results for Different Simulation Methods

Concluding Remarks

- AAD is superior to bump-and-revalue
- Advantages of approximating sensitivity transformation
  - Reasonably accurate
- Approximate market sensitivities with machine learning
  - Training set generation is crucial